Lecture 3
Deriving the Strategic Form from the Extensive Form

Sometimes a game is easiest to solve using the extensive form. Sometimes it is easier to solve using the strategic form. Sometimes using a combination of approaches that breaks the game up into subgames is the best approach. This session shows how to derive the strategic form from the extensive form.

The strategic versus the extensive form

- In the first two lectures we defined games in two ways, in the strategic form and the extensive form. This session shows how to derive the strategic form from the extensive form.
- Recall the strategic form has three elements (players, strategies and expected payoffs), while the extensive form has four (players, moves, information sets and payoffs).
- Since the extensive form of a game is a more comprehensive description than the strategic form, many extensive forms have the same strategic form, but not the other way around.
- For example, the strategic form of every game in which players make multiple moves is identical to a simultaneous move game in which each strategy in the multiple move game is relabeled as a move.
- Depending on the game it is sometimes easiest to solve using the extensive form, the strategic form, or using a combination of approaches that decomposes the game into subgames.

Bottling wine

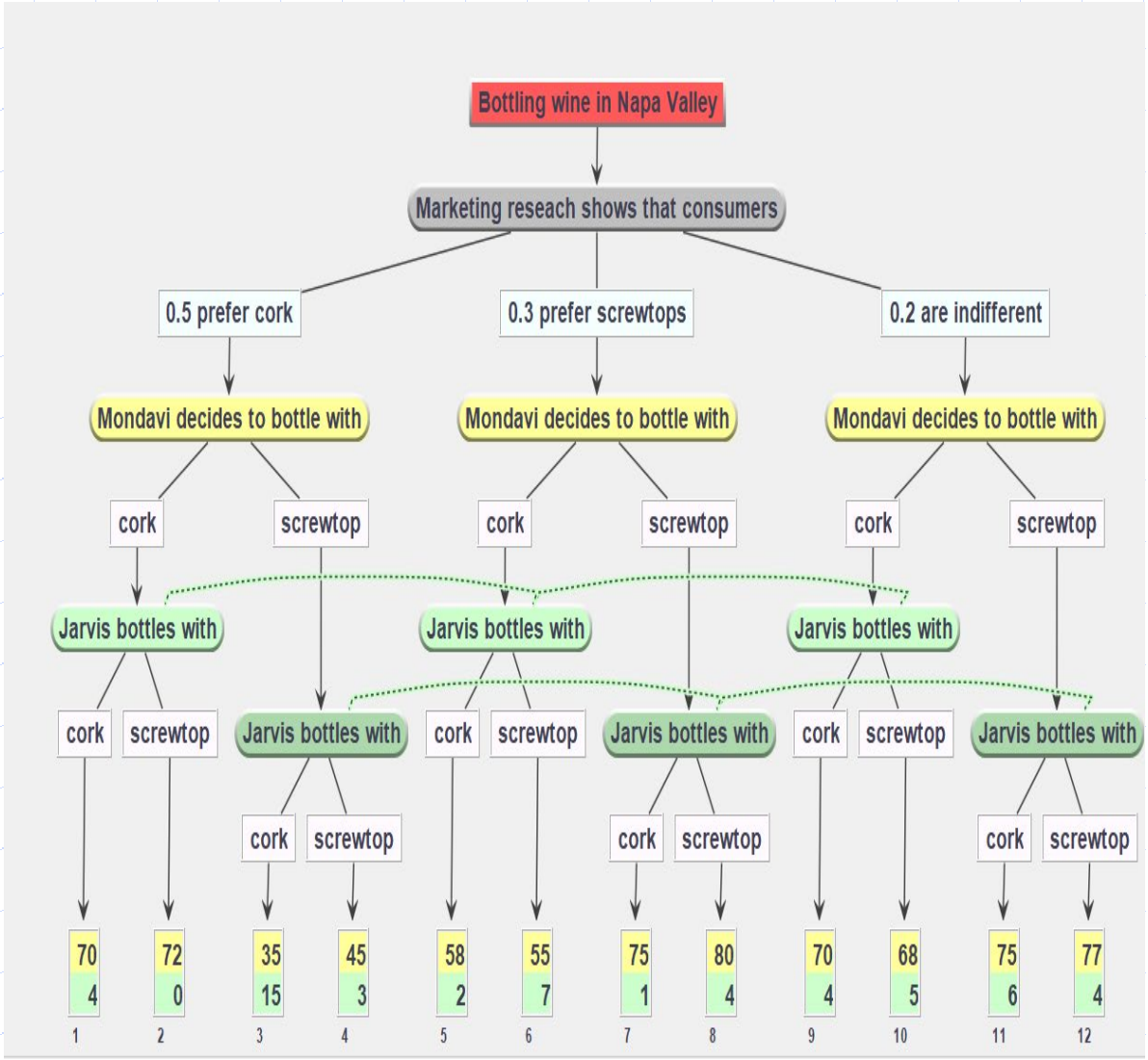
- Corks are traditionally used in bottling wine, but recent research shows that screwtops give a better seal, and hence the reduce the risk of oxidation and tainting. They are also less expensive.
- However, consumers associate screwtops with cheaper varieties of wine, so wineries risk losing brand reputation from moving too quickly ahead of the consumer tastes.
- To illustrate this problem, consider two Napa valley wineries who face the choice of immediately introducing screwtops or delaying their introduction.

