9 Limit Order Markets

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What are Limit Order Markets?

Market microstructure

- Markets exist because individuals benefit from voluntary exchange.
- Competitive equilibrium is a useful modeling tool to parsimoniously capture the fundamentals of trade.
- But can we replace the fiction of a Walrasian auctioneer setting prices with models based on:
 - institutions, or trading rules, designed to facilitate trade
 - where behavior can be modeled as a noncooperative game.
- Focusing on one such institution, three questions frame this lecture:
- What is a limit order market (LOM)?
- ② Do LOM models have empirical content?
 - Can LOM models be tested (falsified)?
 - Note that empirical content does not imply identification.
- 4 How efficient are LOMs in allocating resources?

The order book

- The trading mechanism for a given security in a generic limit order market can be described by:
 - 1 the order book.
 - 2 the rules and procedures for submitting and withdrawing orders.
- At any given instant during business hours, there is:
 - 1 a list of unfilled orders to buy the security
 - another list of unfilled orders to sell the security
- Each limit order on each list consists of:
 - a price
 - a quantity
 - a submission time
- Every order on the sell list is marked with a higher price than every order on the buy list.
- The difference between the lowest unfilled sell order (the ask) and the highest unfilled buy order (the bid) is called the spread.

Orders

- An investor seeking to trade the security in this market can:
 - add to one of the lists by placing a buy (sell) order, which is lower than the offer (higher than the bid). This is called making a limit buy (sell) order.
 - execute a trade by accepting the ask (bid) on the other side of the market. This is called a market buy (sell) order.
- If two unfilled orders have the same price, then the order submitted earlier is executed first.
- Investors wishing to execute only a proportion of another investor's unfilled limit order with their own market order may do so.
- Investors wishing to withdraw their limit orders may so at any before a market order cancels them with a transaction.
- Summarizing limit order markets exhibit *price/time precedence*.

Trading window

Sell	Price: 5800.00		Quantity:	9	Duration:	60000
	Price(Markup)	Qua	ntity (Cum.)	Revenue	\Cost (Cum.)	Duration
Buy	6000.00 (-291.24)	4 (4)		24000.00 (24	000.00)	59603
	5800.00 (-491.24)	9 (13)		52200.00 (78	200.00)	59634
Delete	3800.00 (2491.24)	2 (2)		7600.00 (760	0.00)	59301
Center	200.00 (6091.24)	4 (6)		800.00 (8400	1.00)	59197
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Data on limit order markets

- A limit order market (LOM) for financial securities offers an excellent laboratory analyzing trading mechanisms where there are many players on both sides of the market:
 - The rules governing trading in limit order markets are transparent, and therefore easy to capture with a model (compared to labor markets and transactions in industrial organization).
 - 2 Different units of the securities are perfect substitutes and therefore comparable (in contrast to many real assets).
 - The volume and value of traded securities is huge, inducing traders to perform as well as they can (unlike experimental settings).
 - Reliable data can be obtained from several limit order exchanges because they form part of the contract to which parties agree on both sides (relative to say survey data or information small businesses provide to the government for taxation purposes).
- One deficiency: researchers cannot (usually) observe the traders' identity to track their orders or observe their wealth portfolio.

An LOM Model (Hollifield, Miller and Sandas, 2004)

Valuation

- At time $t \in \{1, 2, ...\}$ just one trader has his only opportunity to submit an order for one (or more generally exogenously determined) unit(s) of an asset.
- Trader t is risk neutral and values the unit at:

$$v_t = u_t + y_t$$

where:

- u_t is independent and identically distributed with support on the real line and probability distribution function G(u).
- y_t is a Martingale, meaning $E_t[y_{t+1}] = y_t$.
- We interpret y_t as the expected liquidation value of an asset that pays no dividends in the meantime.
- Trader t observes both components.



