6 Procurement Contracts

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Institutional Background and Data

Motivation for study

- **More than 10%** of US federal government spending is on procurment.
- In FY 2010 $241 billion or 45% were payments for contracts attracting *a* **single bid**.
- Two important institutional features attracting attention:
    1. A procurement agency (*a buyer*) chooses the extent to which a contract will draw competitive bids: 51% or 1.2 million contracts were awarded without full and open competition in FY 2010.
    2. The final contract price can differ from, and is often **much larger than**, the initially agreed upon price.
- The regulations give the buyer **considerable discretion** in determining contract terms, as well as the extent of competition.

Institutional Background and Data

Data sources and variables

- Data from US government for contracts initiated in FY 2004–2015.
- For each procurement contract, we observe:
  1. solicitation procedure
  2. number of bids
  3. award type (e.g. definitive contracts, purchase orders, delivery orders)
  4. contract pricing type (e.g. firm-fixed-price, cost-plus)
  5. history of price and duration changes
  6. product and service code
  7. commercial availability
  8. contracting agency (e.g. Department of Defense)
  9. identity and attributes of winning contractor, and location of contract
- We augment this with data on:
  1. contracting agencies (from federal human resources data base)
  2. number of establishments by industry (from County Business Patterns).

Institutional Background and Data

Focus of study

- We analyze definitive contracts and purchase orders in information technology (IT) and telecommunications:
  - **Products** include computer hardware, software, and telecommunications equipment.
  - **Services** include IT strategy, architecture, programming, cyber security, Internet service.
- We further restrict our attention to the contracts that satisfy:
  1. The *base maximal contract price* below US 2010 $1 million.
  2. The *base contract price* at least $150,000 in nominal dollars. The Federal Acquisition Regulations (FAR) require the contracts with an anticipated value below $150,000 (and above $3,500) to be set aside for small business concerns.
  3. The *base duration* at least 30 days but no longer than 400 days.
  4. The *final contract end date* before the end of FY 2017.
  5. Procured items produced or the services performed is in the US.
- This yields 17,123 contracts costing US 2010 $6.2 billion in total.

Institutional Background and Data

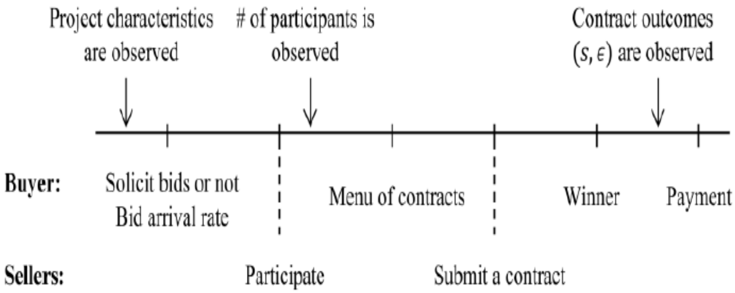
Table 1. Competition for IT contracts (FY 2004 - 2015)

	Obs.	Final Price ($K)		Number of Bids		
		Mean	SD	Mean	Median	Fraction One Bid
Panel A: Competed or not						
Full and open competition	5,030	350.00	234.94	3.02	2	0.35
Set-aside for small business	2,534	343.04	232.24	4.11	3	0.27
No competition by regulation	3,376	423.60	293.81	1.03	1	0.99
No competition by discretion	6,183	359.37	228.49	1.00	1	1.00
Panel B: Solicitation procedures						
Negotiated proposal/quote	4,395	366.63	248.31	2.89	2	0.45
Simplified acquisition	5,964	344.70	229.29	2.49	1	0.58
Other procedures†	143	365.05	228.07	3.42	2	0.43
No solicitation	6,067	386.47	252.77	1.03	1	0.99
Not specified	554	393.12	322.07	1.82	1	0.80

Notes: This table provides summary statistics of all contracts with definitive terms and conditions for IT and telecommunications products or service that initiated during FY 2004-2015 and satisfy the six sample selection conditions as described in Section 2.1. *Final price* refers to the total amount of obligated money to the government per contract as of FY 2018, in 2010 dollars. † Architect-engineer, basic research, and (two-step) sealed bids.