A verbal description of the timeline

1. Before Mondavi does some market research both companies believe that with probability:
   - 1/2 the customers prefer cork
   - 3/10 they prefer screwtop
   - 1/5 they don't care.
2. The research then shows which of these scenarios is true.
3. Mondavi makes its choice between cork and screwtop.
4. Jarvis sees Mondavi's choice but not the results of the market research.
5. Jarvis makes its choice between cork and screwtop.

Extensive form representation of the game

- Mondavi has resources to conduct market research into this issue, but Jarvis does not.
- However Jarvis can retool more quickly than its larger rival, so it can copy what Mondavi does.



The strategies for both players

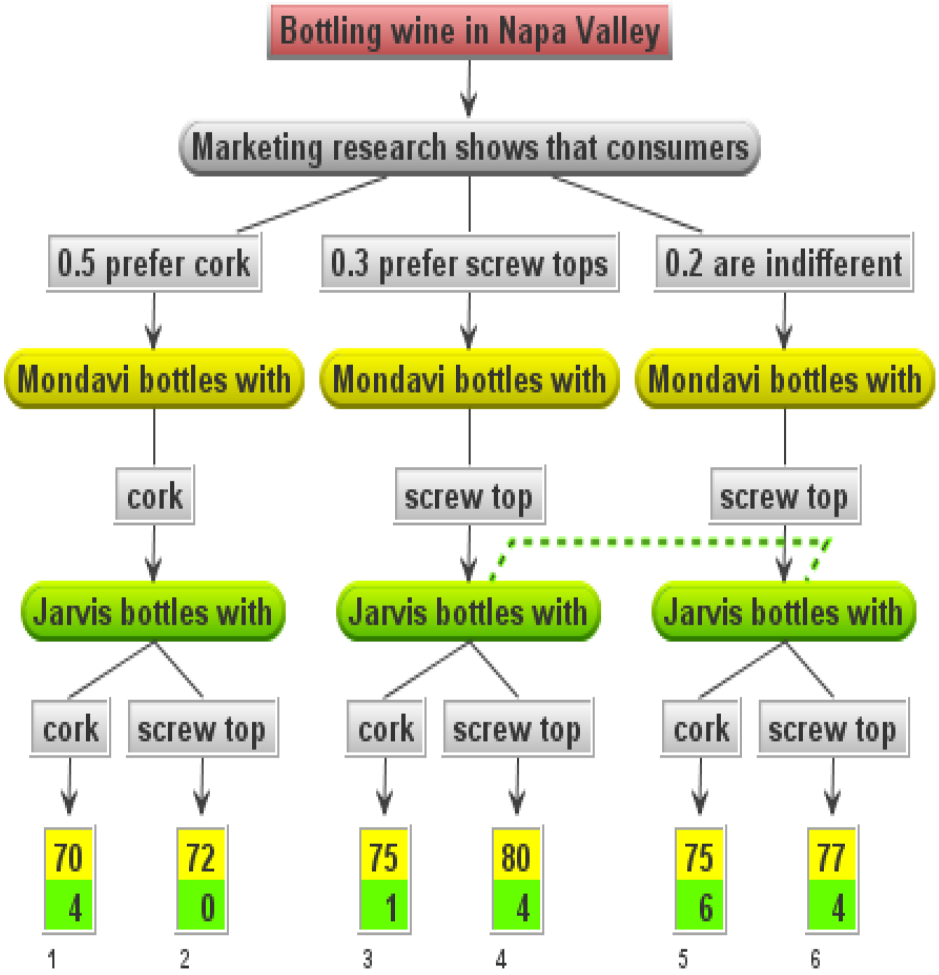
- Mondavi has 8 strategies:
  - There are 3 scenarios each strategy must cover. Consumers prefer cork, screwtops, or do not care.
  - In each of the three cases it makes one of two moves, bottle with cork or screwtop.
  - There are 2*2*2 = 8 strategies in total.
- Jarvis has 4 strategies:
  - There are 2 scenarios each strategy must cover. They observe Mondavi bottling with screwtop or cork before making their own decision.
  - In each scenario Jarvis also make one of two moves, bottling with cork or screwtops.
  - There are 2*2 = 4 strategies in total.

