

Session 5.1

Mixtures revealing Dominated Strategies

Dominated strategies are not always easy to spot. In the example discussed in this session, a (pure) strategy that is not dominated by any pure strategy, is dominated by a mixed strategy. Here tossing a coin, weighted appropriately, and then taking one of two other strategies, depending on the toss, yields a strategy that gives higher expected value than the dominated strategy.

The supermarket's payoffs

- ◆ Green's payoffs are an order of magnitude higher than yellow's.
- ◆ At first glance green playing hours looks like a fairly safe strategy.
- ◆ Green's payoff from playing hours:
 - is higher than service if yellow plays price
 - is higher than price if yellow plays service
 - is higher than service if yellow plays hours.
- ◆ But green playing hours is a **dominated strategy!**

	Price	Service	Hours
Price	1 65	4 50	7 51
Service	2 50	5 55	8 52
Hours	3 60	6 50	9 55

Payoffs: Corner store franchise Supermarket | Inactive | Nature

Strategies dominated by a mixture

◆ Green's hours strategy is dominated by a mixture of the price and service strategies.

◆ Let π denote the probability that green chooses a price strategy, and $(1-\pi)$ denote the probability that green chooses a service strategy.

◆ This mixture dominates the hours strategy if the following three conditions are satisfied:

$$\pi 65 + (1-\pi) 50 > 51 \quad \text{or} \quad \pi > 1/15$$

$$\pi 50 + (1-\pi) 55 > 52 \quad \text{or} \quad 3/5 > \pi$$

$$\pi 60 + (1-\pi) 50 > 55 \quad \text{or} \quad \pi > 1/2$$

◆ Hence all mixtures of π satisfying the inequalities:

$$1/2 < \pi < 3/5$$

dominate the hours strategy.

Eliminating a column strategy

- ◆ Yellow is confident green will not play hours.
- ◆ Using the principle of iterated dominance yellow eliminates that strategy from consideration.
- ◆ Yellow has dominant strategy in this reduced game of playing hours.
- ◆ Green best replies with price.

		Price		Service	
Price	1	65	50		
	2	50	55		
	3	60	50		
Service	4	6	3		
	5	3	3		
	6	3	3		
Hours	7	15	12		
	8	15	12		
	9	15	12		

Payoffs	Inactive	Nature
Corner store franchise		
Supermarket		

A unique Nash equilibrium

- ◆ In general, a NE solved by iterative dominance is unique.
- ◆ This is illustrated here, as indicated by the best reply yes/no notation in the payoff cells.
- ◆ But the main lesson is that if iterated dominance solves the NE, then no coordination is required by the players, just a moderate degree of sophistication.

	Price	Service	Hours
Price	1 no 65 yes	4 no 50 no	7 yes 51 no
Service	8 no 50 no	5 no 55 yes	6 no 52 no
Hours	15 yes 60 yes	12 yes 50 no	9 no 55 no

Payoffs: Corner store franchise Supermarket

Inactive

Nature