Model

Figure 1: Timeline of procurement process



Model

Assumptions about seller costs and private information

- There are two seller types $k \in \{0, 1\}$.
- The proportion of type $k = 1$ sellers in the population is $\pi \in (0, 1)$.
- The expected total cost to a type k seller of completing the project is:

$$c_k \equiv \gamma_k + \int c(\mathbf{s}) f_k(\mathbf{s}) d\mathbf{s}$$

where type 1 sellers are low cost, meaning:

$$\gamma_1 < \gamma_0 \text{ and } \int c(\mathbf{s}) f_1(\mathbf{s}) d\mathbf{s} < \int c(\mathbf{s}) f_0(\mathbf{s}) d\mathbf{s}$$

and:

  - γ_k is *hidden information* known only to the seller
  - $\mathbf{s}$ is contractible with:

$$\int f_0(\mathbf{s}) d\mathbf{s} \neq \int f_1(\mathbf{s}) d\mathbf{s} \text{ for some } s \in S \text{ but share a common support.}$$

Model

Actions and preferences of buyer

- The buyer is a *risk neutral cost minimizer*.
- She pays η to *solicit competitive bids* by setting $y = 1$, or alternatively makes an offer to a *default seller* $(y = 0)$.
- Let $n \in \{1, 2, \ldots\}$ denote the number of bids from sellers.
- If $y = 1$, she chooses search intensity $\lambda \in \mathcal{R}^+$, the arrival rate of a Poisson distribution for n.
- She incurs (additional) search costs of $\kappa\lambda$.
- Given n, the buyer forms a menu of J contracts $\{p_{jn}, q_{jn}(\mathbf{s})\}_{j=0}^{J-1}$.
- Here p_{jn} denotes a *base* price, and $q_{jn}(\mathbf{s})$ a *price adjustment*.
- There is a *lower* bound M on the variable component $q_{jn}(\mathbf{s}) - c(\mathbf{s})$.
- She chooses the contract some seller has bid, say the i^{th}, paying:

$$p_{in} + q_{in}(\mathbf{s}) + (\kappa\lambda + \eta)\, y$$

Model

Actions and preferences of seller

- Sellers approached by the buyer can bid by choosing one item on the menu, or decline all of them.
- Sellers receive a payoff of zero:
  - from opting out of the procurement process.
  - if they buyer does not select them.
- Sellers discount (enlarge) positive (negative) deviations from full insurance contracts for *liquidity* concerns (cost of working capital).
- The payoff to a type k seller from winning a contract $\{p_{in}, q_{in}(\mathbf{s})\}$ is:

$$p_{in} - \gamma_k + \psi\left[q_{in}(\mathbf{s}) - c(\mathbf{s})\right], \tag{1}$$

where $\psi(\cdot) : \mathcal{R} \to \mathcal{R}$ is continuous with:

  - $\psi(0) = 0$, $\psi'(0) = 1$
  - $\psi'(r) > 0$ and $\psi''(r) \leq 0$ for all $r \in \mathcal{R}$.

Screening Contracts for Risk Averse Sellers

The optimal menu

Theorem

The seller induces a separating equilibrium with menu $\{p_{1n}, q_{1n}(\mathbf{s})\} = \{p_n, c(\mathbf{s})\}$ *and* $\{p_{0n}, q_{0n}(\mathbf{s})\} = \{p, r(\mathbf{s}) + c(\mathbf{s})\}$, *priority going to the former, where:*

$$p_n \equiv \gamma_1 + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\gamma_0 - \gamma_1 - \int \psi[r(\mathbf{s})]\,[1 - l(\mathbf{s})]\, f_0(\mathbf{s})ds\right)$$

$$p \equiv \gamma_0 - \int \psi[r(\mathbf{s})] f_0(\mathbf{s})ds$$

$$r(\mathbf{s}) \equiv \begin{cases} \psi'^{-1}\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-l(\mathbf{s})\min\{\pi,\tilde{\pi}\}}\right) & \text{if } l(\mathbf{s}) \leq \tilde{l}(\min\{\pi, \tilde{\pi}\}) \\ M & \text{if } l(\mathbf{s}) > \tilde{l}(\min\{\pi, \tilde{\pi}\}) \end{cases}$$

and $\tilde{\pi}$ *and* $\tilde{l}(\cdot)$ *are defined in Kang and Miller (2022).*