Mondavi has a dominant strategy

- Considering each of Mondavi's 3 scenarios:
  - Suppose consumers prefer cork . . .
    - If Mondavi bottles with cork, its payoff is either 70 or 72.
    - If Mondavi bottles with screwtop, its payoff is either 35 or 45. Since 35 < 70 and 45 < 72, bottle with cork in this case.
  - Suppose consumers prefer screwtop . . .
    - If Mondavi bottles with cork its payoff is either 58 or 55.
    - If Mondavi bottles with screwtop its payoff is either 75 or 80. Since 58 < 75 and 55 < 80, bottle with screwtop here.
  - Suppose consumers don't care. . .
    - If Mondavi bottles with cork its payoff is either 70 or 68.
    - If Mondavi bottles with screwtop its payoff is either 75 or 77. Since 70 < 75 and 68 < 77, bottle with screwtop here too!
- In short, follow demand, then cost, and ignore small players.

Eliminating Mondavi as a player

- We can simplify the problem Jarvis has by drawing its decision problem when Mondavi follows its dominant strategy.
- For all intents and purposes, this representation removes Mondavi from the game.



Solving for Jarvis

- Since 4 > 0, Jarvis bottles with cork if Mondavi does.
- The expected value of using screwtops when Mondavi does is:

  $$(0.3*4 + 0.2*4)/(0.2 +0.3) = 4.0$$

  while the expected value of retaining corking when Mondavi switches is:

  $$(0.3 + 0.2*6)/(0.2 +0.3) = 3.0$$

- Therefore Jarvis always follows the lead of Mondavi.

Rivals as a source of information

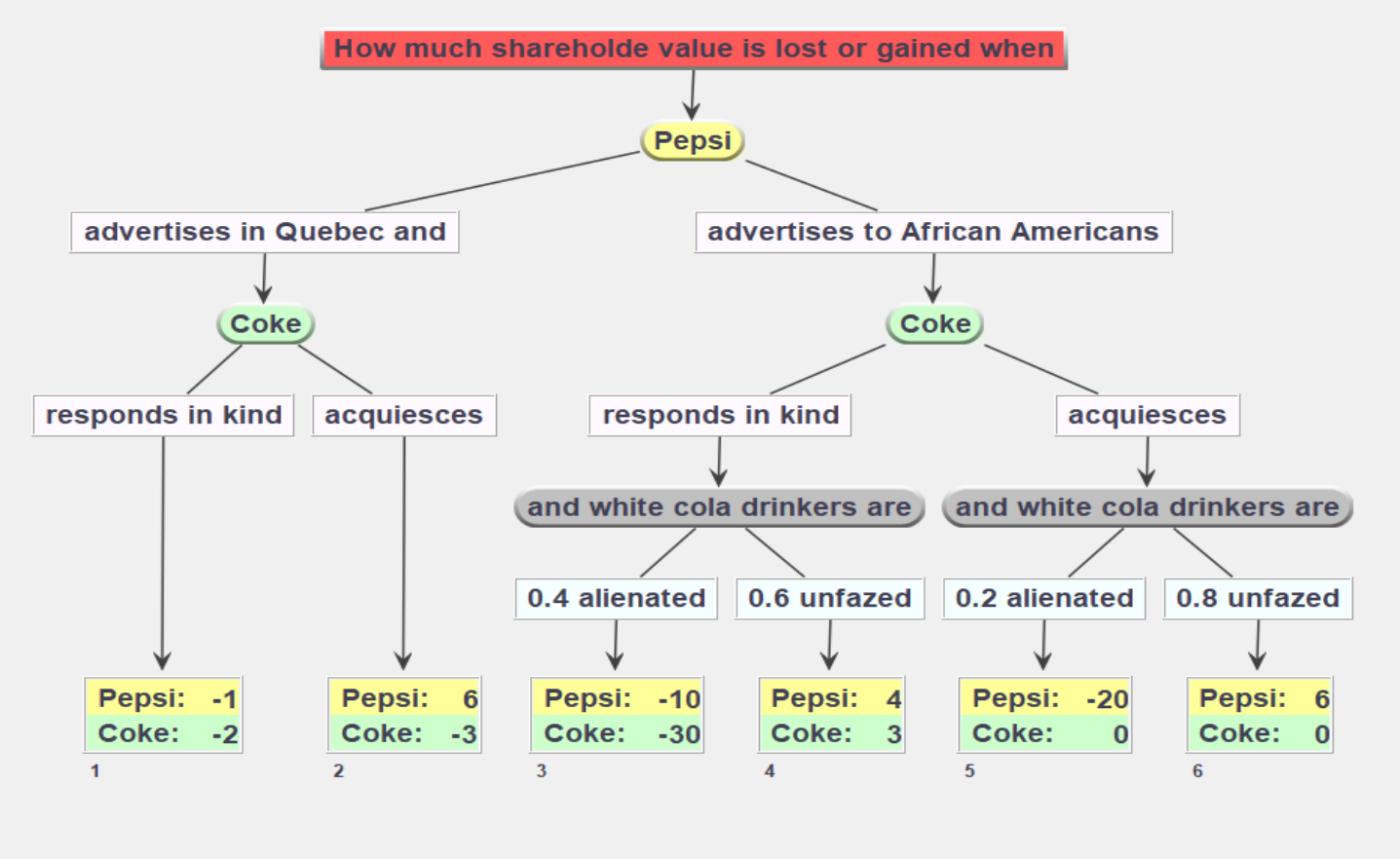
- The solution to this game shows that rivals can be a valuable source of information.
- Although Jarvis could undertake its own research into bottling, it eliminates these costs by piggybacking off Mondavi's extensive marketing research.
- Nevertheless Jarvis receives a noisy signal from Mondavi. Jarvis cannot tell whether consumers prefer screwtops or are indifferent.
- How much would Jarvis be prepared to pay to conduct its own research, and receive a clear signal?

