Nash Equilibrium

The next two chapters complete our discussion about solving simultaneous move games. We begin Chapter 9 with several examples showing that the principles of dominance, weak dominance, and iterative dominance, do not necessarily isolate a unique strategy for each player. Indeed applying these principles might not reduce the set of strategies at all! In such cases stronger behavioral assumptions are required for making predictions. We now assume that each player optimizes her own criterion function, taking as given her strategic environment, which includes the probabilities associated with uncertain events and the choices of the other players. More formally, we define a best reply as a strategy that maximizes a player's expected utility given the strategies selected by all the other players. A Nash equilibrium has the defining property every player selects a best reply.

This definition implies that the solution to a dominance solvable game is a Nash equilibrium. However we establish by example that the Nash equilibrium is not always unique in this case. In another example we present, the first three principles have no predictive power, but there is a unique Nash equilibrium. These two examples demonstrate that the set of Nash equilibrium strategies is not always a subset of those strategies that remain after applying the three principles of dominance, nor vice versa. What is known in general about the existence and multiplicity of Nash equilibrium?

The question in answered in Chapter 10. We start by distinguishing between pure and mixed strategies. Selecting a pure strategy amounts to making a deterministic choice. A mixed strategy is defined by a probability distribution over all the strategies, where strictly positive probabilities are placed on at least two of them. Playing a mixed strategy means randomly selecting a strategy according to the defined probability distribution. Chapter 10 shows there are plausible reasons for playing mixed strategies in some games. The general result is that not all games have Nash equilibrium in pure strategies, but every finite game has at least one Nash equilibrium in pure or mixed strategies.

The principles for deriving the solutions to simultaneous games can be ranked by the degree of sophistication required of players, from the most plausible and least demanding, to the least convincing and most complex. The order is dominance, weak dominance, iterative dominance, and Nash equilibrium. This naturally raises questions about whether the theoretical notions of complexity are reflected in experimental outcomes. Does experimental evidence confirm solutions to dominance solvable games more frequently than solutions to games requiring higher ordered principles? And in a mixed strategy equilibrium, are the sample moments of the experimental outcomes comparable to the probabilities defining the equilibrium? These empirical matters are investigated in both chapters.