Session 4.3
Dominated Strategies Defined

This session is the second in triplet on the concept of dominance:

1. Dominant strategies
2. Dominated strategies
3. Iterative dominance.

This session provides formal definition of dominated strategies. What is a dominated strategy? We devote the next session to explaining iterative dominance, and tying the three parts together.

Dominated strategies

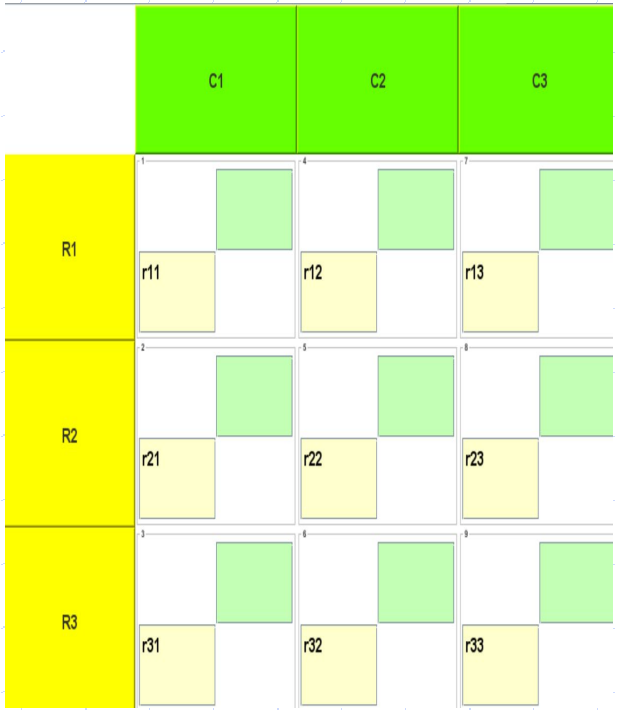
- A strategy is dominated if there is another strategy the player could take that always gives a higher expected payoff regardless of what the other players in the game do.
- You should never play a dominated strategy: your expected payoff is for sure higher by playing a strategy that dominates it.
- As in the case of dominant strategies, it is not necessary to know anything about the payoffs of the other players to determine whether or not a strategy is dominated.
- Inequalities about your own payoffs determine whether or not a strategy is dominated.

A payoff matrix to the row player

- In the matrix row picks a strategy of R1, R2 or R3, while column picks a strategy of C1, C2 or C3.
- For i = 1, 2, 3 and j =1, 2, 3 the payoff to row is rij if row selects Ri and column selects Cj.
- Only the payoffs of row are shown, because whether or not a strategy is dominated does not depend on the payoffs of column.

Domination by a pure strategy

- R2 is a dominated strategy if the following conditions are met:
  - r11 > r21 and r12 > r22 and r13 > r23 and at least one of the inequalities is strict.
- R2 is a dominated strategy if the following conditions are met:
  - r31 > r21 and r32 > r22 and r33 > r23 and at least one of the inequalities is strict.
- Note that it is not necessary for both R1 and R3 to dominate R2. If either R1 or R3 dominates R2, then R2 is a dominated strategy.



Domination by a mixed strategy

- The preceding slide shows how a pure strategy can be used to prove that another strategy is dominated.

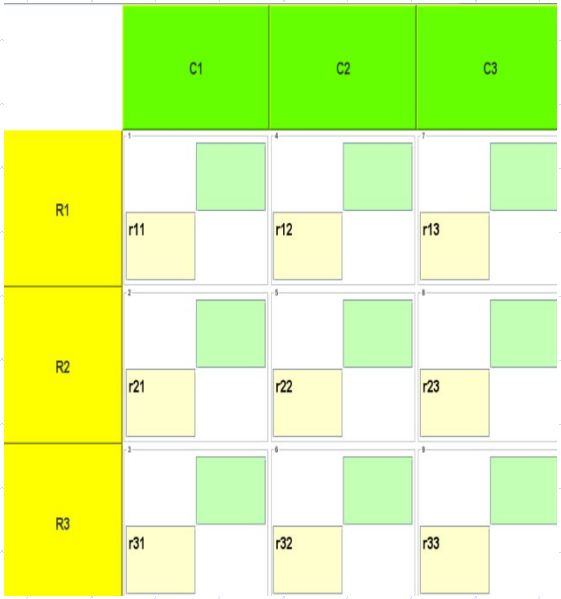
- A mixed strategy might also be used to reveal a dominated strategy.

- Suppose there exists a probability p, that is 0 < p < 1, and:

  p r11 +(1 - p)r31 > r21

  p r12 +(1 - p)r32 > r22

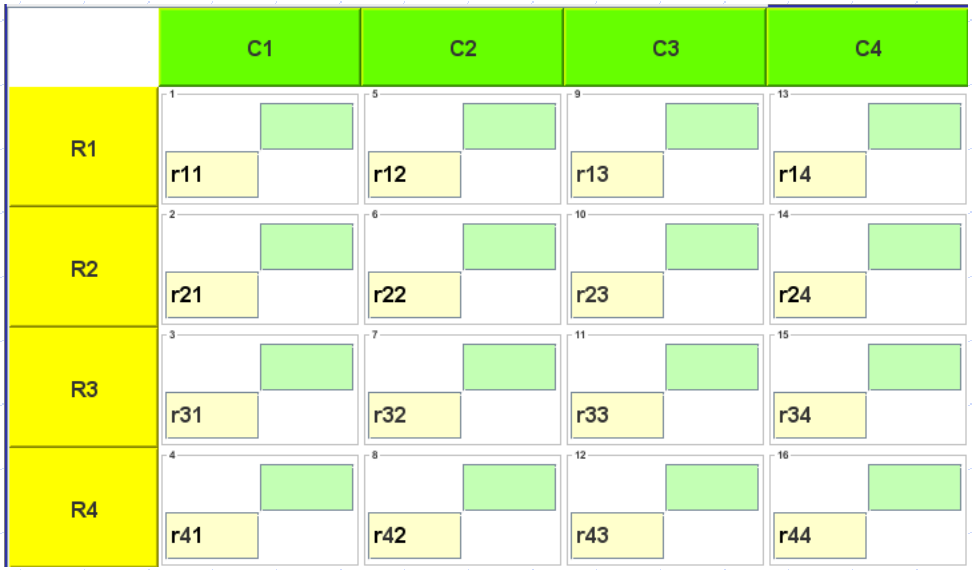
  p r13 +(1 - p)r33 > r23 and at least one of the inequalities is strict.



- Then R2 is a dominated strategy, because randomly picking R1 with probability p and R3 with probability (1 - p) yields a higher payoff than picking R3, regardless of what the column player does.

Extending the definition

- Again, this definition readily extends to include extra:
  - column strategies, such as C4 with payoffs to the row payoffs r14, r24, and r34.
  - row strategies such as R4 with payoffs r41, r42, and r43.
  - players (who require a little more notation).



Summary

- A dominated strategy yields lower expected payoffs than at least one other strategy regardless of what the other players do.
- Therefore if a dominant strategy exists, every other strategy is dominated.
- Like dominant strategies, dominated strategies are determined using very little information about expected payoffs:
  - No information about the payoffs (or the behavior) of the other players is used.
  - Only inequalities, not magnitudes, are used.
- Never play a dominated strategy.