Comparing the Transfer from the Buyer to Seller

Decomposition of the seller's transfer

- We can show that $T_U(n) < T(n) < T_{FIC}(n)$ and $\Gamma < 0$ where:
  1. the *full information ultimatum offer* transfer is
  $$T_U(n) = c_1 + (1-\pi)^n (c_0 - c_1)$$
  2. the *first price sealed bid* (firm-fixed-price) transfer is
  $$T_{FP}(n) = T_U(n) + (1-\pi)^{n-1} \pi (\gamma_0 - \gamma_1)$$
  3. the expected transfer under the *optimal menu* is
  $$T(n) = T_{FP}(n) + (1-\pi)^{n-1} \Gamma$$

  where $\Gamma \equiv T(1) - T_{FP}(1)$ is defined as the difference between the expected transfer under the optimal menu and the first price sealed bid (firm-fixed-price) transfer, c_0, when $n = 1$:

$$\Gamma = \int (1-\pi) \{r(\mathbf{s}) - \psi[r(\mathbf{s})]\} - \pi\psi[r(\mathbf{s})] [1 - I(\mathbf{s})] f_0(\mathbf{s}) d\mathbf{s} < 0$$

- For any real number λ, define the convex function:

$$U(\lambda) \equiv \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} T(n+1) + \kappa\lambda$$

- Note $U(\lambda) + \eta$ is the *expected total cost with positive search effort* λ.
- Let $U(0)$ denote the *expected cost of noncompetitive procurement*.
- Suppose $U(\lambda)$ attains its global minimum at λ^*:
  1. Then competitive bids are solicited if and only if:

$$U(\max\{0, \lambda^*\}) + \eta \leq U(0).$$

  2. If $\lambda^* > 0$, there is competitive bidding if and only if:

$$\begin{aligned} \eta &\leq U(0) - \sum_{n=0}^{\infty} \frac{\lambda^{*n} e^{-\lambda^*}}{n!} T(n+1) - \kappa\lambda^* \qquad (2) \\ &= (1 - e^{-\lambda^* \pi}) [(1-\pi)(c_0 - c_1) + \pi(\gamma_0 - \gamma_1) + \Gamma] - \kappa\lambda^*. \end{aligned}$$

Identification

Model primitives and data generating process

- The primitives of the model are:
  - $f_\pi(\pi) : (0,1) \to \mathcal{R}^+$ (density of proportion of low-cost sellers)
  - $\gamma_k(\pi) : \Pi \to \mathcal{R}^+$ (initial costs)
  - $c(\mathbf{s}) : \mathcal{S} \to \mathcal{R}^+$ (cost changes as a function of contract outcomes)
  - $F_{ks}(\mathbf{s}) : \mathcal{S} \to [0,1]$ (distribution of contract outcomes)
  - $\psi(r) : \mathcal{R} \to \mathcal{R}$ with $\psi'(r) > 0$ and $\psi''(r) < 0$ and normalizations $\psi(0) = 0$ and $\psi'(0) = 1$ (liquidity preferences)
  - $F_\eta(\eta) : \mathcal{R} \to [0,1]$ with $F_\eta'(\eta) > 0$ (solicitation costs)
  - $\kappa(\pi) : \Pi \to \mathcal{R}^+ > 0$ (unit search cost)
- We assume the data generating process of the model records:
  - whether contract is competitive $y \in \{0,1\}$
  - number of bids, n
  - winning contract type, $k \in \{0,1\}$
  - contract outcomes, $\mathbf{s}$
  - base price of winning contract p_{kn}
  - price changes $q_k(\mathbf{s})$

