

**ASSIGNMENT 5 (9 points)**  
*(Due Sunday November 23)*

**There are three multipart questions and two experiments.**

**Question 1 (3 points)** . . . *rust belt*

Consider an industry with a weak board of directors (representing fragmented and scattered shareholders), an organized workforce and entrenched management. The union and management simultaneously make demands on the firm's resources (Low, Medium or High). If the sum of their demands is less than or equal to the total resources of the firm, shareholders receive the residual. If the sum exceeds the firm's total resources, then the firm is bankrupted by industrial action. To achieve a bigger share of the gains from trade, both sides court disastrous consequences. This is sometimes called a game of chicken, or attrition. The payoff matrix for employees (yellow) and managers (green) is illustrated below as a percentage of wealth of the firm.

		Management		
		Low	Medium	High
Union	Low	25	50	75
	Medium	50	50	0
	High	75	0	0

- (a) Using the arrow notation, show the best replies in this game. (For example, copy this diagram into your answer sheet, and paste your best reply arrows on to the matrix.)
- (b) Is there a pure strategy Nash equilibrium? How many are there? What are they?
- (c) Copy the game, and have your **experimental group** play it four (or more) times. Show the empirical distribution and the best responses?
- (d) Briefly discuss the value of coordinating the union and management. Who misses out? Who has the longest horizon, shareholders, union members or management? Does that help explain the running head for this question?

**Question 2 (3 points)** . . . *taxing small businesses*

Taxation authorities are typically evaluated by the amount of revenue they collect.

In this problem the taxpayer owes 4. He could *report truthfully* and pay 4 but might reduce his taxes by 2 through *negligent accounting* and 0 through *fraud*.

It costs the IRS 1 to *check for irregularities* (which reveal negligent accounting but not fraud), and 2 to reveal both negligent accounting and fraud.

The third strategy of IRS is to *pass over*; this costs the IRS nothing but exposes the authority to reduced revenue. The penalties for being caught are harsh, 2 for negligence, and 8 for fraud.

Summarizing, this is a two-player game between the *IRS* and a *taxpayer* in which each player has three strategies.

- (a) Using comlabgames display the matrix representation of the strategic form of this game. Here are two hints: if the taxpayer commits fraud, and the IRS audits, then the payoff to each is  $-12 = -4 - 8$  and  $10 = 4 + 8 - 2$  respectively. Alternatively, if the taxpayer engages in negligent accounting and the IRS passes over, then the payoffs are  $-2 = 4 - 2$  and  $2$  respectively. Your job is to create the bi-matrix game representation, label the strategies and fill in cells with the payoffs. Here is another hint showing what part of the matrix should look like:

	Audit	Check Irregularities
Commit Fraud	$-12$ $10$ $0$	$0$ $-1$ $0$
Negligent Accounting	$-6$ $4$ $0$	$-6$ $5$ $0$

- (b) Play the game in your **experimental group** at least five times and report the results using a snapshot that shows the cell outcomes.
- (c) What is the empirical distribution of strategies of each player (the proportion of times each player, taxpayer and IRS, chooses one of three strategies).
- (d) What is the best response of each player to the empirical strategy of the other one.

**Question 3 (3 points) . . . a continuation**

Continuing with the taxation game . . .

- (e) Illustrate the best replies with arrows on the bi-matrix game you have displayed in Question 2 using a separate copy if you like.
- (f) Prove there is no pure strategy NE. Picking on two of the cells, explain in words that if the cell is a best reply for the taxpayer (IRS), then it is not a best reply for the IRS (Taxpayer).

There is a mixed strategy NE for this game (since there is no pure strategy). To solve for the mixed strategy NE let  $p_1$  denote the probability that the taxpayer reports truthfully, let  $p_2$  denote the probability the taxpayer engages in some negligent accounting, and  $p_3 = 1 - p_1 - p_2$  the probability he commits fraud. Similarly let  $q_1$  denote the probability that the IRS audits, let  $q_2$  denote the probability the IRS checks irregularities, and  $q_3 = 1 - q_1 - q_2$  the probability the IRS passes over.

- (g) Give the mixed strategy of the IRS what is the expected payoff to the taxpayer from committing fraud? From reporting truthfully?
- (h) Using a similar logic, what is the expected payoff to the taxpayer engaging in negligent accounting?
- (i) If the taxpayer is indifferent between engaging in negligent accounting, or committing fraud, and reporting truthfully then the expected payoff from the three strategies must be equal. Solve for  $(q_1, q_2, q_3)$ , the mixed strategy of the IRS that equalizes the three expected values for the taxpayer.
- (j) Solve for  $(p_1, p_2, p_3)$  in a similar way to obtain the equilibrium strategy for the taxpayer.
- (k) What is the probability of reaching each of the cells? Answer this question by displaying them in your game matrix. How likely is the taxpayer to be caught for fraud?
- (l) Compare the mixed strategy equilibrium you have found with the outcomes of the empirical distribution you reported on in the previous question.