Prices

- Traders buy and sell on a discrete price grid $\{\ldots, p_{j-1}, p_j, p_{j+1}, \ldots\}$.
- The difference $p_{j+1} p_j$ is called the tick size.
- ullet Denote by $\left\{p_{0t}^{(b)}, p_{1t}^{(b)} \ldots
 ight\}$ the buy prices trader t can choose from:
 - $p_{0t}^{(b)}$ is the lowest limit order sell offer (the ask price).
 - Trader t submits a market buy order by selecting $p_{0t}^{(b)}$.
 - $p_{kt}^{(b)}$ is k ticks below $p_{0t}^{(b)}$.
 - Trader t submits a limit buy order by selecting $p \in \left\{p_{1t}^{(b)}, p_{2t}^{(b)} \ldots \right\}$.
- ullet Similarly $\left\{p_{0t}^{(s)},p_{1t}^{(s)}\ldots
 ight\}$ are sell prices, and trader t can submit a:
 - market sell order by selecting the (highest limit order) bid price $p_{0t}^{(s)}$
 - limit sell order $p_{kt}^{(s)}$ that is k ticks above $p_{0t}^{(s)}$.
- The difference $p_{0t}^{(b)} p_{0t}^{(s)}$ is called the spread (ask price less bid price).
- Prices on and inside the spread can be selected by a buyer or seller.

Choices, cancellations and executions

- Let $d_{kt}^{(b)} \in \{0,1\}$ and $d_{kt}^{(s)} \in \{0,1\}$ for $k \in \{0,1,2,\ldots\}$, where $d_{kt}^{(b)} = 1$ means t submits a buy order at price $p_{kt}^{(b)}$.
- Assume t submits at most one order, implying:

$$\sum_{k=0}^{\infty} \left(d_{kt}^{(b)} + d_{kt}^{(s)} \right) \le 1$$

- Suppose $d_{\iota \star}^{(s)} = 1$ and let:
 - $r_{k,t,t+ au}^{(s)}=1$ if the order is cancelled t+ au (otherwise $r_{k,t,t+ au}^{(s)}=0$). $q_{k,t,t+ au}^{(s)}=1$ if the order is filled at t+ au (otherwise $q_{k,t,t+ au}^{(s)}=0$).

 - $r_{\nu_{++++}}^{(b)}$ and $q_{\nu_{++++}}^{(b)}$ be similarly defined.

Optimization

- Assume $r_{k \ t \ t+\tau}^{(s)}$ and $r_{k.t.t+\tau}^{(b)}$ are independent exogenous processes.
- Execution is endogenous (because another trader is involved):

 - $q_{0,t,t}^{(s)}=1$ because market orders execute immediately.
 If $q_{k.t.t+\tau}^{(s)}=1$ for some $\tau>0$ then $d_{0,t+\tau}^{(b)}=1$. (Every trade fills a limit order with a market order.)
 - Price precedence implies if $q_{k,t,t+\tau}^{(s)'}=1$ and $d_{k't'}^{(s)}=1$ for some k' < k and $t' < t+\tau$ then $q_{k',t',\rho}^{(s)}=1$ or $r_{k',t',\rho}^{(s)}=1$ for some $\rho \leq t+\tau$.
- Trader t chooses $d_t \equiv \left(d_{0t}^{(b)}, d_{0t}^{(s)}, d_{1t}^{(b)}, d_{1t}^{(s)}, \ldots\right)$ to maximize:

$$E_{t} \left\{ \sum_{k=0}^{\infty} \sum_{\tau=0}^{\infty} \left[\begin{array}{c} d_{kt}^{(b)} \prod_{\rho=0}^{\tau} \left(1 - r_{kt,t+\rho}^{(b)}\right) q_{kt,t+\tau}^{(b)} \left(v_{t+\tau} - p_{kt}^{(b)}\right) \\ + d_{kt}^{(s)} \prod_{\rho=0}^{\tau} \left(1 - r_{kt,t+\rho}^{(s)}\right) q_{kt,t+\tau}^{(s)} \left(p_{kt}^{(s)} - v_{t+\tau}\right) \end{array} \right] \right\}$$

Equilibrium

Existence and uniqueness

- This is (isomorphic to) a perfect information game:
 - Each trader t observes the value of yt and all the outstanding limit orders (comprising the limit order book).
 - Traders move sequentially each trader is fully informed about the moves of previous agents.
- If the game has a finite horizon, then it is straightforward to establish that (generically) a unique equilibrium exists.
- Let $\widehat{d}_{kt}^{(b)}$ and $\widehat{d}_{kt}^{(s)}$ denote equilibrium $d_{kt}^{(b)}$ and $d_{kt}^{(s)}$ choices.
- Similarly let $\widehat{q}_{kt}^{(b)}$ and $\widehat{q}_{kt}^{(s)}$ denote equilibrium $q_{kt}^{(b)}$ and $q_{kt}^{(s)}$ executions.