The value of independent research

- Another use of the extensive form is to predict what would happen if the information available to each player changed.
- Since Mondavi has a dominant strategy, it would not benefit at all know what Jarvis is going to do before making its own production decision.
- In the next assignment you asked what would happen if Jarvis could obtain the information that Mondavi has:
  - How does the extensive form change if Jarvis also obtains the information? (Hint: what happens to the dotted lines?
  - How much would it be willing to pay?

The extensive form of the Cola game

- The diagram depicts the extensive form of the Coke-Pepsi game when Pepsi moves first:



The Strategy Space for Pepsi

- What happens if we try to solve the game with its strategic form?
- Pepsi is a first mover, and makes no further moves in the game.
- Furthermore nature (consumer demand tastes) follow Pepsi's move. That is Pepsi moves without knowing nature's moves.
- Therefore, each of Pepsi's two moves corresponds to precisely one strategy:
  1. Advertise in Quebec.
  2. Advertise to African Americans.

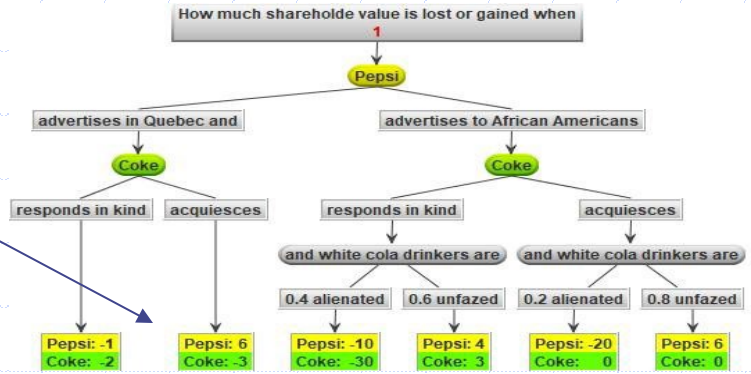
The Strategy Space for Coke

- Coke, like Pepsi, only makes one move in this game.
- However Coke moves second, and can see the move Pepsi has taken before it makes its own.
- Therefore Coke can condition on what Pepsi does in formulating its own strategy.
- Thus Coke has four strategies:
  1. Always acquiesce.
  2. Always respond in kind.
  3. Acquiesce if Pepsi advertises in Quebec and respond in kind if Pepsi advertises to African Americans.
  4. Acquiesce if Pepsi advertises to African Americans and respond in kind if Pepsi advertises in Quebec.

