

Session 4.1

Dominant strategies defined

We have introduced and used examples to illustrate the concepts of:

1. Dominant strategies
2. Dominated strategies
3. Iterative dominance.

This session provides formal definition of dominant strategies. What is a dominant strategy? We devote the next two sessions to explaining dominated strategies and iterative dominance.

Dominant strategies

- ◆ Strategies that are optimal for a player regardless of what the other players do are called **dominant**.
- ◆ Although a player's payoff might depend on the choices of the other players, when a dominant strategy exists, the player has no reason to introspect about the objectives of the other players in order to make his own decision.
- ◆ Similarly, when a dominant strategy exists, the player does not need to know the behavior of the other players to form his or her best response to the probability distribution characterizing their choices.

A payoff matrix to the row player

- ◆ In the matrix the **row** player picks a strategy of R1, R2 or R3, while the **column** player picks a strategy of C1, C2 or C3.
- ◆ For $i = 1, 2, 3$ and $j = 1, 2, 3$ the payoff to the row player is r_{ij} if the **row** player selects R_i and the **column** player selects C_j .
- ◆ Only the payoffs of the **row** player are shown, because whether a strategy is dominant or not does not depend on the payoffs of the **column** player.

	C1	C2	C3
R1	r11	r12	r13
R2	r21	r22	r23
R3	r31	r32	r33

Illustrating the inequalities

◆ R1 is a **dominant** strategy if the following sets of conditions are met:

- $r_{11} > r_{21}$ and $r_{12} > r_{22}$ and $r_{13} > r_{23}$ and at least one of the inequalities is strict.
- $r_{11} > r_{31}$ and $r_{12} > r_{32}$ and $r_{13} > r_{33}$ and at least one of the inequalities is strict.

	c1	c2	c3
R1	1 r11	4 r12	7 r13
R2	2 r21	5 r22	8 r23
R3	3 r31	6 r32	9 r33

Applying the definition to a bigger matrix

- ◆ This definition readily extends to include extra:
 - **column strategies**, such as C4 with payoffs to the row payoffs r14, r24, and r34.
 - **row strategies** such as R4 with payoffs r41, r42, and r43.
 - **players** (which would require a little more notation).

◆ The matrix below adds an extra row strategy and an extra column strategy.

	C1	C2	C3	C4
R1	1 r11	5 r12	9 r13	13 r14
R2	2 r21	6 r22	10 r23	14 r24
R3	3 r31	7 r32	11 r33	15 r34
R4	4 r41	8 r42	12 r43	16 r44

Summary

- ◆ A **dominant** strategy yields higher expected payoffs than **all the other strategies** regardless of what the other players do.
- ◆ Dominant strategies are determined using very little information about expected payoffs:
 - No information about the payoffs (or the behavior) of the other players is used.
 - Only inequalities, not magnitudes, of the player's own payoffs are used.
- ◆ Always play a dominant strategy if it exists.