Estimates

Table 4: Model fit

	Data	Model
Extent of competition		
Probability of competitive solicitation	0.340	0.348 [0.332, 0.360]
Expected number of bids	1.635	1.609 [1.453, 1.656]
Probability of a firm-fixed-price contract		
No competitive solicitation	0.960	0.961 [0.954, 0.965]
Competitive solicitation with one bid	0.958	0.940 [0.932, 0.953]
Competitive solicitation with multiple bids	0.971	0.988 [0.983, 0.991]
Contract price (in thousand 2010 USD)		
Final	363.7	363.4 [358.5, 371.8]
Base, firm-fixed-price, no solicitation	337.0	334.2 [330.6, 341.6]
Base, firm-fixed-price, solicitation with one bid	329.2	335.3 [332.1, 342.8]
Base, firm-fixed-price, solicitation with multiple bids	336.1	340.5 [329.6, 345.1]
Base, other	359.7	352.0 [335.4, 363.4]
Adjustments, firm-fixed-price	24.5	25.1 [21.8, 27.9]
Adjustments, other	88.5	55.9 [41.8, 120.1]

Notes: Numbers in brackets are 95% confidence intervals based on bootstrapping.

Estimates

Table 5: Some comparisons

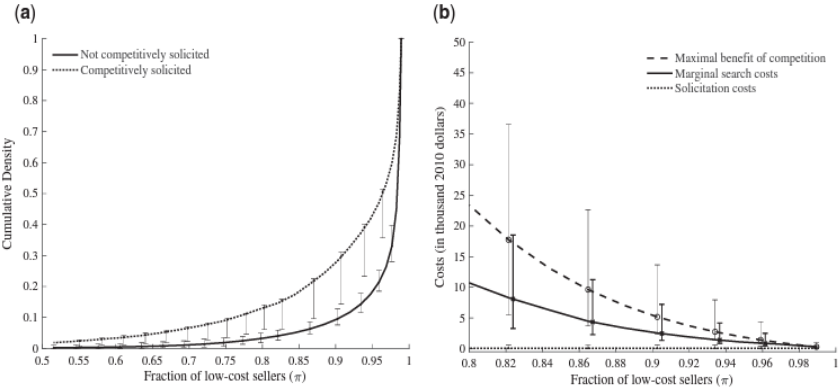


FIGURE 2

The fraction of low-cost sellers and procurement costs

Notes: Based on the estimated parameters, Panel (a) shows the cumulative distribution function of π conditional on whether or not the contract is competitively solicited, averaged across sample observations, and Panel (b) provides the buyer's marginal search costs and solicitation costs, as well as an upper bound of the benefit of competition, as defined in (6.1). The error bars represent the 95% confidence intervals based on bootstrapping.

Estimates

Table 5: Estimating the role of observed heterogeneity

	All contracts			Mean differences:	
	Mean	Median	SD	Product vs. services	Commercially available vs. not
Fraction of low-cost sellers	0.940	0.963	0.065	0.097	0.031
	(0.004)	(0.004)	(0.004)	(0.008)	(0.006)
Project costs of low-cost sellers	360.87	244.69	141.81	−27.19	−11.23
	(3.54)	(4.78)	(2.04)	(6.80)	(5.38)
Project cost difference	40.91	20.37	46.55	−2.02	19.24
	(30.63)	(19.37)	(32.27)	(36.00)	(38.55)
Maximal benefits of competition[a]	4.51	1.16	13.17	−6.61	−0.46
	(1.12)	(0.72)	(5.09)	(2.56)	(1.91)
Marginal search costs	1.70	0.56	4.65	−3.73	−0.32
	(0.53)	(0.43)	(1.20)	(1.29)	(0.77)
Solicitation costs	0.06	0.06	0.02	−0.01	0.003
	(0.14)	(0.14)	(0.07)	(0.05)	(0.03)
Conditional soliciting costs	−0.01	−0.01	0.02	−0.01	0.01
	(0.06)	(0.23)	(0.14)	(0.06)	(0.03)

Notes: This table provides summary statistics of the distribution of the mean values of the fraction of low-cost sellers, sellers' project costs, and the buyer's search and solicitation costs, integrated over π and evaluated at each realization of $(\mathbf{x}_i, \mathbf{z}_i)$ and the estimated parameters. It also provides the mean differences between contracts for products and those for services, as well as the differences in means between contracts for commercially available versus unavailable products and services. All cost estimates are in thousand 2010 dollars. Numbers in parentheses are bootstrap standard errors. [a]See (6.1) for the definition.