Matching strategy pairs to expected payoffs

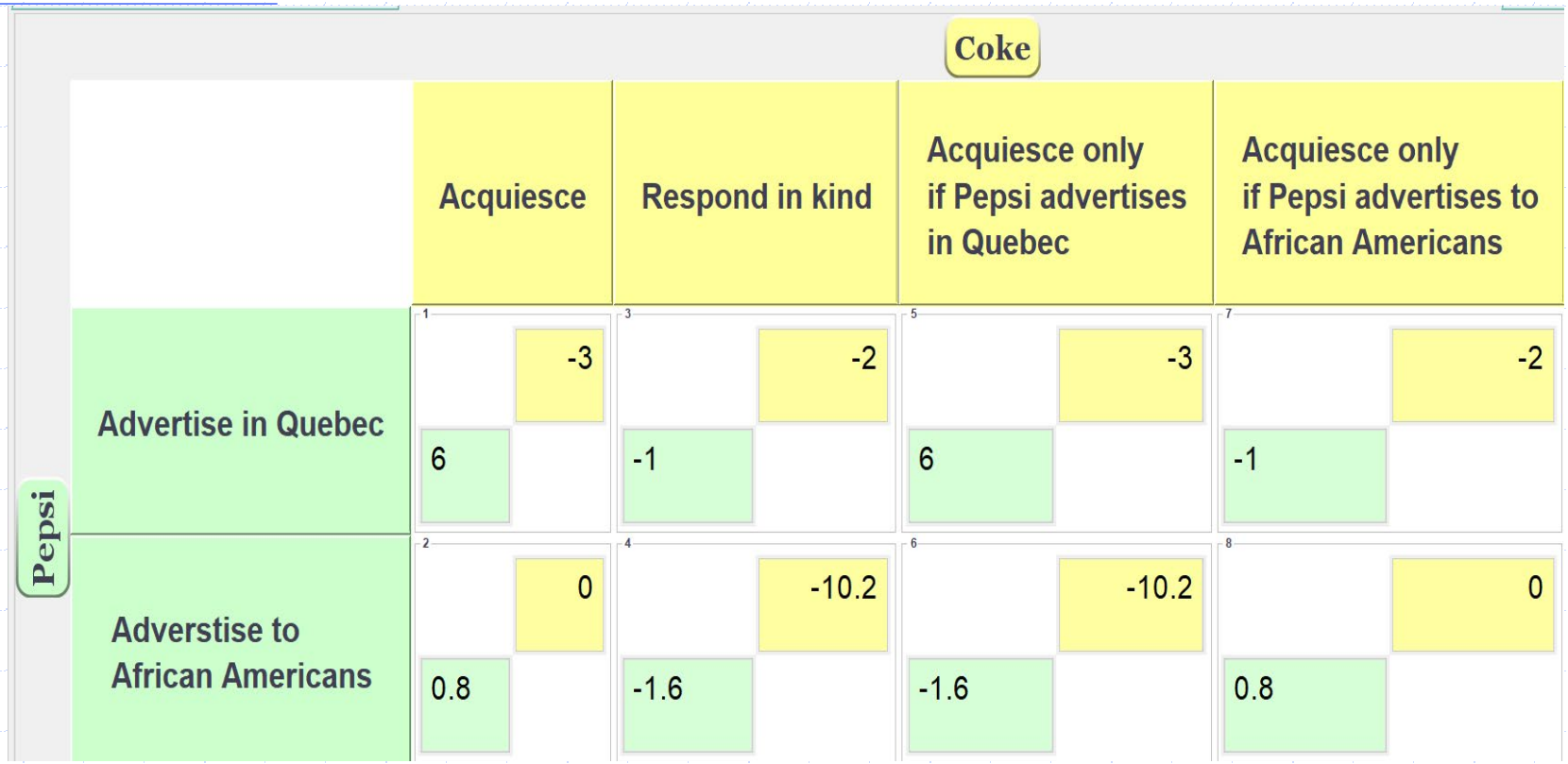
- For each strategy pair we calculate the expected payoff to both of the players.

