

Lecture 6

Designing and Solving Games

This lecture shows why the extra information conveyed by the extensive form is sometimes crucial in determining the solution. There is a unique pure strategy NE to TQM, but it is not convincing. There is also a mixed strategy NE that is far more plausible. This session shows how to derive both.

Then we review the main principles we have discussed throughout the course with a view to thinking about how it can be used to help understand strategic situations.

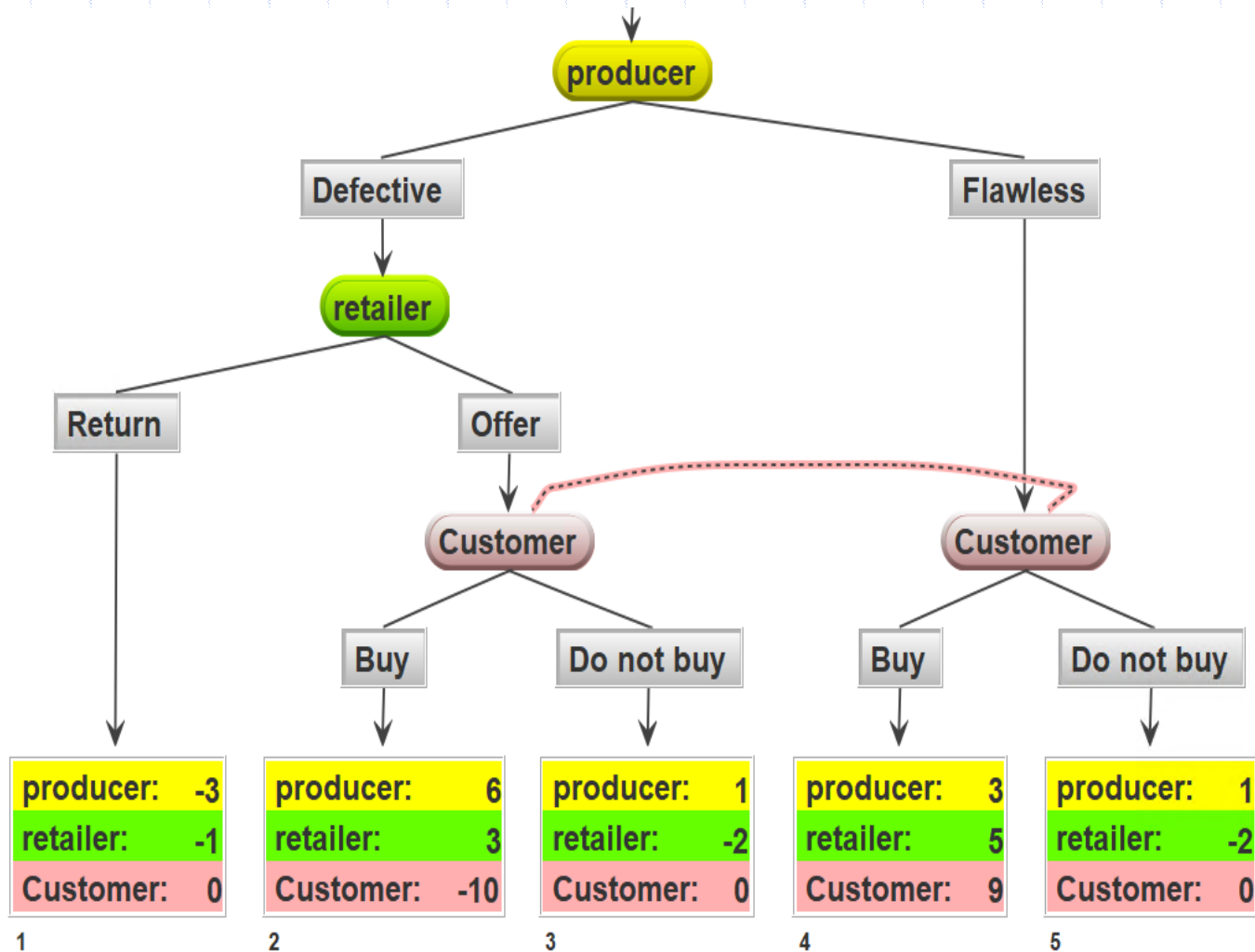
Perfection

- ◆ Strategies involve making implicit threats, but not all threats are credible.
- ◆ It would be mistaken to suppose that whenever a pure strategy NE exists, that is a more plausible outcome than a mixed strategy NE.
- ◆ Other principles may be more credible, such as conjecturing what would happen off the equilibrium path.
- ◆ A NE is defined as **perfect** if vanishingly small deviations from the NE converge to it (without jumping).

