# Lecture 2
# The Extensive Form

This lecture introduces a second way of representing strategic situations, using the extensive form. It has a tree structure to illustrate whose turn it is to move, with dotted lines to show what they know at the time, and payoffs at the bottom so players can anticipate what they will receive. We develop this paradigm with an example.

Key words and phrases:

players, actions, information, payoffs.

# The strategic versus the extensive form

- In the first lecture we defined a game using its strategic form. It has three elements:
  1. Players . . . . Who is involved?
  2. Strategies . . . . A complete list of moves in all possible situations
  3. Expected payoffs . . . . Expected value of the outcome contingent on the strategic choice every player makes

- A more comprehensive way of representing a game is to use the extensive form. It has four elements:

  1. Players . . . . Who is involved?
  2. Moves . . . . What can they do at different points?
  3. Information . . . . How much do they know when they move?
  4. Payoffs . . . . How do they value the outcomes?

# Analyzing strategic interactions begins with the extensive form

- The extensive form of a game is more intuitive than the strategic form.
- There are many examples of games with different extensive forms that have the same strategic form (but not the other way around).
- To study strategic interactions, it is sometimes more useful to start with the extensive form of a game rather than the strategic form.

# A cola war

- After struggling through the Great Depression of the 1930s Pepsi finds its soft drink sales are stalled in the 1940s.
- Coke is the industry leader, and its products command a premium price over Pepsi's.
- The country is at war, but remains segregated along racial lines, with blacks economically and socially disadvantaged.

# Players: who is involved?

- How many major players are there, and whose decisions we should model explicitly?
- Can we consolidate some of the players into a team because they pool their information and have common goals?
- Should we model the behavior of the minor players should be modeled directly as nature, using probabilities to capture their effects on the game?
- Does nature play any other role in resolving uncertainty, for example through a new technology that has chance of working?

# Who are the main players in this episode of the cola war?

- Pepsi shareholders
- Coke shareholders
- White cola demanders (treated as nature)
- Black cola demanders (treated as nature)

# Actions or moves: What can they do?

- Each node designates whose turn it is. It could be a player or nature. The initial node shows how the game starts, while terminal nodes end the game.
- A branch join two nodes to each other. Branches display the possible choices for the player who should move, and also the possible random outcomes of nature's moves.
- Tracing a path from the initial node to a terminal node is called a history. A history is uniquely identified by its terminal node.

# What are the choices facing the main players in the cola war?

- Pepsi could:
  1. target its product line to African American consumers
  2. pursue another strategy, such as expanding its operations in Canada.

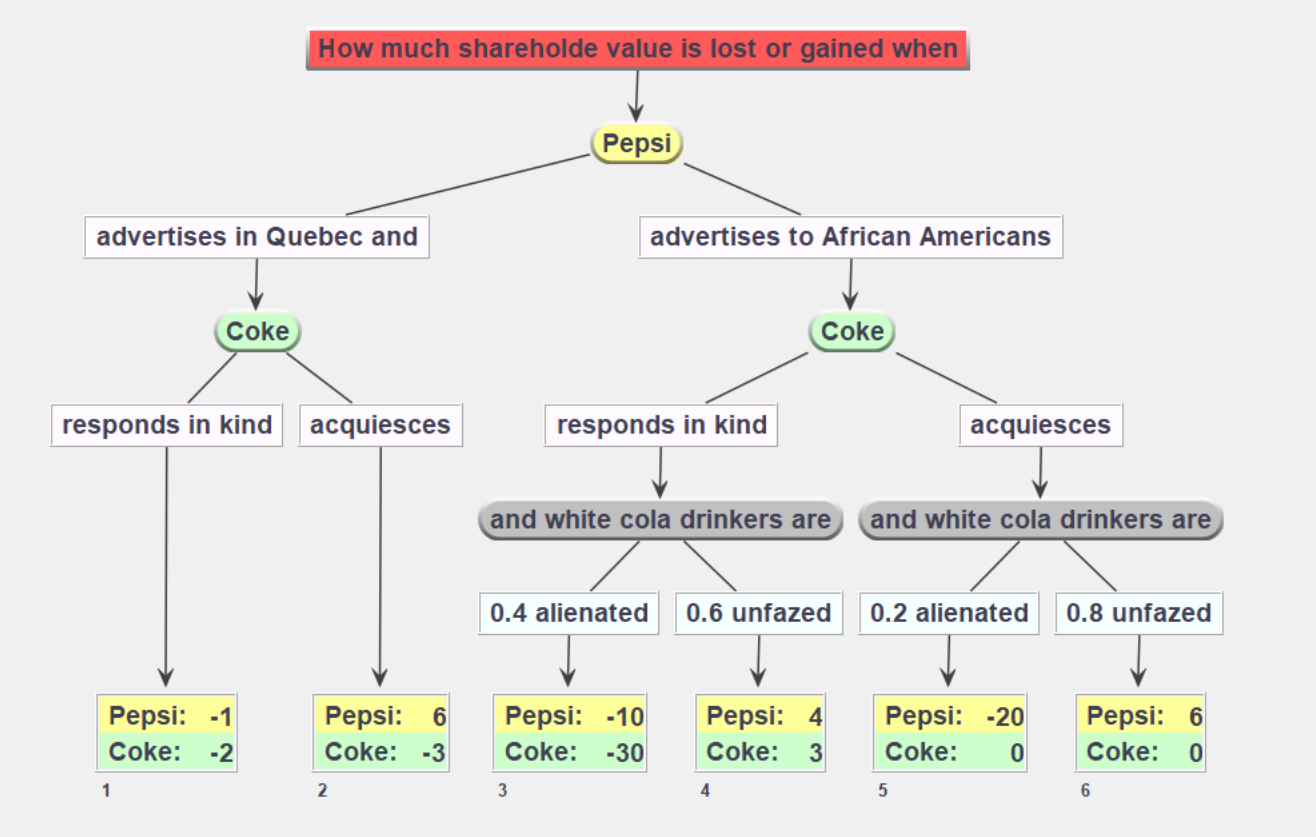
- Coke could :
  1. respond aggressively to any marketing initiative taken by Pepsi
  2. respond passively to any marketing initiative taken by Pepsi.

# Valuing outcomes: What are the payoffs?

- Payoffs capture the consequences of playing a game.
- They represent the utility or net benefit to each player from a game ending at any given terminal node.
- Payoffs show how resources are allocated to all the players contingent on a terminal node being reached.