- First, suppose Pepsi advertises in Quebec and Coke always acquiesces. That is they both choose their first strategy. Then the payoff to:
  1. Pepsi is 6
  2. Coke is -3



- Second, suppose Pepsi advertises to African Americans and Coke always acquiesces. Then the expected payoff to:
  1. Pepsi is 6*0.8 - 20*0.2 = 4.8 – 4 = 0.8
  2. Coke is 0*0.8 - 0*0.2 = 0

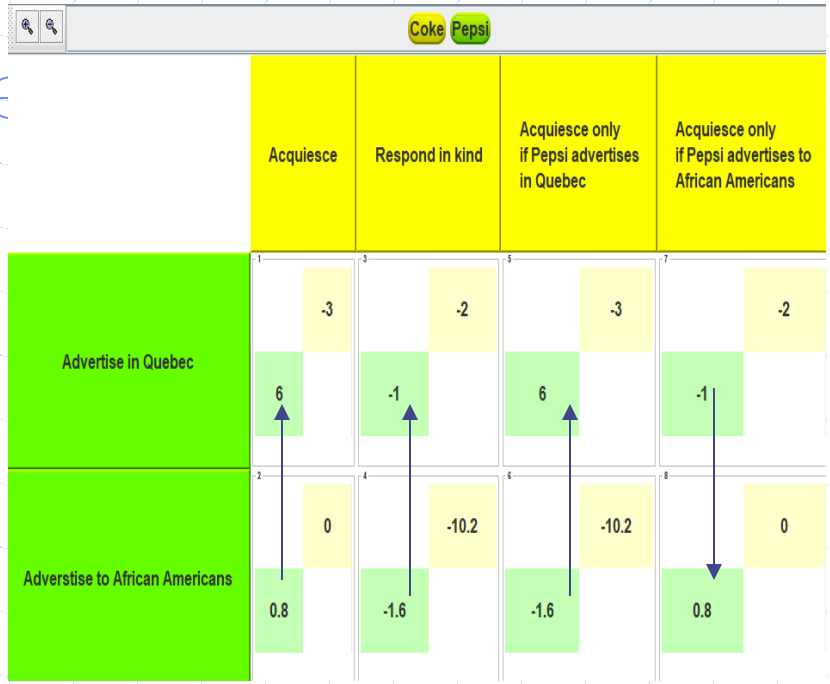
The Strategic Form Illustrated

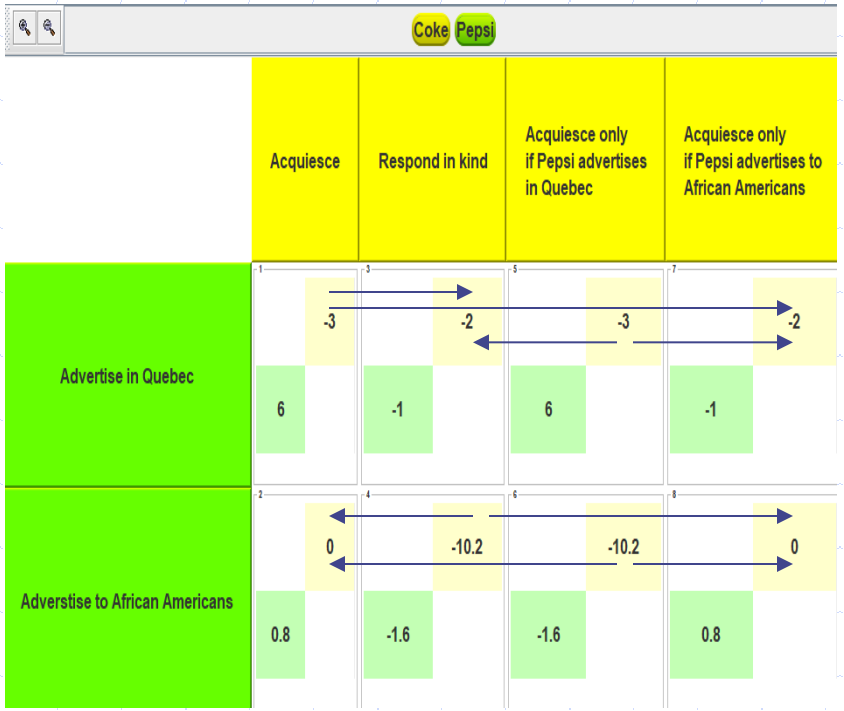


Pepsi \ Coke	Acquiesce	Respond in kind	Acquiesce only if Pepsi advertises in Quebec	Acquiesce only if Pepsi advertises to African Americans
Advertise in Quebec	1: -3 / 6	3: -2 / -1	5: -3 / 6	7: -2 / -1
Adverstise to African Americans	2: 0 / 0.8	4: -10.2 / -1.6	6: -10.2 / -1.6	8: 0 / 0.8

- The bi-matrix is created by forming rows from Pepsi's strategies, columns from Coke's strategies and filling in the expected payoffs in the cell corresponding to each strategy pair.