Estimates

Table 6: Counterfactual analyses

Panel A: Why so little competition?	Change in number of bids
Seller cost distribution	
What if the fraction of low-cost sellers (π) were 0.5?	+2.931 [2.901, 3.256]
What if the cost differences ($c_2 - c_1$) were doubled?	+0.690 [0.490, 0.744]
Buyer's ability to negotiate	
What if the buyer offered full-insurance contracts only?	+1.659 [0.834, 3.497]
Search and solicitation costs	
What if κ were halved?	+0.573 [0.372, 0.679]
What if η were halved?	+0.009 [0.006, 0.072]

Panel B: Effects of policies to mandate more competition

	Base	Minimum search intensity ($\lambda \geq 2$)	
		No	Yes
Number of bids	1.614 [1.418, 1.639]	+0.021 [0.014, 0.211]	+0.786 [0.779, 0.920]
Transfer	367.39 [359.64, 371.82]	-0.03 [-0.222, -0.005]	-1.42 [-1.996, -0.471]
Search costs	0.81 [0.218, 1.052]	+0.03 [0.004, 0.148]	+2.02 [0.550, 2.651]
Solicitation costs	0.02 [-0.128, 0.030]	+0.12 [0.015, 0.828]	+0.12 [0.015, 0.828]

Note: Both counterfactual policies in Panel B mandate competitive solicitation. The difference is that the first one requires no minimum search efforts, while the second one requires that search efforts are at least two so that the expected number of bids is two or more. Bootstrap 95 percent confidence intervals are provided in brackets.

A1 $\mathbf{s}$, π, and η are mutually independent.

A2 $F_\pi(\pi)$ is strictly increasing for all $\pi \in \Pi$.

A3 $\Pi \subset (0, \tilde{\pi})$, and $I(\mathbf{s}) \leq \tilde{I}(\pi)$ for all $(\mathbf{s}, \pi) \in S \times \Pi$.

A4 $\gamma_1(\pi)$ is non-increasing in $\pi \in \Pi$.

A5 $\gamma_0(\pi) - \gamma_1(\pi)$ is non-increasing in $\pi \in \Pi$.

A6 Either $\Psi_0(\pi) \leq \gamma_0'(\pi)$ for all $\pi \in \Pi$, or $\Psi_0(\pi) \geq \gamma_0'(\pi)$ for all $\pi \in \Pi$ where $\Psi_0(\pi) \equiv$:

$$\int \left(\psi'' \left[\psi'^{-1} \left(\frac{1-\pi}{1-\pi I(\mathbf{s})} \right) \right] \right)^{-1} \frac{(1-\pi)\left[I(\mathbf{s})-1\right]}{\left[1-\pi I(\mathbf{s})\right]^3} f_0(\mathbf{s}) d\mathbf{s}.$$

- Also define $v(I, \pi)$ as interior solution in r to FOC (**??**):

$$\psi'(r) = (1-\pi)/(1-\pi I) \tag{3}$$

- Assuming **A3** implies $v(I(\mathbf{s}), \pi) = q_{0n}(\mathbf{s}) - c(\mathbf{s})$ for all $\pi \in \Pi$.
- Write $p(\pi)$ and $p_n(\pi)$ for base prices p and p_n on π respectively.

Identification

Monotonicity

- Identification proof exploits monotonicity of $p(\pi)$ and $p_n(\pi)$.
- As π increases, there is a greater chance of selecting a low-cost seller.
- The buyer can reduce the base price for the low-cost contract if IR_1 does not bind.
- To satisfy IC_1 while reducing this base price, the buyer makes high-cost contract less attractive to low-cost sellers by increasing its volatility.
- Whether this makes high-cost contract more or less attractive to high-cost sellers depends on the other parameter values.