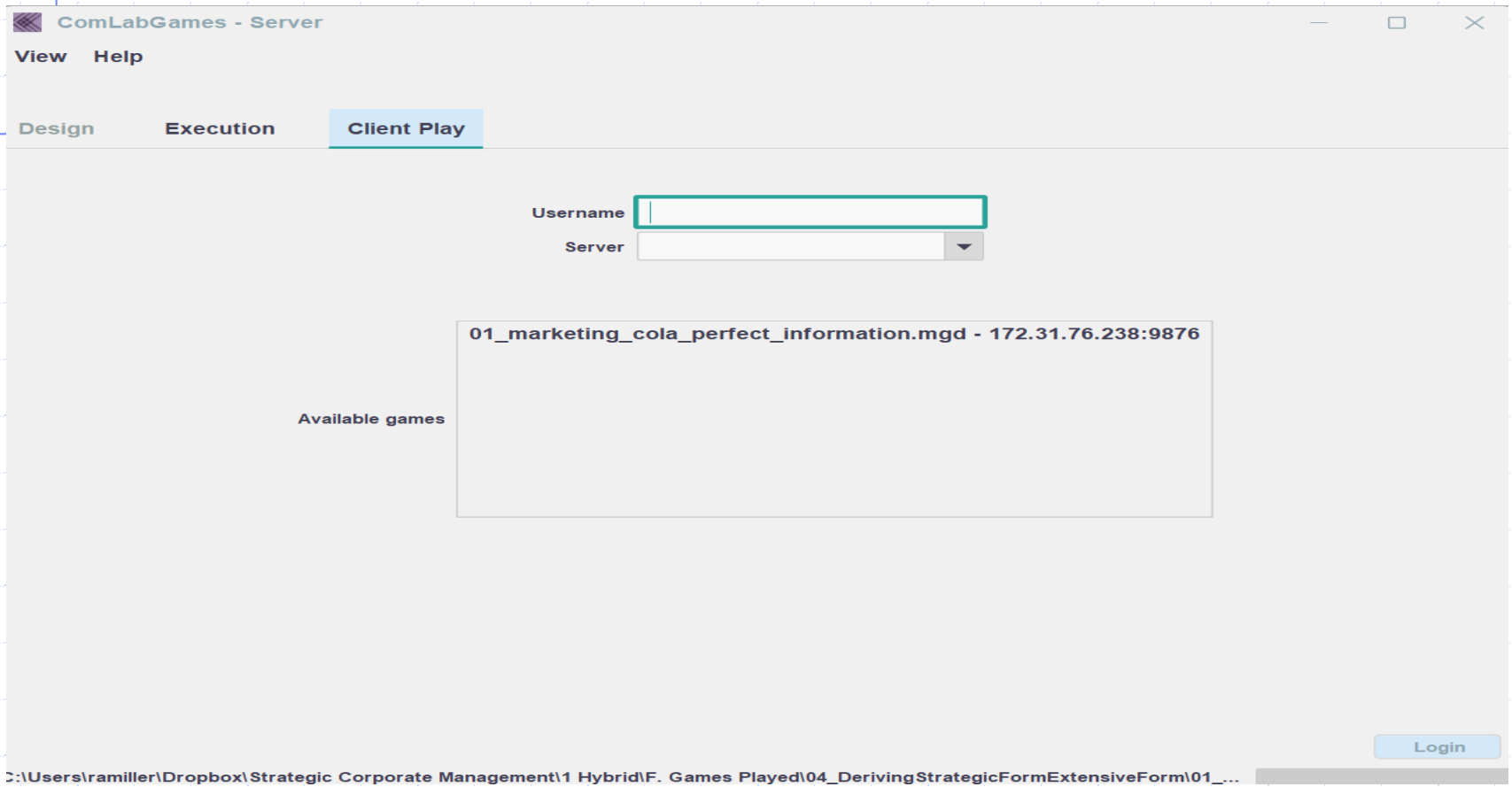
# The extensive form of the game when Pepsi is a first mover

- The diagram depicts the extensive form of the Coke-Pepsi game when Pepsi move first:



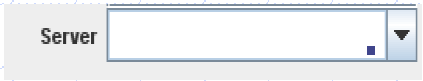
# Login instructions

1. Click on Client Play
2. Click on Available games: “01_marketing_cola_perfect…”

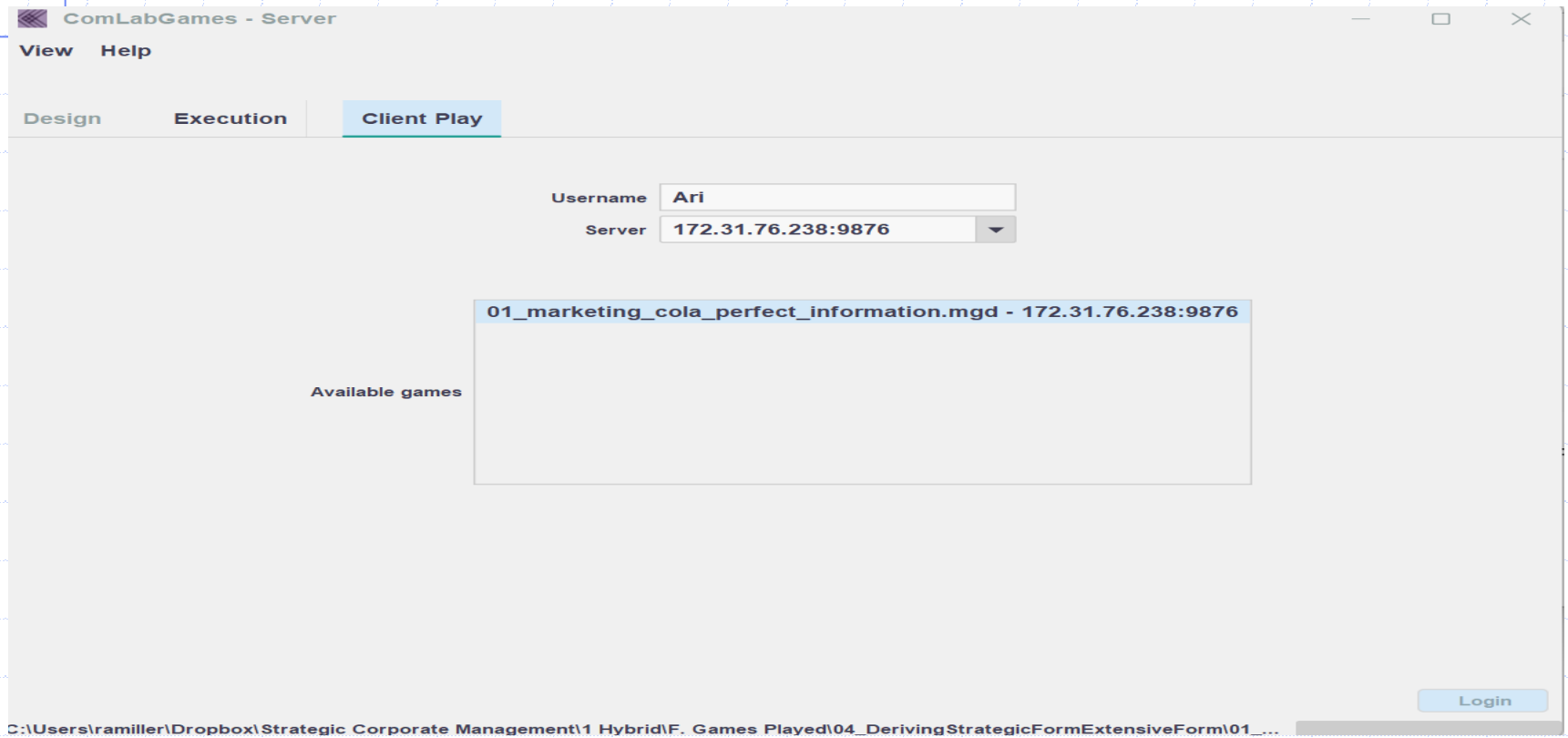


# Continuing login instructions

1. Server address that starts with: 172…. will appear automatically after selecting "01_marketing…". If not type 172.31.76.238 in