Equilibrium

Conditional choice probabilities, execution probabilities and picking off risks

- To characterize the equilibrium choices, define:
 - conditional choice probabilities of submission:

$$\lambda_{kt}^{(b)} \equiv \int \widehat{d}_{kt}^{(b)} dG(u)$$

execution probabilities:

$$\psi_{kt}^{(b)} \equiv E_t \left[\sum_{\tau=0}^{\infty} \widehat{q}_{kt,t+\tau}^{(b)} \prod_{\rho=1}^{\tau} \left(1 - r_{kt,t+\rho}^{(b)} \right) \right]$$

• picking-off risk:

$$\zeta_{kt}^{(b)} \equiv E_t \left[\sum_{k=0}^{\infty} (y_{t+\tau} - y_t) \, \widehat{q}_{kt,t+\tau}^{(s)} \right]$$

• Thus trader t chooses d_t to maximize:

$$\sum_{k=0}^{\infty} \left\{ d_{kt}^{(b)} \left[\psi_{kt}^{(b)} \left(v_t - \rho_{kt}^{(b)} \right) + \xi_{kt}^{(b)} \right] + d_{kt}^{(s)} \left[\psi_{kt}^{(s)} \left(\rho_{kt}^{(s)} - v_t \right) - \xi_{kt}^{(s)} \right] \right\}$$

A revealed preference argument (Lemma 1, HMS 2004)

- Suppose $v_t = u + y_t$ and $v'_t = u' + y_t$.
- ullet If $d_{kt}^{(b)}\left(u
 ight)=d_{k't}^{(b)}\left(u'
 ight)=1$, then one can show:

$$\left[\psi_{kt}^{(b)} - \psi_{k't}^{(b)}\right]\left(v_t - v_t'\right) = \left[\psi_{kt}^{(b)} - \psi_{k't}^{(b)}\right]\left(u - u'\right) \ge 0$$

• Define $\theta_t^{(b)}(k, k')$ as the *threshold valuation* of a trader indifferent between submitting $p_{kt}^{(b)}$ versus $p_{k't}^{(b)}$:

$$heta_{t}^{(b)}\left(k,k'
ight) = p_{kt}^{(b)} + rac{\left[p_{kt}^{(b)} - p_{k't}^{(b)}
ight]\psi_{k't}^{(b)} + \xi_{k't}^{(b)} - \xi_{kt}^{(b)}}{\psi_{k't}^{(b)} - \psi_{kt}^{(b)}}$$

- Similar expressions can be defined for traders indifferent between:
 - selling at two different prices
 - buying a unit versus selling a unit (at a higher price)
 - trading at some price versus not trading at all.



Monotonicity of threshold valuations (Lemmas 2 and 3, HMS 2004)

• The revealed preference argument implies:

$$\theta_t^{(b)}(k, k+1) > \theta_t^{(b)}(k+1, k+2)$$

An analogous argument applies to the sell side:

$$\theta_t^{(s)}(k, k+1) < \theta_t^{(s)}(k+1, k+2)$$

• Using similar reasoning we can show:

$$\theta_t^{(b)}\left(k, k+1\right) > \theta_t^{(s)}\left(k', k'+1\right)$$

• These inequalities characterize the equilibrium submission strategy.

Empirical content of revealed preference

- Consider a market where:
 - The tick size is one unit: $p_{L+}^{(b)} p_{0+}^{(b)} = k$.
 - The market buy price is one hundred: $p_{0+}^{(b)}=100$.
 - There is no common component: $y_t \equiv 0$.
 - Traders submit orders at $p_{0_1}^{(b)}$, $p_{1_+}^{(b)}$ and $p_{2_+}^{(b)}$.
 - By definition $\psi_{0t}^{(b)} = 1$.
 - Also assume $\psi_{1+}^{(b)}=0.7$ and $\psi_{2+}^{(b)}=0.6$.
- Using the formula for calculating threshold valuations:

$$\theta_t^{(b)}(0,1) = 100 + 0.7/(1 - 0.7) = 102.33$$

 $\theta_t^{(b)}(1,2) = 99 + 0.6/(0.7 - 0.6) = 105.00$

• Since $\theta_t^{(b)}(1,2) > \theta_t^{(b)}(0,1)$ the monotonicity condition is violated.