Quality control

- ◆ Manufacturers do not consistently produce flawless products despite legions of consultants who have advised them against this policy.
- ◆ Retailers help guard against flawed products by returning some of the defective items sent, and lending their brand to the ones they retail.
- ◆ Consumers cannot judge product quality as well as retailers and producers, since each one experiences only a **tiny fraction** of the **end product**.
- ◆ What is an acceptable **defect rate**, how often should retailers **return** defective items, and what are the implications for consumer **demand**?

Total quality management



TQM in strategic form

		buy		do not buy	
		offer	return	offer	return
defective	1	6 YES 3 YES -10 NO	-3 NO -1 NO 0 YES	1 YES -2 NO 0 YES	-3 NO -1 YES 0 YES
	3	3 NO 5 YES 9 YES	3 YES 5 YES 9 YES	1 YES -2 YES 0 NO	1 YES -2 YES 0 NO

Payoffs: Customer (yellow), Producer (green), Retailer (red)
 Others: _____
 Nature: _____

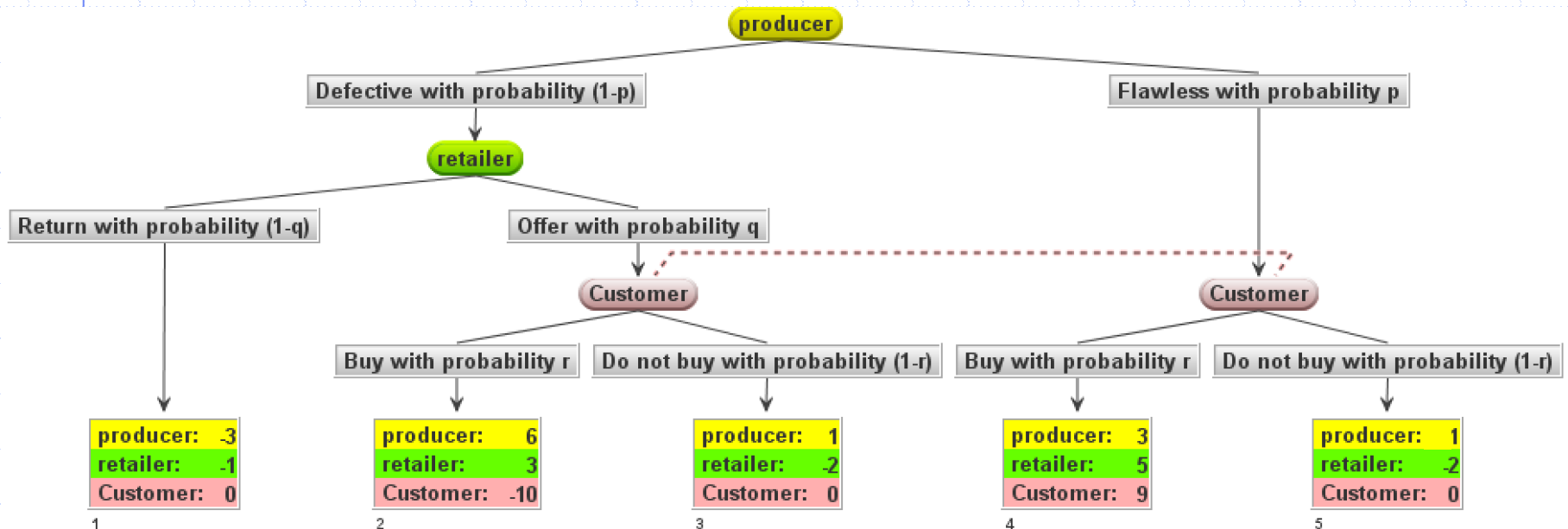
- ◆ There are two strategies for each player. Having derived the strategic form of the game, we can locate the pure strategy Nash equilibriums.
- ◆ There is a unique pure strategy Nash equilibrium, in which the producer only manufactures flawless products, the retailer only sells flawless products and the customer always buys the product.