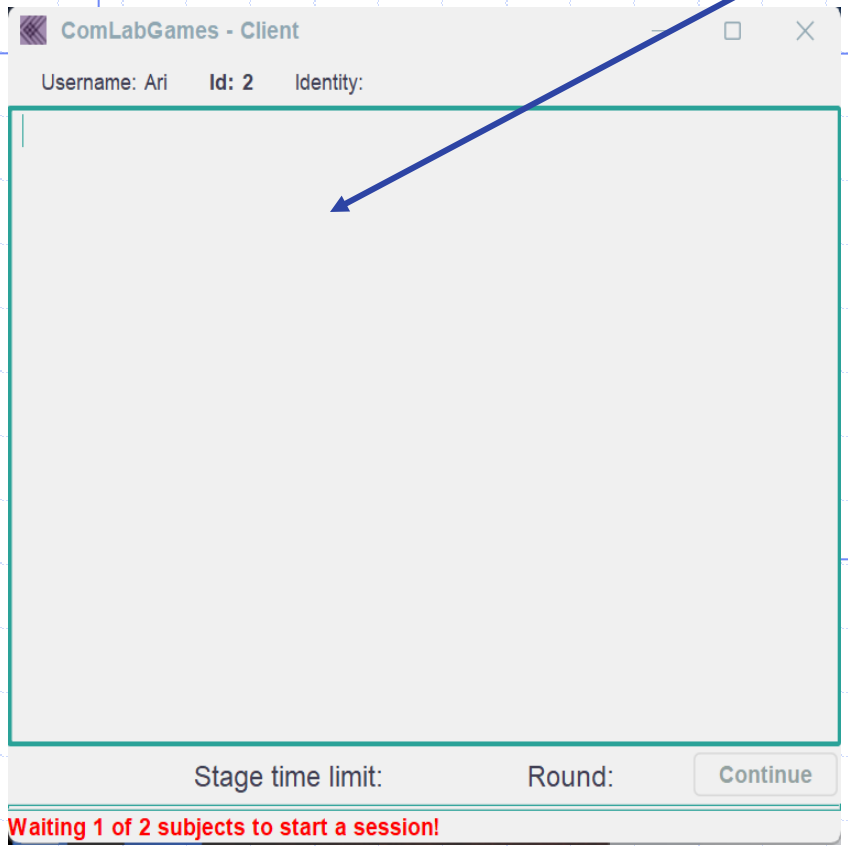


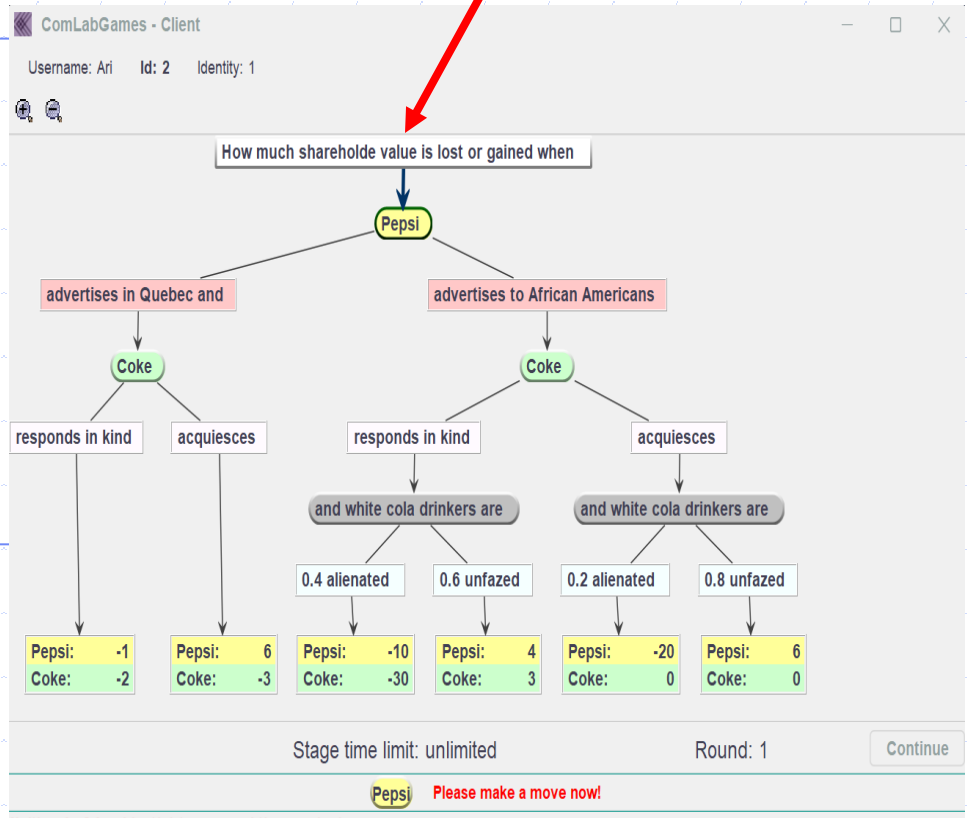

2. Write a username.
3. Click on "Login".



# Game window

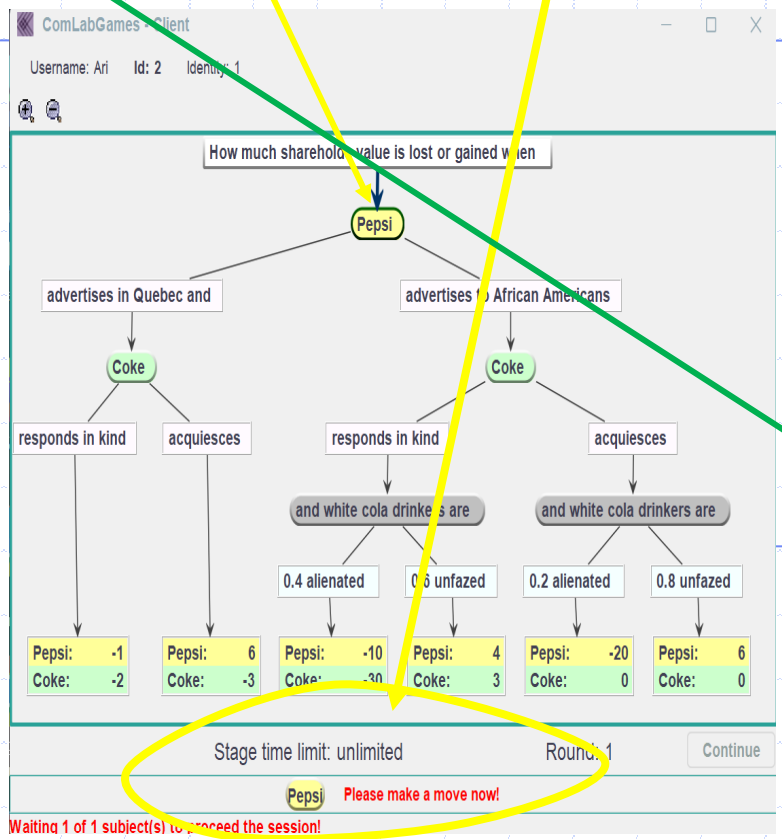
1. Two subjects need to login to start the marketing cola game. The first subject to login will see a blank screen with username: Username: Ari
2. When required number of subjects login the game replaces the blank window.