Testing strategy

Given a typical inequality implied by the model, say:

$$\theta_{t}^{(b)}(k, k+1) - \theta_{t}^{(b)}(k+1, k+2) > 0$$

for any z_t in the information set of trader t:

$$E\left[\theta_{t}^{(b)}\left(k,k+1\right)-\theta_{t}^{(b)}\left(k+1,k+2\right)|z_{t}\right]>0$$

which implies:

$$E\left\{ \left[\theta_{t}^{(b)}\left(k,k+1\right)-\theta_{t}^{(b)}\left(k+1,k+2\right)\right]\left|z_{t}\right|\right\} > 0$$

• The test statistic is based on:

$$T^{-1} \sum_{t=1}^{T} \left\{ \left[\widetilde{\theta}_{t}^{(b)}(k, k+1) - \widetilde{\theta}_{t}^{(b)}(k+1, k+2) - LB \right] |z_{t}| \right\}$$

where:

- $oldsymbol{\widetilde{ heta}}_t^{(b)}(k,k+1)$ is a consistent estimator for $heta_t^{(b)}(k,k+1)$
- and $0 < LB \le \theta_t^{(b)}(k,k+1) \theta_t^{(b)}(k+1,k+2)$ for all (t,k,z_t) .

Overview of test procedure

- To implement the test we must first identify the
 - $\textbf{ § find subsequences of conditional choice submission probabilities } \left\{\lambda_{jt}^{(b)}\right\}_{j} \text{ and } \left\{\lambda_{jt}^{(s)}\right\}_{j} \text{ that are strictly positive }$
 - ② estimate the execution probabilities $\psi_{kt}^{(b)}$ and $\psi_{kt}^{(s)}$ for the elements in the subsequence
 - \odot estimate the y_t process
 - lacktriangledown estimate the picking-off risk $\xi_{kt}^{(b)}$ and $\xi_{kt}^{(s)}$
 - **5** form the threshold values $\theta_t^{(b)}(k, k')$ and $\theta_t^{(s)}(k, k')$.
 - **1** test the inequalities that apply to $\theta_t^{(b)}(k, k')$ and $\theta_t^{(s)}(k, k')$.
- Note the null hypothesis is a joint hypothesis combining all six steps.
- Therefore the *size* of the test (the probability of being in the tail of a test statistic) is affected by all the sources of sampling variation.

Notes on implementation in HMS (2004)

- The sample comprises data on Ericsson taken from the Stockholm Automated Exchange system in 1991- 92.
- $\bullet \text{ We focus on } \left\{ p_{0t}^{(b)}, p_{1t}^{(b)}, p_{2t}^{(b)}, p_{3t}^{(b)} \right\} \text{ and } \left\{ p_{0t}^{(s)}, p_{1t}^{(s)}, p_{2t}^{(s)}, p_{3t}^{(b)} \right\}.$
- We conducted the tests of strictly positive submission probabilities, strictly positive differences in execution probabilities, and monotonicity in threshold valuations separately.
- Consequently the critical values for the tests aren't adjusted properly for sampling error in prior stages.
- We cannot reject the (separately tested) hypotheses that for probabilities of:

submission
$$\lambda_{jt}^{(b)}>0$$
 and $\lambda_{kt}^{(b)}>0$ for $j\in\{0,1,2,3\}$ execution $\psi_{jt}^{(b)}>\psi_{j+1,t}^{(b)}$ and $\psi_{jt}^{(s)}>\psi_{j+1,t}^{(s)}$ for $j\in\{0,1,2\}$

Empirical Content of LOM models

Where does the model succeed?

• For several components of $|z_t|$ the estimated differences:

$$E\left\{ \left[\theta_{t}^{\left(b\right)}\left(j,j+1\right)-\theta_{t}^{\left(b\right)}\left(j+1,j+2\right)\right]\left|z_{t}\right|\right\}$$

are positive and significant, as the model predicts.

On the sell side:

$$E\left\{ \left[\theta_{t}^{(s)}\left(1,2\right)-\theta_{t}^{(s)}\left(0,1\right)\right]\left|z_{t}\right|\right\}$$

is positive and significant, but:

$$E\left\{ \left[\theta_{t}^{(s)}\left(2,3\right)-\theta_{t}^{(s)}\left(1,2\right)\right]\left|z_{t}\right|\right\}$$

is positive but not significant.

• Summarizing, the null hypothesis of monotonicity is not rejected when buy and sell thresholds are considered separately.