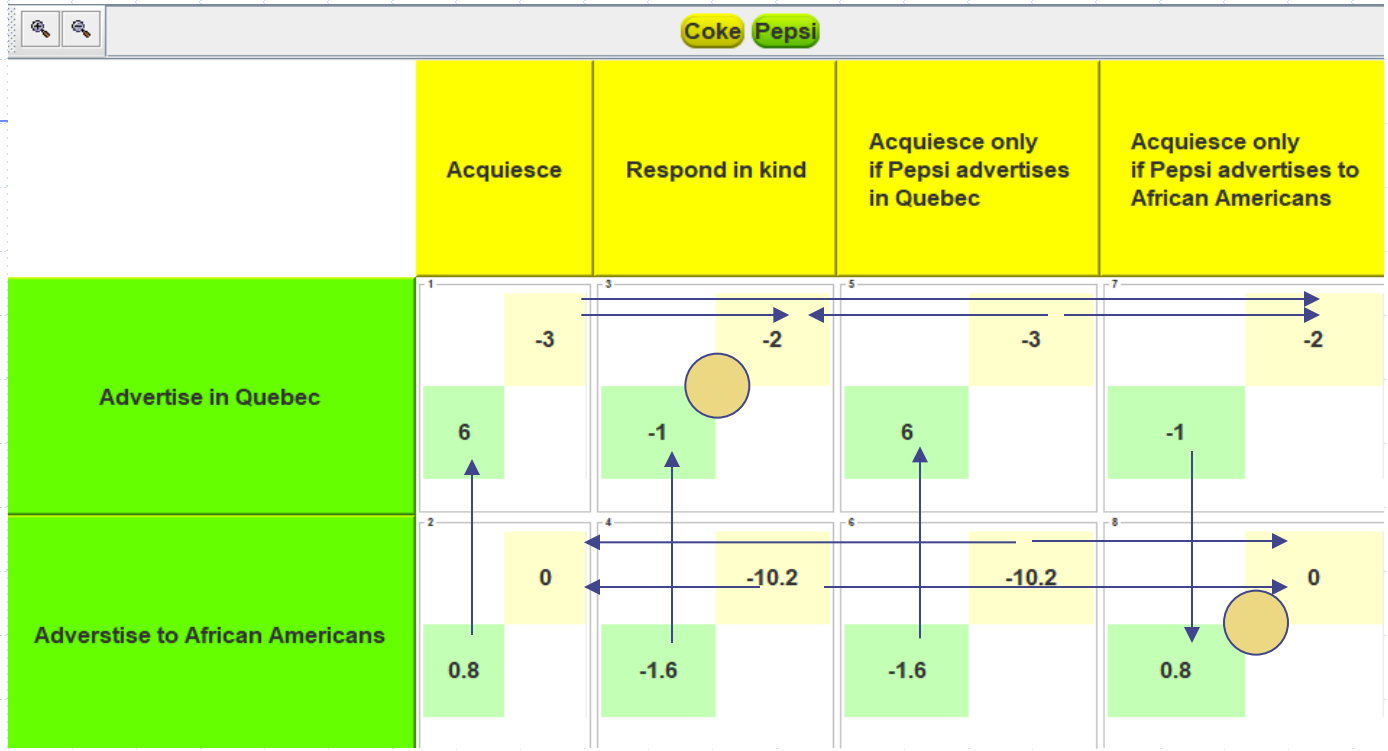
Solutions to the Strategic Form





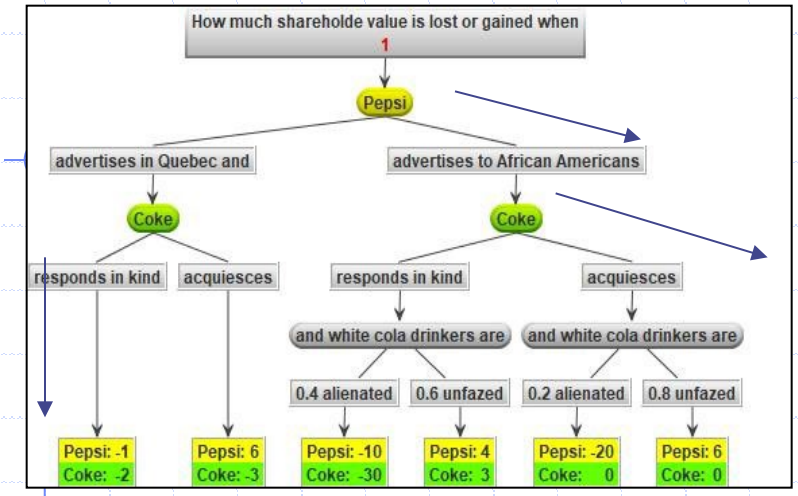
- To find the Nash equilibrium(s), draw best response arrows:
  1. vertical for the row player (Pepsi)
  2. horizontal for the column player (Coke).