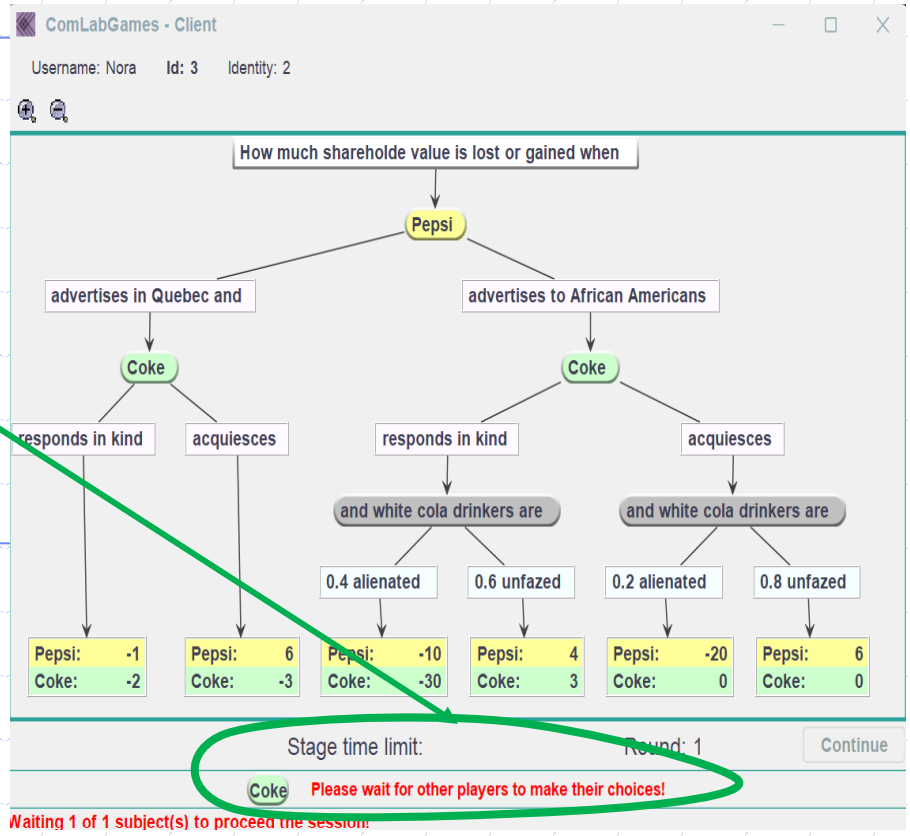


# Viewing marketing Cola game

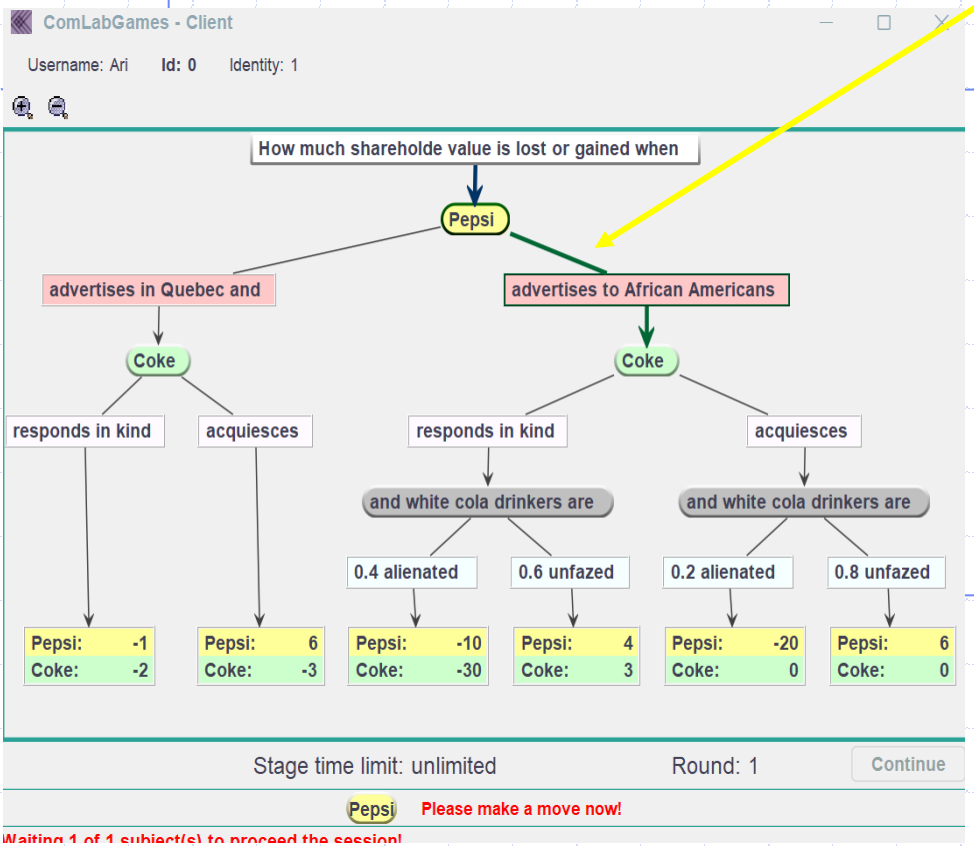
1. In this game Pepsi moves first. Pepsi node is circled in blue to indicate the first move: between "advertises in Quebec" and "advertises to African American". His possible choices are blinking between pink and white.
2. Coke has to wait for Pepsi to move first in this game.

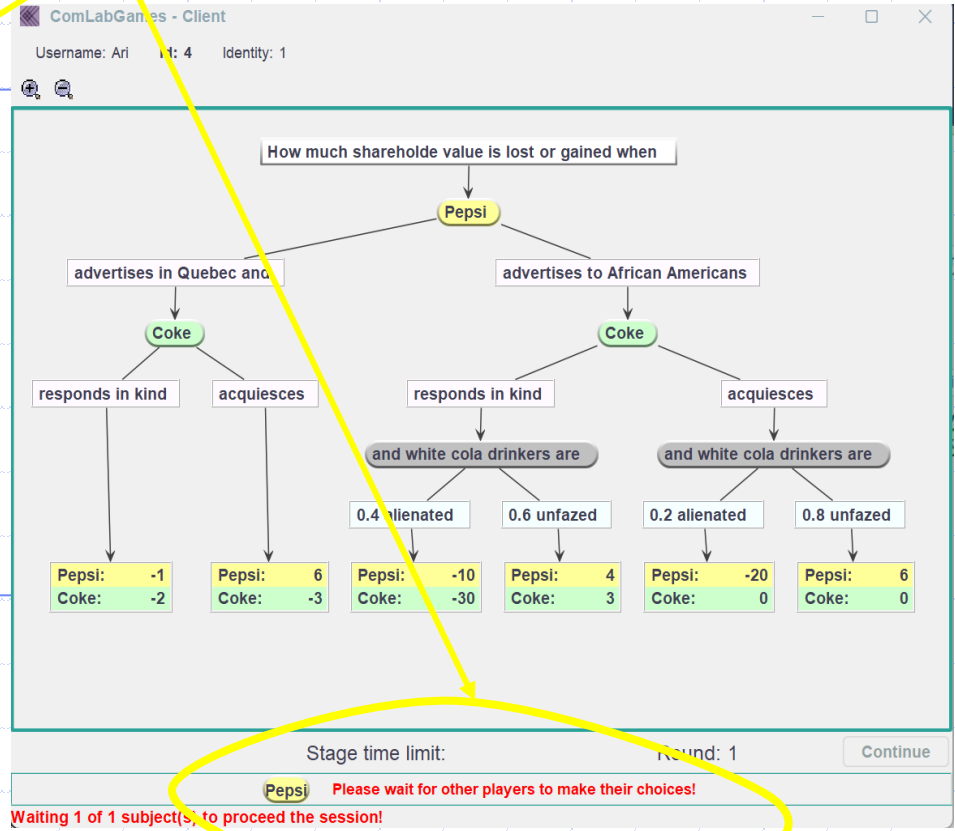


# Pepsi moves

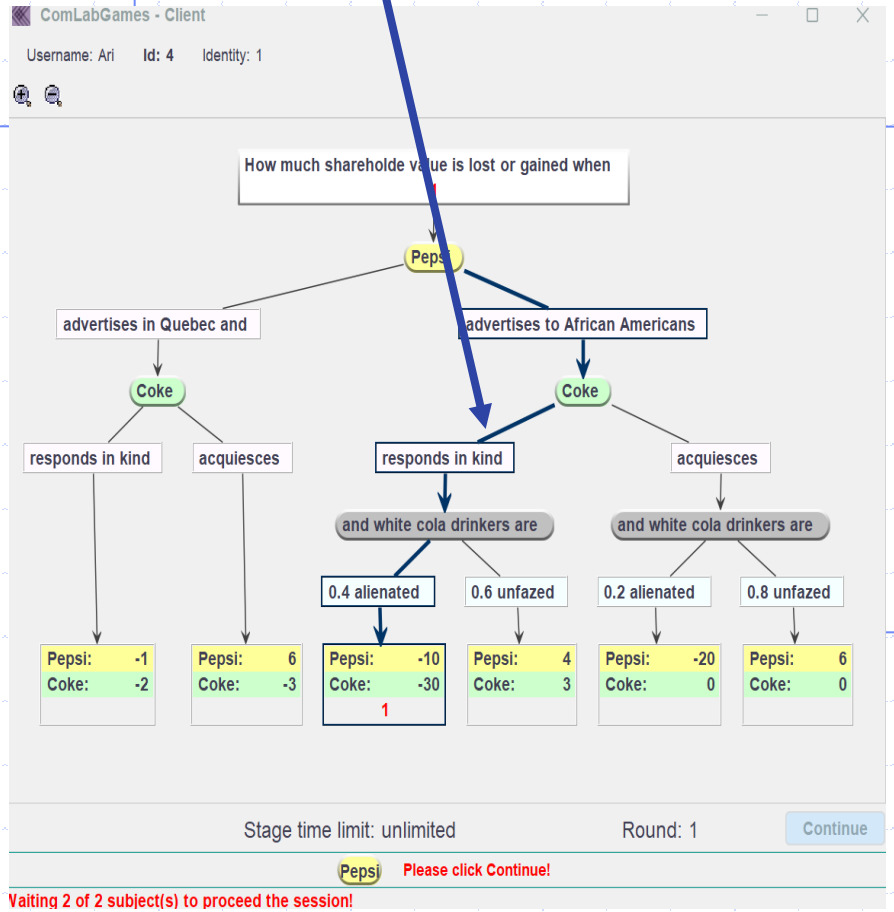
1. To select a move a cursor (mouse) to the choice and click on that choice. advertises to African Americans was selected.
2. Once the selection is made Pepsi waits for Coke's decision.