Test Results

Where does the model fail?

- Contrary to the predictions of the model, the sample:
 - gives negative point estimates of:

$$E\left\{ \left[\theta_{t}^{\left(b\right)}\left(2,3\right)-\theta_{t}^{\left(s\right)}\left(2,3\right)\right]\left|z_{t}\right|\right\}$$

- rejects the null hypothesis that buyer threshold valuations are higher than seller threshold valuations.
- Thus rejections only occur for investors who are almost indifferent between placing a high limit sell order versus a low limit buy order.
- According to our parameter estimates:
 - investors placing high sell limit orders should place low buy limit orders instead.
 - ② investors placing low buy limit orders should place high limit sell orders instead.
- Note that transaction probabiliites are low for these groups. Could a model of rational innattention explain this feature of the data?

Estimating an LOM model

Adapting the model to continuous time

- We now assume traders arrive sequentially at rate:
 - $\Pr{\text{Trader arrives in interval }[t, t + \Delta t] | x_t} = \lambda(t; x_t) dt$
 - ullet where x_t is an exogenous vector of state variables
- Following the same notation as before:
 - $d_{kt}^{(b)} \in \{0,1\}$ and $d_{kt}^{(s)} \in \{0,1\}$ for $k \in \{0,1,2,\ldots\}$
 - with the same constraint $\sum_{k=0}^{\infty} \left(d_{kt}^{(b)} + d_{kt}^{(s)} \right) \leq 1$.
- As above we assume traders are risk neutral with valuations
 - differ in their private valuation $v_t = y_t + u_t$
 - ullet where u_t is distributed independently with $\Pr\left(u_t \leq u \,| x_t\,
 ight) \equiv G\left(u \,| x_t\,
 ight)$
- If and when he has the opportunity the trader:
 - can submit an order to trade one unit
 - \bullet pays c_0 to placing an order
 - pays a further ce if the order executes.



Estimating an LOM model

An estimation strategy

- Note that G(u|x) must be estimated to obtain estimates of the gains from trade $L(z_t)$.
- One strategy is to:
 - 1 Follow the same procedure as above to
 - \bullet determine orders with positive submission probabilities $\lambda_{jtj}^{(b)}$ and $\lambda_{jt}^{(s)}$
 - Then estimate
 - ullet their execution probabilities $\psi_{kt}^{(b)}$ and $\psi_{kt}^{(s)}$ (nonparametrically)
 - \bullet the y_t process
 - the picking-off risks $\xi_{kt}^{(b)}$ and $\xi_{kt}^{(s)}$
 - Apply a competing risk hazard framework to jointly estimate:
 - the arrival rate of traders $\lambda(t; x_t)$
 - and G(u|x), the distribution of their valuations.

Estimating an LOM model

Estimating the arrival of treaders and the distribution of private valuations

- Briefly, we partition each time interval [t, t + dt] by each possible event, and estimate the probability of its occurrence:
- For example a crude partition of events is that in [t, t + dt):
 - A market buy order arrives:

$$\Pr\left\{\widehat{d}_{0t}^{(b)}=1 \text{ in } \left[t,t+dt\right)|z_{t}\right.\right\}=\left\{1-G\left[\theta_{t}^{(b)}\left(0,1\right)|x\right]\right\}\lambda\left(t;x_{t}\right)dt$$

There is a market sell order:

$$\Pr\left\{\widehat{d}_{0t}^{(s)} = 1 \text{ in } [t, t + dt) | z_t \right\} = G\left[\theta_t^{(s)}(0, 1) | x\right] \lambda(t; x_t) dt$$

Either a limit order arrives or there is no order:

$$\begin{aligned} &\operatorname{Pr}\left\{\widehat{d}_{0t}^{(b)}+\widehat{d}_{0t}^{(s)}=0 \text{ in } \left[t,t+dt\right)|z_{t}\right.\right\} \\ &= &\left.1-\lambda\left(t;x_{t}\right)dt\right. \\ &\left.+\left\{G\left[\theta_{t}^{(b)}\left(0,1\right)|x\right]-G\left[\theta_{t}^{(b)}\left(0,1\right)|x\right]\right\}\lambda\left(t;x_{t}\right)dt \end{aligned}$$

How Efficient is an LOM?