Lemma

(i) If **A3** *holds then* $\partial |v(I, \pi)| / \partial \pi > 0$. *(ii) If* **A3–A5** *hold then* $\partial p_n(\pi) / \partial \pi < 0$ *for all* $n \in \{1, 2, \ldots\}$. *(iii) If* **A3** *and* **A6** *hold then* $p(\pi)$ *is monotone.*

- Since the equilibrium menu is separating, $f_0(\mathbf{s})$ and $f_1(\mathbf{s})$ are directly identified from the distributions for the contract outcomes
- Hence the likelihood ratio $l(\mathbf{s}) \equiv f_0(\mathbf{s}) / f_1(\mathbf{s})$ is too.
- In equilibrium price changes to a low-cost seller equate with his cost changes: $c(\mathbf{s}) = q_{1n}(\mathbf{s})$.

- Denote by $\pi^*(p)$inverse of (strictly monotone) $p(\pi)$.
- Define the composite function $v^*(l(s), p) \equiv v[l, \pi^*(p)]$.
- $v^*(l, p)$ is identified off the high-cost contracts.
- Note $\frac{\partial v(l,\pi')}{\partial l} = \frac{\partial v^*(l,p')}{\partial l}$ for all (l, π', p') satisfying $p' = p(\pi')$.
- Holding π constant, totally differentiate (3) with respect to l, substitute $\frac{\partial v^*(l,p')}{\partial l}$ for $\frac{\partial v(l,\pi')}{\partial l}$ in the result, and rearrange to obtain:

$$\psi''(r) = \left[\frac{\partial v^*(l, p)}{\partial l}\right]^{-1} \frac{1 - \psi'(r)}{1 - l} \psi'(r). \tag{4}$$

- Noting $\psi'(0) = 1$ (4) has a unique solution of $\psi'(r)$.
- Furthermore $\psi(0) = 0$ implies $\psi(r)$ is solved too and identified off $v^*(l, p)$.

Stepwise Identification

3. Distribution of the project-type

- Since $\psi(q)$ is identified, realizations of π for high-cost contracts are identified from the FOC:

$$\pi = \frac{1 - \psi'\left[q_{0n}(\mathbf{s}) - c(\mathbf{s})\right]}{1 - \psi'\left[q_{0n}(\mathbf{s}) - c(\mathbf{s})\right] l(\mathbf{s})}.$$

- This identifies $f_{\pi|y,n,k}(\pi|y,n,0)$
- Noting high-cost contracts occur with probability $(1-\pi)^n$:

$$f_{\pi|y,n,k}(\pi|y,n,1) = \frac{\Pr(k=0|y,n)}{\Pr(k=1|y,n)} \frac{\left[1-(1-\pi)^n\right]}{(1-\pi)^n} f_{\pi|y,n,k}(\pi|y,n,0). \tag{5}$$

$$\Rightarrow f_{\pi|y,n}(\pi|y,n) = \frac{f_{\pi|y,n,k}(\pi|y,n,0)}{(1-\pi)^n \int (1-\pi')^{-n} f_{\pi|y,n,k}(\pi'|y,n,0)\, d\pi'}. \tag{6}$$

- Identifying $f_\pi(\pi)$ follows from identification of $f_{\pi|y,n}(\pi|y,n)$, because (y,n) is observed.

Stepwise Identification

4. Base prices as a function of pi

- As π realizations of high-cost contracts are identified, so is $p(\pi)$.
- What about $p_n(\pi)$?
- Let $G_{p_n|y}(p|y)$ denote the *cdf* for p_n conditional on $y \in \{0, 1\}$.
- Note p_n is strictly decreasing in π, and the inverse of $F_\pi(\pi)$ exists
- Therefore the inverse of $G_{p_n|y}(p|y)$ exists, so for $y \in \{0, 1\}$:

$$p_n(\pi) \equiv G^{-1}_{p_n|y}\left[1 - F_{\pi|y,n,k}(\pi\,|y, n, 1)\,|y\right].$$

- Hence $p_n(\pi)$ is identified because:
    - $f_{\pi|y,n,k}(\pi|y, n, 1)$ is identified (from previous slides)
    - $G_{p_n|y}(p_n|y)$ is identified directly off the data generating process.