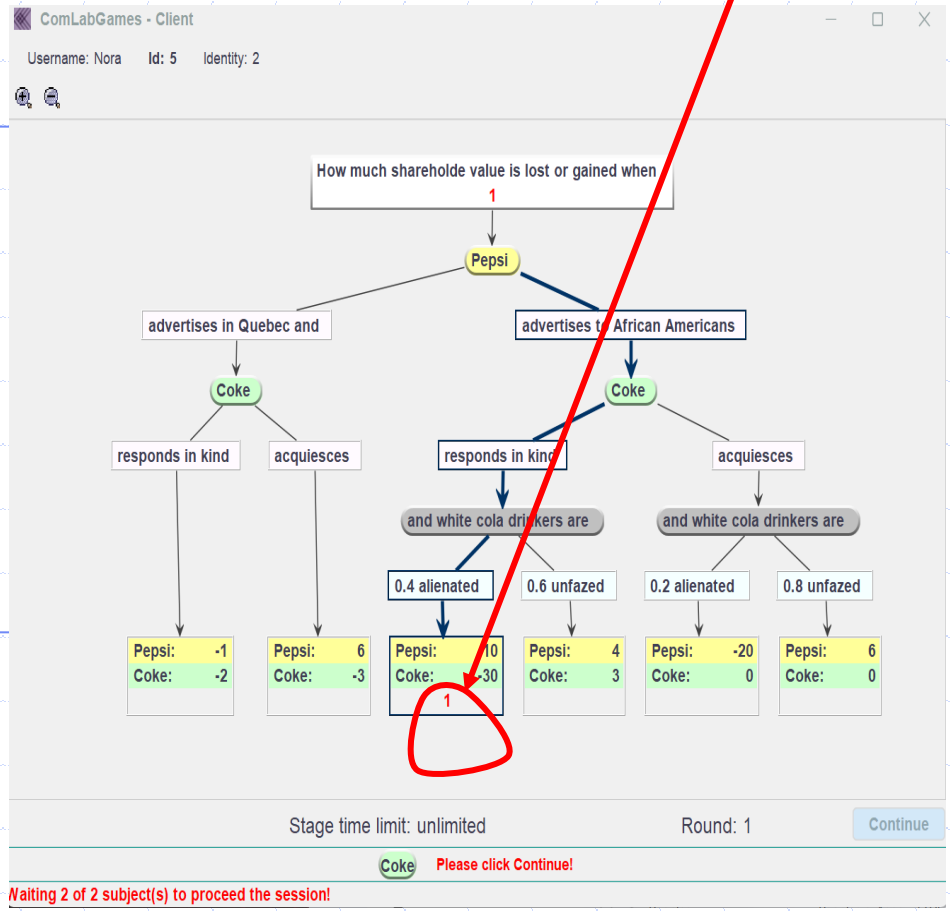


# Game outcome

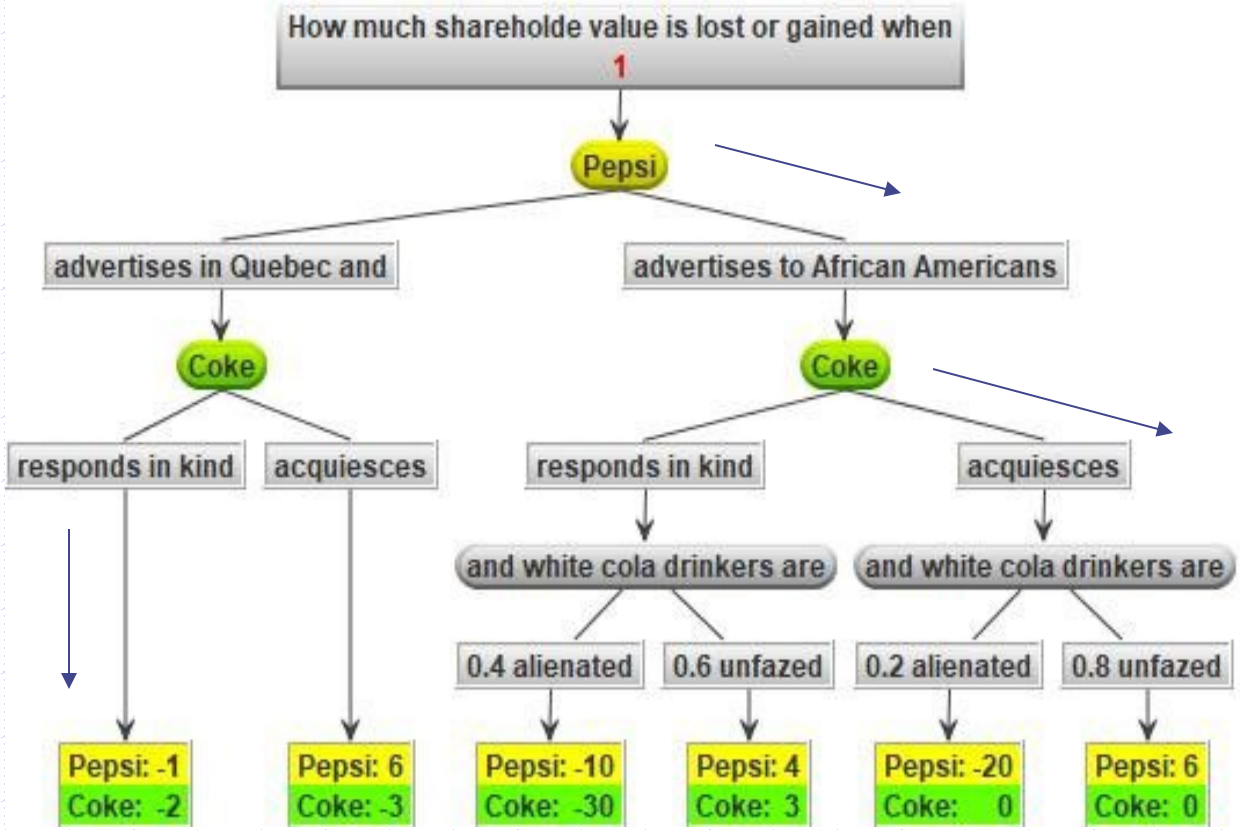
1. Both players observe the selection they made.
2. The path is highlighted in blue.
3. The node selected is circled in blue and the counter in red indicates the number of times the node was selected.



# Solving the game when Pepsi moves first

- The arrows show what each player will do:



# Simultaneous move games

- In many situations, you must decide all your moves without knowing what your rivals are doing, and their situations are similar to yours.
- Even if the moves are not literally taking place at the same moment, but all are made before anybody can react, the moves are effectively simultaneous.
- A game where no player can make a choice that depends on the moves of the other players is called a simultaneous move game.
- In simultaneous move games the strategic and extensive forms essentially represent the same details.