Equilibrium gains from trade from behind the Rawlsian (1971) veil of ignorance

- The gains from trade do not depend on the transaction price or the picking off risk, which are transfers between buyer and seller.
- When the buyer places a limit order at t and the seller places a market order at $t + \tau$ cancelling the buy order, the gains from trade are:

$$u_{t+\tau}-u_t-2\left(c_0+c_e\right)$$

More generally, the expected gains from a new trader arriving at t are:

$$V = E \left\{ \begin{array}{l} \sum_{k=0}^{\infty} \widehat{d}_{kt}^{(b)}\left(u_{t}, z_{t}\right) \left[\psi_{kt}^{(b)}\left(z_{t}\right)\left(u_{t} - c_{e}\right) - c_{0}\right] \\ -\sum_{k=0}^{\infty} \widehat{d}_{kt}^{(s)}\left(u_{t}, z_{t}\right) \left[\psi_{kt}^{(s)}\left(z_{t}\right)\left(c_{e} + u_{t}\right) + c_{0}\right] \end{array} \right\}$$

How Efficient is an LOM?

Maximal gains from exchange from behind the Rawlsian (1971) veil of ignorance

- We compare the expected gains from trade in an LOM with the potential gains from exchange, obtained by choosing between:
 - immediately executing a new order
 - or placing the order in inventory
 - where orders in the inventory are subjected to cancellation risk
 - to maximize the expected gains from exchange.
- We can categorize the reasons why limit order markets do not realize all the potential gains from exchange.
 - 1 Limit orders are not executed when they should be.
 - Traders do not submit orders when they should.
 - Trader submits a "wrong sided" order that executes.
 - Traders submit orders when they should not.
- Our estimates possibly understate the efficiency of LOMs, because we did consider coordination problems between investors arriving at the market at different times when computing the maximal gains.

The Vancouver Stock Exchange (VSE)

Structural estimates from Hollifield, Miller, Sandas and Slive (2006)

	вно	ERR	WEM
		Gains	
	Maximum gains as a	a % of the common value	
	9.07	8.61	6.75
		% of the common value	
ower bound	7.88	8.09	6.08
Jpper bound	8.45	8.31	6.40
verage	8.16	8.20	6.24
		minus current gains	
ower bound	0.62	0.30	0.35
Jpper bound	1.20	0.52	0.67
verage	0.91	0.41	0.51
		a % of maximum gains	
ower bound	86.79	93.97	90.07
Jpper bound	93.13	96.57	94.81
verage	89.96	95.27	92.44
		ition of Losses	
	No execution a	s a % of total losses	
ell side	32.32	31.20	33.05
Buy side	40.10	39.01	41.85
ubtotal	72.42	70.21	74.90
		as a % of total losses	
ell side	2.24	0.62	0.41
Buy side	1.98	0.15	0.71
Subtotal	4.22	0.77	1.12
		as a % of total losses	
ell side	0.86	0.02	0.39
Buy side	0.20	0.05	0.63
ubtotal	1.06	0.07	1.02
		ssions as a % of total losses	
ell side	9.81	11.87	10.30
luy side	12.49	17.07	12.66
ubtotal	22.30	28.94	22.96
otal	100.00	100.00	100.00
		poly Gains	
	Monopoly gains as a		
	5.02	5.57	4.18
	Monopoly gains as		
	55.31	61.87	
	162.65	147.23	149.41
	Current gains as: 162.65	a % of monopoly gains 147.23	N / E 1

The Vancouver Stock Exchange

Received history

- A brief history of the VSE:
 - Incorporated 1906, and fully automated in 1990.
 - Trading increased from C\$4 billion in 1991 to \$6.7 billion in 1993.
 - Merged with Canadian Venture Exchange (CDNX) in 1999.
 - Subsequently absorbed into the Toronto Stock Exchange (TSE).
- VSE had an unsavory reputation reminiscent of the wild west:
 - In 1989, Forbes magazine christened it "scam capital of the world".
 - A 1994 report refers to "shams, swindles and market manipulations".
 - The summary judgement of Investopedia.com is that "the VSE is an example of one of the world's less successful stock exchanges."
- The historical narrative of VSE is puzzling:
 - Our analysis paints a glowing picture of capitalism at work.
 - Why did the trading volume grow substantially after automation?
 - Several European exchanges merged when the VSE was absorbed:
 - Is this evidence of unsuccessful exchanges?
 - Was this driven by the electronic exchange technology?