Why is the pure strategy Nash equilibrium unconvincing?

- ◆ But is this Nash equilibrium convincing?
- ◆ Sure you can't eliminate any dominated strategies.
- ◆ If, however, the producer does manufacture a defective item, the retailer, but not the consumer will know, and makes more by offering the item for sale.
- ◆ Can the retailer convince consumers that they really will return defective products?

Is there a mixed strategy equilibrium?

- ◆ Let q denote the probability that the retailer offers a defective product item sale.
- ◆ Let r denote the probability the customer buys the item.
- ◆ Let p be the probability of producing a flawless item.



Solving for r , the probability of buying

- ◆ If $0 < q < 1$, then the retailer is indifferent between offering a defective product and returning it.
- ◆ In that case:

$$3r - 2(1 - r) = -1$$

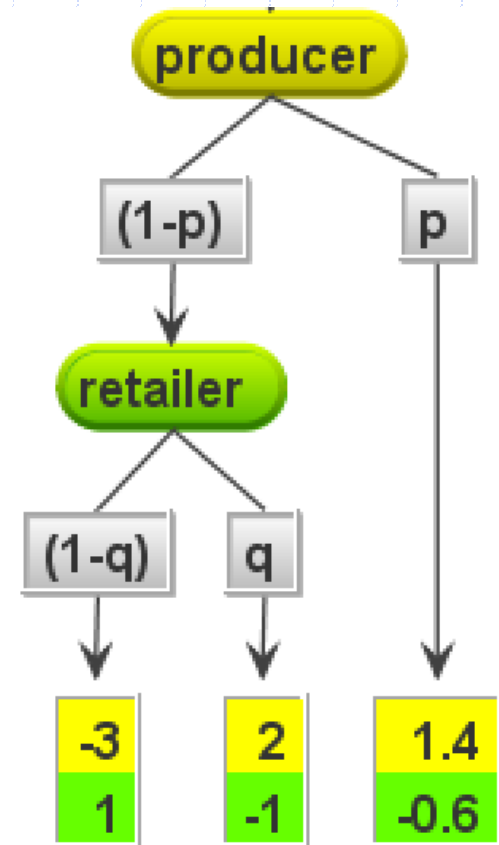
$$\Rightarrow 3r - 2 + 2r = -1$$

$$\Rightarrow 5r = 1$$

$$\Rightarrow r = 0.2$$

How to solve for p and q

- ◆ Once we substitute for $r = 0.2$ in the shopper's decision, we are left with the diagram:
1. q is chosen so that the producer is indifferent between production methods;
 2. p is chosen so that the shopper is indifferent between buying and not buying.



Solving q , the probability of offering the product

- ◆ The producer will only mix between defective and flawless items if the benefit from both are equated:

$$[6r + (1 - r)]q - 3(1 - q) = [3r + (1 - r)]$$

$$\Rightarrow 2q - 3 + 3q = 1.4$$

$$\Rightarrow 5q = 4.4$$

$$\Rightarrow q = 0.88$$

Solving for p , the probability of producing a flawless product

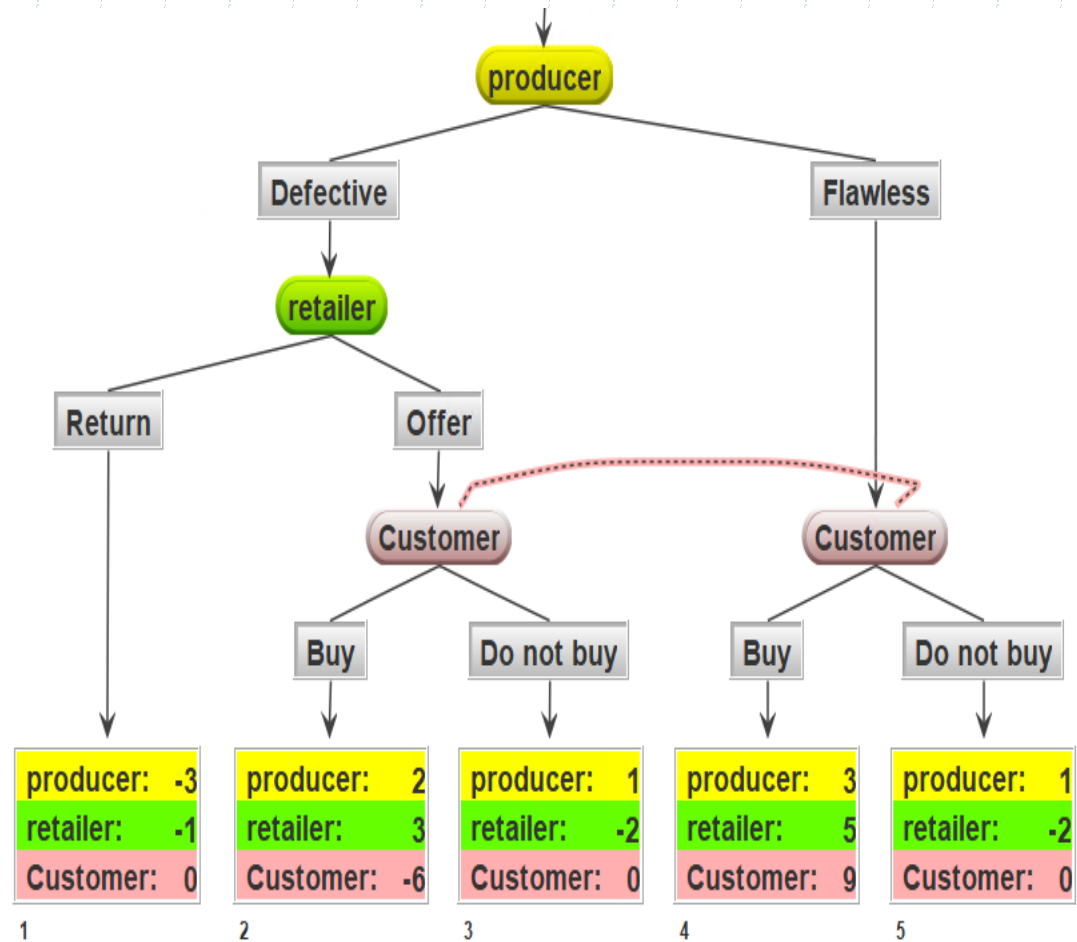
- ◆ Investigating the cases above shows that in a mixed strategy equilibrium $r = 0.2$ and $q = 0.88$.
- ◆ Since the shopper is indifferent between buying the item versus leaving it on the shelf, there are no expected benefits of acquiring the item:

$$9p - 10(1 - p)q = 0 \Rightarrow (9 + 10q)p = 10q$$

$$\Rightarrow p = 44/89$$

Offering a partial refund

We now modify the game slightly. If the customer buys a defective product, she receives partial compensation.



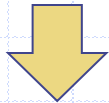
A different outcome

- ◆ In this case if the shopper buys the best reply of the manufacturer is to specialize in the production of flawless goods.
- ◆ Recognizing this, the shopper picks a pure strategy of buying.
- ◆ Realizing that the shopper will buy everything she is offered, the retailer never returns its merchandise to the manufacturer (and indeed there is never any reason too).

A practical guide to designing games

- ◆ *Extensive form . . .*

Players, moves, payoffs, information sets.



- ◆ *Strategic form . . .*

Players, strategies, expected payoffs.

- ◆ *Not a fine-grained reproduction of reality!
A tool for thinking about strategy.*

- ◆ Complexity . . .

Simple enough to solve with some thought.

Hard enough for experimental subjects to make mistakes.

Perfect information games

Simultaneous move games

} are usually too easy to solve.