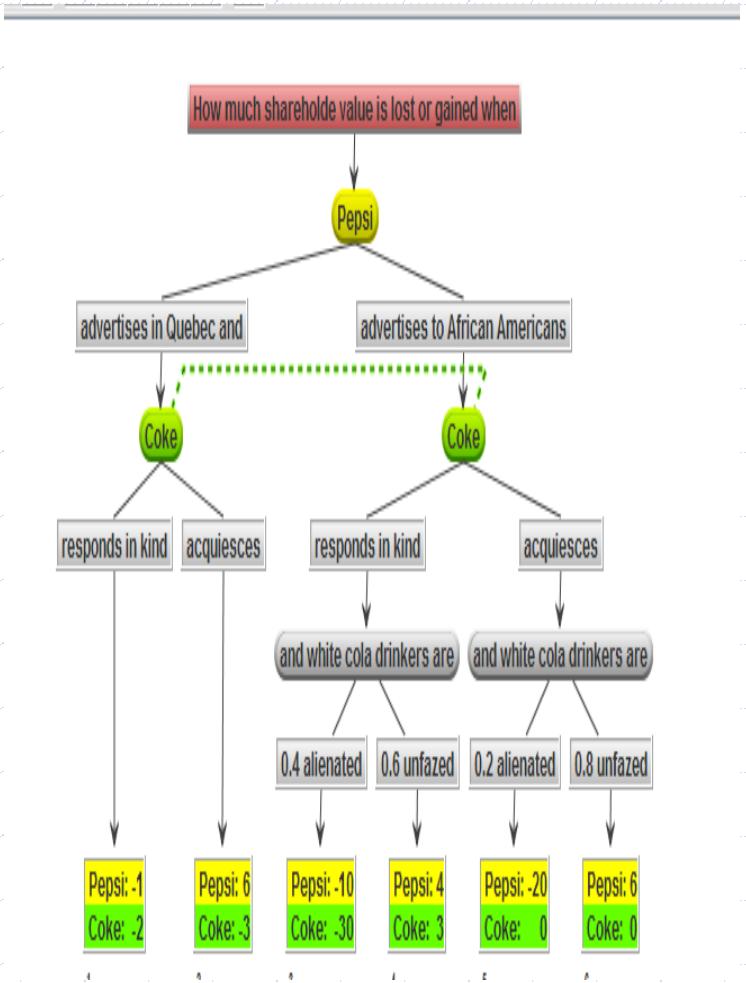
# Information: What do they know?

- Each non-terminal decision node is associated with an information set.
- If a decision node is not connected to a dotted line, the player assigned to the node knows the partial history.
- If two nodes are joined by a dotted line, they belong to the same information set, and the two sets of branches defining the player's choice set, must be identical.
- A player cannot distinguish between partial histories leading to nodes that belong to the same information set.

# Changing the information available to Coke

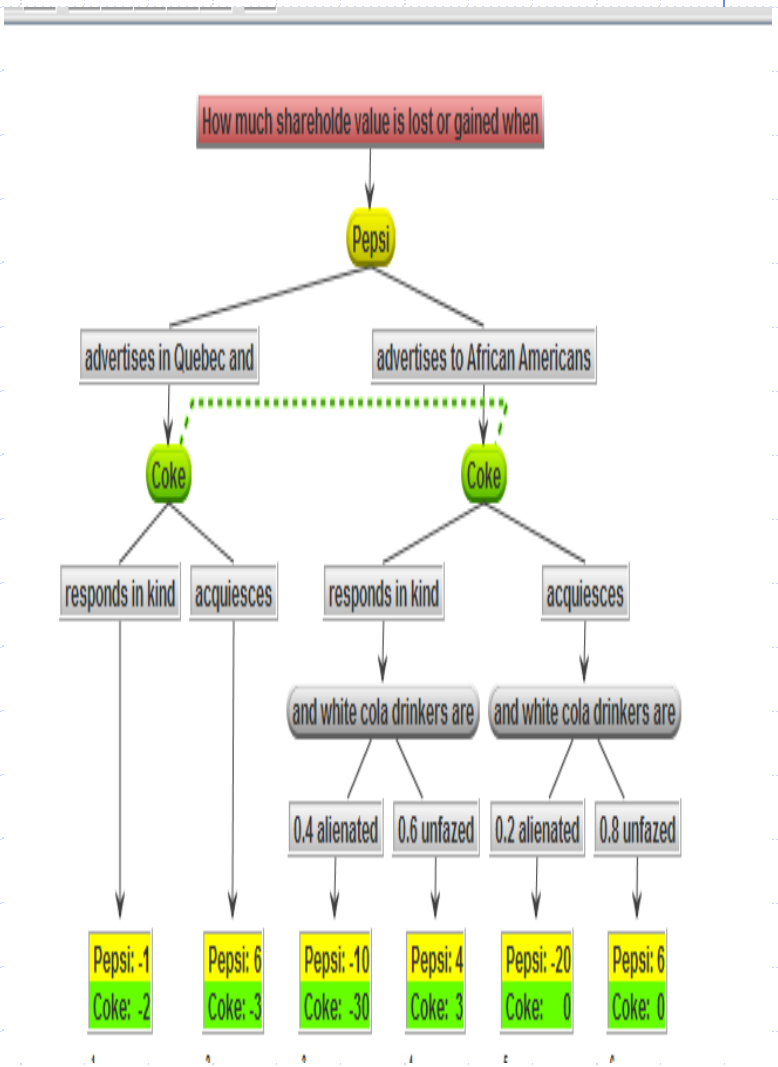
- Drawing a dotted line connecting the two decision nodes for Coke, is the way to show that Coke cannot see which market Pepsi enters, before Coke chooses whether to acquiesce or not.

- Nodes connected by dotted lines belong to the same information set.



# Illustrating the extensive form when Pepsi is not a first mover

- If Pepsi is not a first mover, then Coke does not know what Pepsi does as it makes its own choice.
- This is called a simultaneous move game because neither player can see what the other has done before moving.
- The extensive form for this new game cannot be solved using backwards induction.



# The strategies for Pepsi and Coke in this simultaneous move game

- As in the perfect information game Pepsi has just two strategies:
  1. Advertise in Quebec.
  2. Advertise to African Americans.

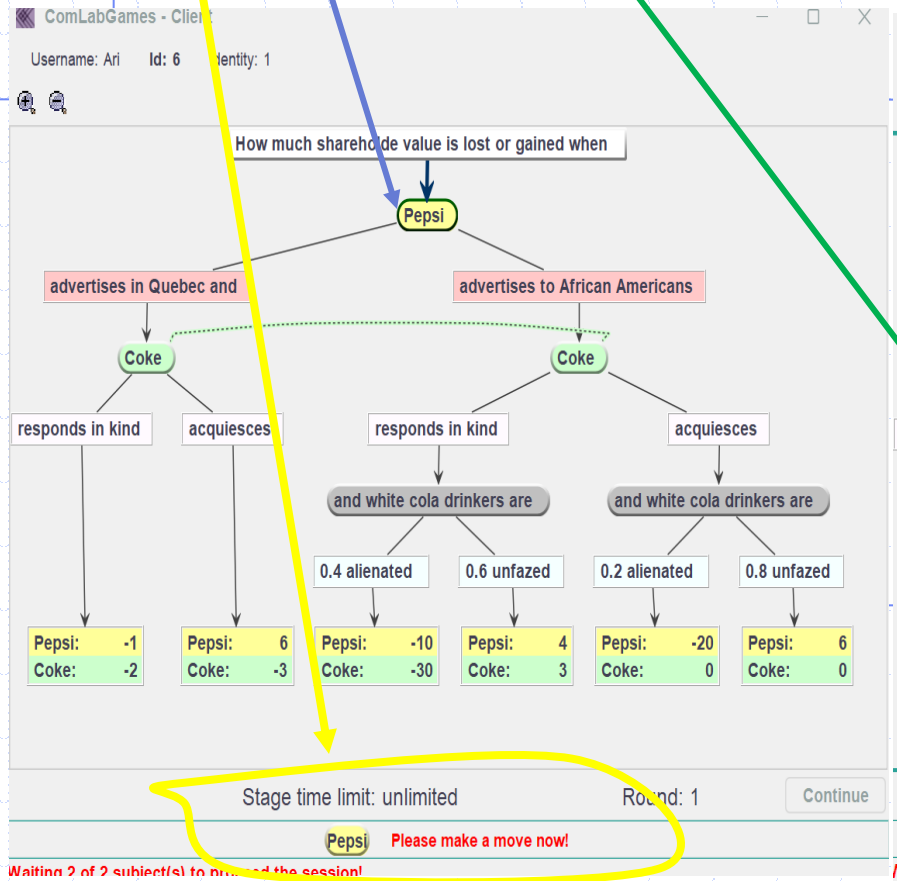
- In contrast to the perfect information game Coke cannot condition on what Pepsi has done in this game.