- Substitute:
  - $p_n(\pi)$ for p_n in (**??**)
  - $p(\pi)$ for p in (**??**)

  . . . and manipulate resulting equations giving the expressions for $\gamma_0(\pi)$ and $\gamma_1(\pi)$.
- Thus, using their FOC's, for $n \in \{2, 3, \ldots\}$:

$$
\begin{aligned}
\gamma_1(\pi) &= \frac{1-(1-\pi)^n}{1-(1-\pi)^{n-1}} p_n(\pi) - \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^{n-1}} p_1(\pi) \\
\gamma_0(\pi) &= p(\pi) + \int \psi\left(\psi'^{-1}\left[\frac{1-\pi}{1-\pi l(\mathbf{s})}\right]\right) f_0(\mathbf{s}) ds
\end{aligned}
$$

- Multiple equations overidentify $\gamma_0(\pi)$ and $\gamma_1(\pi)$.

- Rearranging the FOC for optimal search intensity $\lambda^o(\pi)$:

$$\kappa(\pi) = \pi e^{-\pi\lambda^o(\pi)} \left\{ \begin{array}{l} (1-\pi)\left[c_0(\pi) - c_1(\pi)\right] \\ +\pi\left[\gamma_0(\pi) - \gamma_1(\pi)\right] + \Gamma(\pi) \end{array} \right\} \text{ if } \lambda^o(\pi) > 0$$

$$\kappa(\pi) \geq \pi \left\{ \begin{array}{l} (1-\pi)\left[c_0(\pi) - c_1(\pi)\right] \\ +\pi\left[\gamma_0(\pi) - \gamma_1(\pi)\right] + \Gamma(\pi) \end{array} \right\} \text{ if } \lambda^o(\pi) = 0$$

- We have already identified:
  - $\gamma_0(\pi) - \gamma_1(\pi)$ and $c_0(\pi) - c_1(\pi)$
  - $\Gamma(\pi) = \int (1-\pi)\{r(\mathbf{s}) - \psi[r(\mathbf{s})]\} - \pi\psi[r(\mathbf{s})]\left[f_0(\mathbf{s}) - f_1(\mathbf{s})\right] ds$
- Therefore a lower bound for $\kappa(\pi)$ is identified when $\lambda^o(\pi) = 0$.
- Otherwise $\kappa(\pi)$ is point identified because $\lambda^o(\pi)$ is identified from:

$$\begin{aligned} \lambda^o(\pi) &= \sum_{n=0}^{\infty} n \Pr(n+1|\pi, y=1) \\ &= \frac{\sum_{n=0}^{\infty} n f_{\pi|y,n}(\pi|1, n+1)\Pr(n+1|y=1)}{\sum_{n=0}^{\infty} f_{\pi|y,n}(\pi|1, n+1)\Pr(n+1|y=1)}. \end{aligned}$$

- The buyer solicits competitive bids if and only if $\eta \leq \Omega(\pi)$ defined as:

$$\Omega(\pi) \equiv \left[1 - e^{-\lambda^{o}(\pi)}\right] \left\{ \begin{array}{l} (1-\pi)\left[c_0(\pi) - c_1(\pi)\right] \\ +\pi\left[\gamma_0(\pi) - \gamma_1(\pi)\right] + \Gamma(\pi) \end{array} \right\} - \kappa(\pi)\lambda^{o}(\pi$$

- Variation in π induces variation in $\Omega(\pi)$, partially identifying $F_\eta(\eta)$, because $F_\eta[\Omega(\pi)] = \Pr(y = 1|\pi)$, and both $\Pr(y = 1|\pi)$ and $\Omega(\pi)$ are identified (from the previous results).
- For example when $\lambda^*(\pi) \leq 0$ then $\lambda^{o}(\pi) = 0$, and hence $\Omega(\pi) = 0$, implying $F_\eta(0)$ is identified.
- Thus $F_\eta(\eta)$ is identified on the range of $\Omega(\pi)$, defined:

$$\Upsilon \equiv \{\widetilde{\eta} \in \mathcal{R} : \widetilde{\eta} = \Omega(\widetilde{\pi}) \text{ for some } \widetilde{\pi} \in \Pi\}.$$