Session 1.1
Defining Nash Equilibrium

This session formally defines a game in strategic form and then defines a pure strategy. A Nash equilibrium is defined by a strategy for each player that is a best reply to all the other players.

Finite games in strategic form

- At the heart of the definition of a NE is the concept of a strategy.
- A strategy is a full set of instructions to a player, telling her how to move at any point she might reach in the game.
- Strategies are based on the player's information at the times she makes her moves: they cannot depend on things she doesn't know yet.
- Strategies are exhaustive: they include directions about moves the player should make should she reach any of her assigned nodes.
- In this context the word finite indicates there are (only) a finite number of players, and each player only has a finite number of (pure) strategies.

Defining the strategic form

- Three elements define the strategic form of a game:

1. Players . . . Suppose there are N players, who we label 1,2, . . .,n , . . . N.

2. Strategies . . . Suppose player n has S_n strategies which we label s_n = 1,2, . . .,S_n. Also write s_{-n} for the strategy of every player apart from player n.

3. Payoffs . . . . In general the payoff of player n depends on his own strategy, as well as the strategy of every everybody else. We label the payoff to n as $u_n(s_n, s_{-n})$.

Pure strategy Nash equilibrium defined

- Consider an N player finite game, where s_n is the strategy of the n^{th} player where 16 n6 N.
- A strategy for everybody $s = (s_1 , ... , s_N)$ is called a strategic profile.
- Now ask whether s_n is the best reply of n to s_{-n} for all n? If so, nobody has an incentive to unilaterally deviate from their assigned strategy.
- In terms of the notation we have developed, suppose $\rho = (\rho_1 , ... , \rho_N)$ is a strategic profile.
- Then ρ is a pure strategy Nash equilibrium if and only if for each n =1,2, . . . N and every s_n:

$$u_n(\rho_n, \rho_{-n}) \geq u_n(s_n, \rho_{-n}).$$

Some history

- Examples of NE were developed almost 200 years ago, by Antoine Cournot (1801 – 1877), Joseph Bertrand (1882 – 1900), and Nobel laureate John Nash (1928 – 2015).

- More recently, several other economists have won Nobel prizes for their contributions that invent, refine, enhance, and extend the NE concept, including:
  - John Haysani, Bengt Holmstrom, Paul Milgrom, James Mirrlees, Roger Myerson, Reinhard Selten, William Vickrey, Robert Wilson

- NE and its refinements are now the most widely accepted tools in economics for predicting strategic interaction.