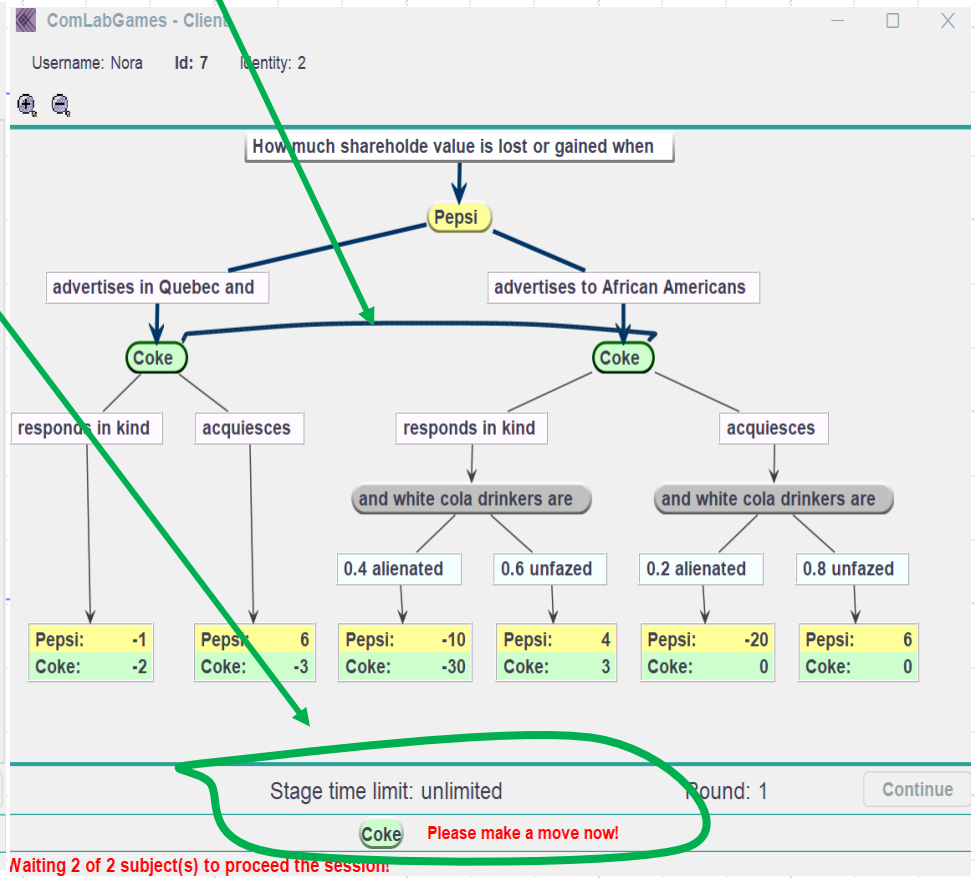
- It is as if they moved simultaneously: hence the name.

- Thus Coke also only has two strategies:
  1. Respond in kind.
  2. Acquiesce.

# Viewing the simultaneous move Cola game

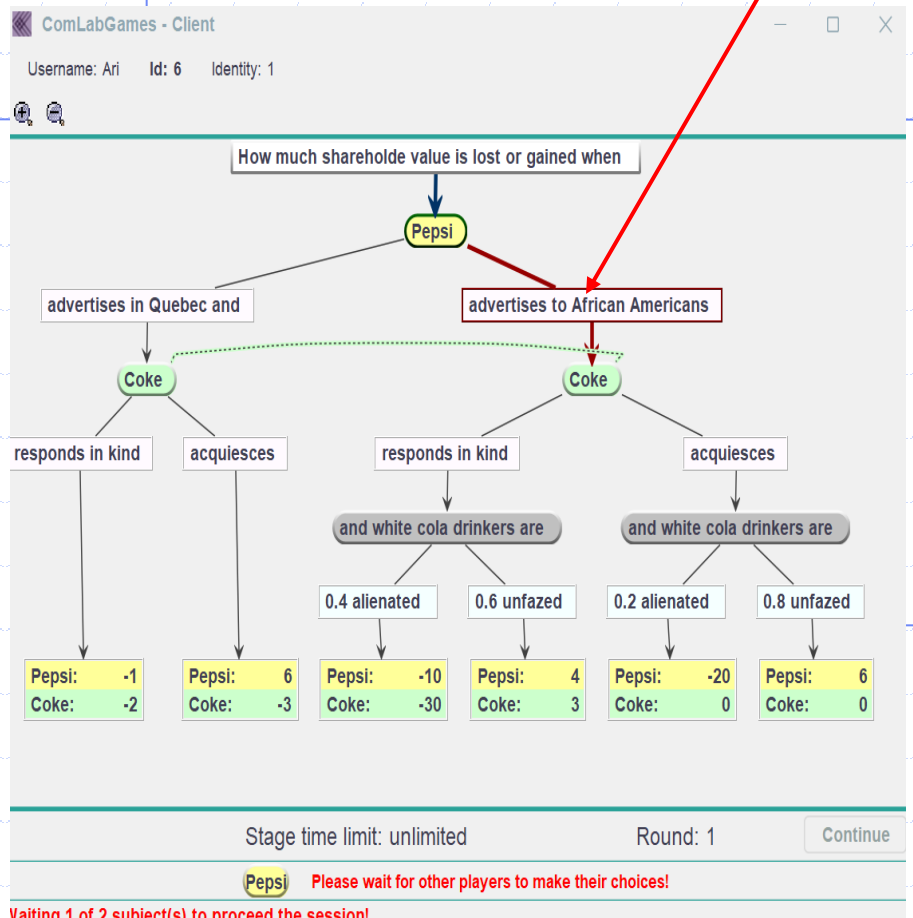
1. Pepsi and Coke can move at the same time.
2. Pepsi decision node is circled in blue and Coke nodes are circled in blue. Coke does not know Pepsi's decision when she makes a choice (dotted lines which are colored in blue.

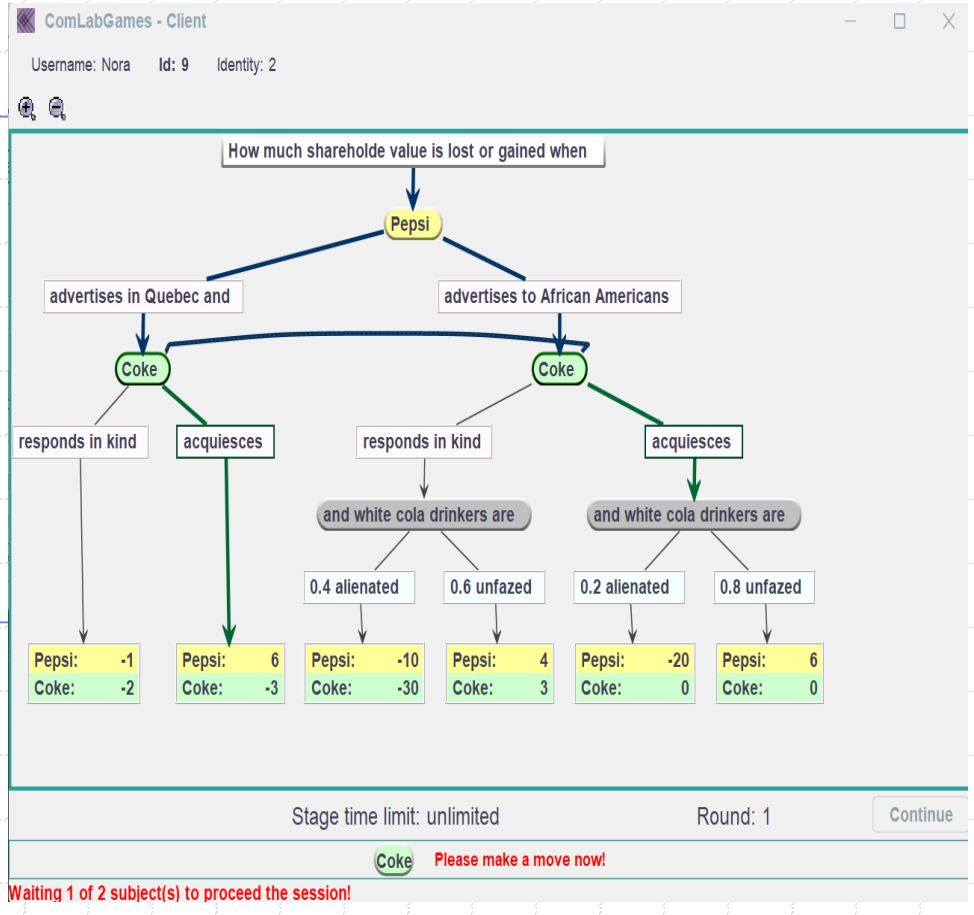


# Coke and Pepsi move

1. To select a strategy move the cursor to the choice and click on that choice. advertises to African America... was selected by Pepsi.
2. Coke does not know what Pepsi selected and Coke selected "acquiesces" in this example.