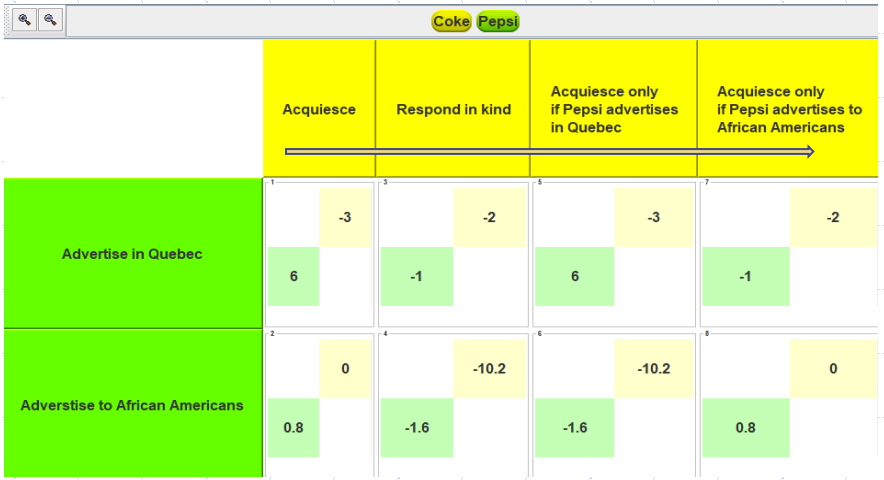
Solutions to the Strategic Form



- A Nash equilibrium is a cell with no arrows leading out.
- In those cells each player's strategy is a best response.
- Here there are two such cells, indicated by the circles.
- There are two Nash equilibria in this game!

One of the Nash equilibrium is implausible





- Suppose Pepsi "advertises to African Americans".
- Why would Coke "respond in kind"? This is implausible.
- For if Coke plays the far right strategy, it cannot lose!
- For this reason Coke's fourth strategy is called a (weakly) dominant strategy.
- The plausible Nash equilibrium is called subgame perfect and exactly corresponds to the solution found by applying backwards induction to the extensive form!

Comparing the solutions of the strategic and extensive forms

- Coke playing the far right strategy, acquiescing only if Pepsi advertises to African Americans (the fourth strategy), guarantees itself the highest payoff regardless of what Pepsi does.
- For this reason Coke's fourth strategy is called a (weakly) dominant strategy.
- If Coke plays this strategy, then the best strategy of Pepsi is to advertise to African Americans.
- The plausible Nash equilibrium is called subgame perfect and exactly corresponds to the solution we found by applying the backwards induction to the extensive form!

Deriving the strategic form of a game from its extensive form

- The cola wars case study shows it is useful to know how to solve games using both the strategic and extensive forms.

- There are essentially three steps to deriving the strategic form from the extensive form:
  1. List all the (pure) strategies for every player.
  2. Using the first step list all the strategy profiles (a strategy for each player) formed from all possible permutations of the strategies.
  3. For each strategy profile compute the expected payoff for every player.

- In the second step, if for example, player n =1,2,3 has n+1 strategies in a 3 player game, then there are:

  2*3*4 = 24 strategy profiles.

But for solving games?

- Sometimes it easier to solve the game in its strategic form; at other times solving the extensive form is easier.

- The cola wars case study we already hinted that:
  1. in the simultaneous move game it seems easier to solve the game when presented in its strategic form.
  2. when Coke could observe Pepsi's move before making its own choice (in what is called a perfect information game), solving the game in its extensive form looked straightforward.

- Note that NE are defined with respect to the strategic form, not the extensive form.

- The bottom line: *use the extensive to solve those parts of the game that have no dotted lines, and solve using the strategic form when dotted lines complicate matters*.