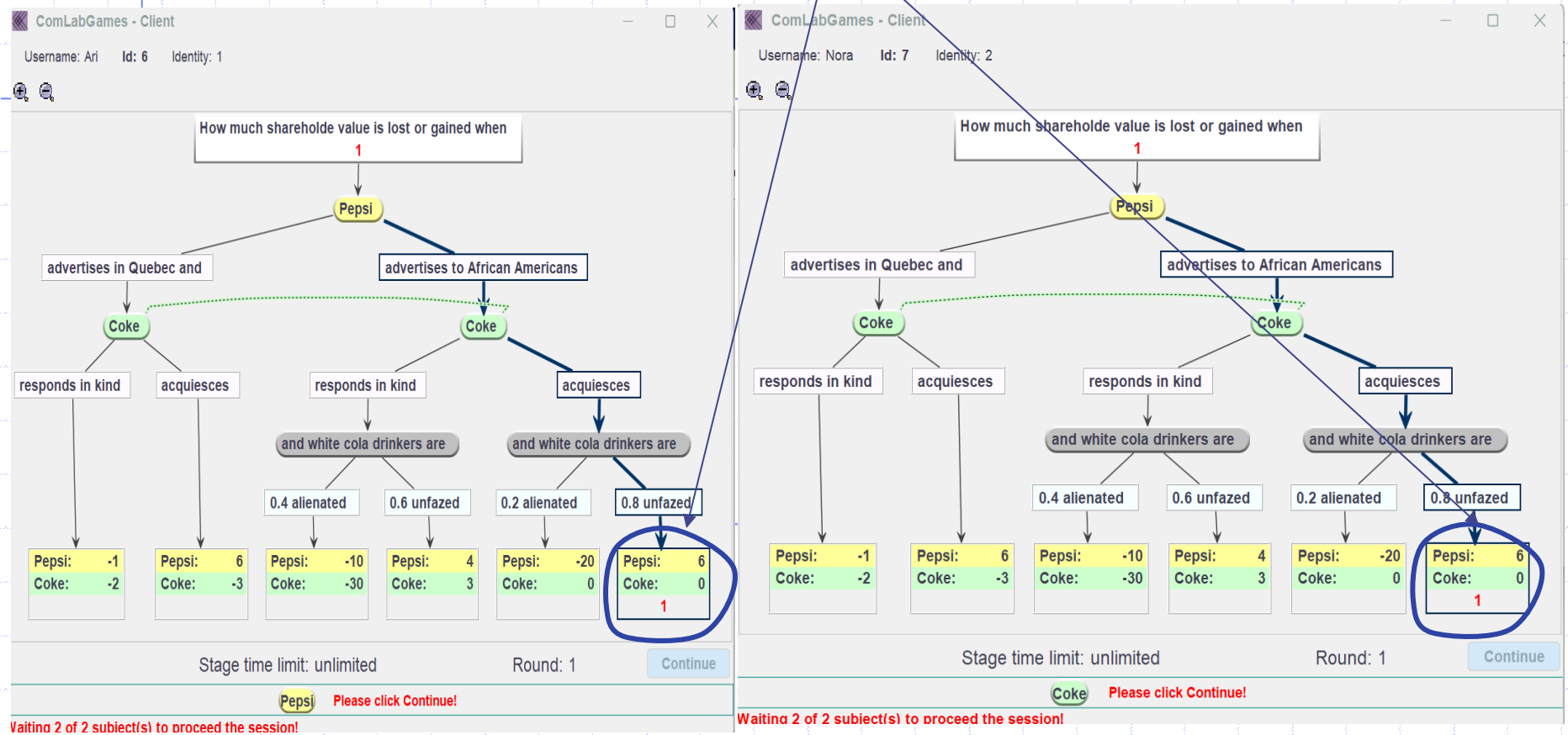


# Outcome of the game

Both players observe their selection:

2. The path is highlighted in blue.
3. The node selected is circled in blue and the counter in red indicates the number of times the node was selected.



# A bi-matrix representation of the simultaneous move game

- Form a 2 by 2 matrix with:
  - column headings for Coke's strategies
  - row headings for Pepsi's strategies.


- In each cell corresponding to a strategy pair, write the expected payoffs. For example:
  - Since Pepsi gets 6 by advertising in Quebec if Coke acquiesces, and Coke gets -3 in that case, the top left cell is filled in with (6,-3).
  - Similarly in the bottom right cell note that in this case:

    0.6 * 4 - 0.4 * 10 = 2.4 – 4 = -1.6 (Pepsi's expected payoff)

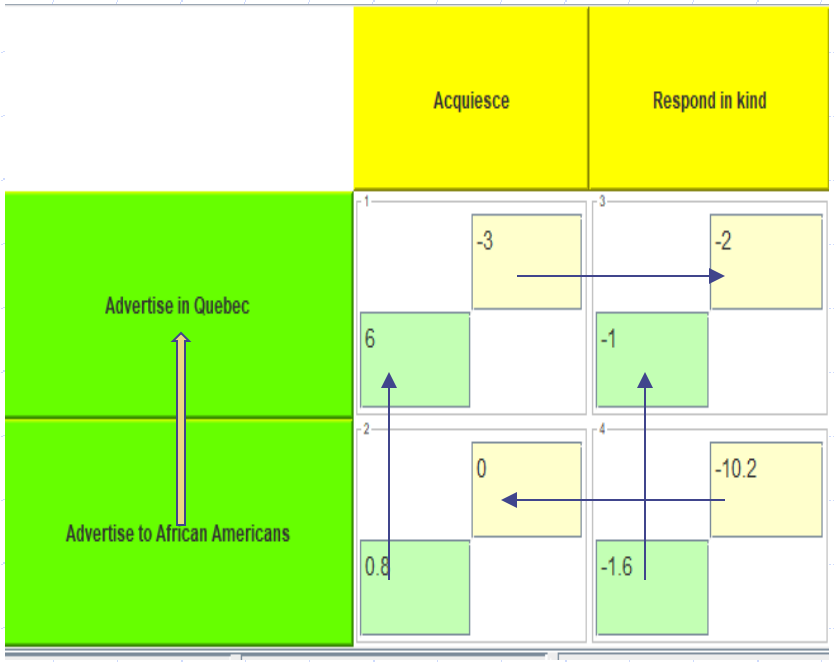
    0.6 * 3 - 0.4 * 30 = 1.8 – 12 = -10.2 (Coke's expected payoff)

# Solving the simultaneous move game

- Regardless of which Coke picks the best strategy for Pepsi is to advertise in Quebec because:

  6 > 0.8 and -1 > -1.6.

- This is called a (strictly) dominant strategy.




- Coke can do this calculation just as easily as Pepsi.
- Consequently Coke anticipates Pepsi will advertise in Quebec.
- Coke replies with "respond in kind" because: -2 > -3.

# Comparing the solutions

- This is exactly the opposite to the solution we obtained for the perfect information game.
- Pepsi increases its payoff by 1.8 from a loss to a profit if it can convince Coke where it has advertised before Coke makes a decision. This is called a first mover advantage.
- More surprising is that Coke also benefits from Pepsi having a first mover advantage: losing nothing in that case, but losing 2 if they move simultaneously.
- In this game when Pepsi communicates with Coke, both firms and African Americans benefit.