SUPPLEMENTARY APPENDIX TO
"WINNING BY DEFAULT: WHY IS THERE SO LITTLE COMPETITION IN GOVERNMENT PROCUREMENT?"

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In this document, we provide the proof of the lemmas and the theorem in Section 5 on the nonparametric identification of the model, the closed-form characterization of the equilibrium contracts and competition of the estimated model, and the details of the simulated GMM estimator.

1. Proof of Lemma 5.1

Lemma 5.1 (i) If **A3** holds then $\partial \left| q(l,\pi) \right| / \partial \pi > 0$.

Proof. Theorem 4.1 characterizes the unique optimal menu of two contracts, defined by the price of the fixed contract:

$$\underline{p}_n = \alpha + \frac{\pi \left(1-\pi\right)^{n-1}}{1-\left(1-\pi\right)^n} \left(\beta - \int \psi[q(s)] \left[1 - l\left(s\right)\right] \overline{f}(s) ds \right), \tag{1}$$

and the variable contract:

$$\overline{p} = \alpha + \beta - \int \psi[q(s)] \overline{f}(s) ds, \tag{2}$$

$$q(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\} l(s)}\right) & \text{if } l(s) \leq \tilde{l}(\min\{\pi,\tilde{\pi}\}), \\ M & \text{if } l(s) > \tilde{l}(\min\{\pi,\tilde{\pi}\}). \end{cases} \tag{3}$$

By **A3** $q\left(l,\pi\right)$ satisfies:

$$\psi'\left[q\left(l,\pi\right)\right]\left[1-\pi l\right] = 1-\pi. \tag{4}$$

Totally differentiating (4) with respect to π and making $\partial q\left(l,\pi\right)/\partial \pi$ the subject of the resulting equation:

$$\frac{\partial q\left(l,\pi\right)}{\partial \pi} = \frac{l-1}{\psi''\left[q\left(l,\pi\right)\right]\left(1-\pi l\right)^2}. \tag{5}$$

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By assumption $\psi''(q) < 0$, implying $\partial q(l,\pi)/\partial\pi \gtreqless 0$ when $l \lesseqgtr 1$. From (4) it follows that $q(l,\pi) \gtreqless 0$ when $l \lesseqgtr 1$. Combining both sets of inequalities $\partial q(l,\pi)/\partial\pi \gtreqless 0$ when $q(l,\pi) \gtreqless 0$, as claimed. □

Lemma 5.1 (ii) If **A3** and **A5** hold then $\partial\overline{p}(\pi)/\partial\pi < 0$.

Proof. Totally differentiating the base price for variable contracts, defined in (2), with respect to π, yields:

$$\overline{p}'(\pi) = \alpha'(\pi) + \beta'(\pi) - \int \psi'(q[l(s),\pi]) \frac{\partial q[l(s),\pi]}{\partial\pi}\overline{f}(s)\,ds.$$

Define:

$$m(l,\pi) \equiv \psi'[q(l,\pi)]\frac{\partial q(l,\pi)}{\partial\pi}.$$

From (7) the first order condition for an interior solution can be rewritten as:

$$\psi'(q) = (1-\pi)/(1-\pi l), \text{ or } q(l,\pi) = h[(1-\pi)/(1-\pi l)].$$

Therefore:

$$m(l,\pi) = h'\left(\frac{1-\pi}{1-\pi l}\right)\frac{(1-\pi)(1-l)}{(1-\pi l)^3}.$$

Note $m(l,\pi) \gtreqless 0$ for $l \gtreqless 1$ because $h'(x) < 0$ and by **A3**, $\pi l < 1$. By definition $l(s) > 0$ for all $s \in S$. Therefore:

$$\begin{aligned}
\overline{p}'(\pi) &= \alpha'(\pi) + \beta'(\pi) - \int m[l(s),\pi]\,\overline{f}(s)\,ds \\
&< \alpha'(\pi) + \beta'(\pi) - \int_{\{s:l(s)<1\}} m[l(s),\pi]\,\overline{f}(s)\,ds \\
&\leq \alpha'(\pi) + \beta'(\pi) - \sup_{l\in(0,1)} |m(l,\pi)| \int_{\{s:l(s)<1\}} \overline{f}(s)\,ds.
\end{aligned}$$

Thus $|m(l,\pi)|$ is bounded for $l \in (0,1)$ by:

$$\begin{aligned}
|m(l,\pi)| &\leq \left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right| \left|\frac{(1-\pi)(1-l)}{(1-\pi l)^3}\right| \\
&\leq \sup_{l\in(0,1)} \left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right| \sup_{l\in(0,1)} (1-l) \sup_{l\in(0,1)} \left|\frac{(1-\pi)}{(1-\pi l)^3}\right| \\
&\leq (1-\pi)^{-2} \sup_{l\in(0,1)} \left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right|.
\end{aligned}$$

Therefore:

$$\overline{p}'(\pi) < \alpha'(\pi) + \beta'(\pi) - (1-\pi)^{-2} \sup_{l\in(0,1)} \left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right| \int_{\{s:l(s)<1\}} \overline{f}(s)\,ds < 0.$$

where the second inequality holds by **A5**. □

Lemma 5.1 (iii) If **A3** and **A4** hold then $\partial \underline{p}_n(\pi)/\partial\pi < 0$ for all $n \in \{1, 2, \ldots\}$.

Proof. Rewriting (1) to make the dependence of $\underline{p}_n$ on π explicit:

$$\begin{aligned}\underline{p}_n(\pi) &= \alpha(\pi) + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left[\beta(\pi) - \int \psi\left(q\left[l\left(s\right), \pi\right]\right)\left[1-l\left(s\right)\right]\overline{f}\left(s\right)\right] \\ &\equiv \alpha(\pi) + \Psi_{0,n}\left(\pi\right)\left[\beta(\pi) - \Psi_1\left(\pi\right)\right].\end{aligned}$$

and hence:

$$\underline{p}'_n(\pi) = \alpha'(\pi) + \Psi'_{0,n}\left(\pi\right)\left[\beta(\pi) - \Psi_1\left(\pi\right)\right] + \Psi_{0,n}\left(\pi\right)\left[\beta'(\pi) - \Psi'_1\left(\pi\right)\right]. \tag{6}$$

By **A4** $\alpha'(\pi) \leq 0$. Also:

$$\frac{\partial}{\partial\pi}\ln\left[\Psi_{0,n}\left(\pi\right)\right] = \frac{1 - n\pi - (1-\pi)^n}{\pi\left(1-\pi\right)\left[1-(1-\pi)^n\right]}.$$

The derivative is zero at $n = 1$ and $-\pi^2$ at $n = 2$. Now suppose it is negative for all $n \in \{2, \ldots, n_0\}$. For $n_0 + 1$ the denominator is positive and the numerator is:

$$1 - (n_0+1)\,\pi - (1-\pi)\,(1-\pi)^{n_0} < \pi\,(1-\pi)^{n_0} - \pi < 0.$$

The first inequality follows from an induction hypothesis, and the second one from the inequalities $0 < \pi < 1$. Therefore $\Psi'_{0,n}(\pi) \leq 0$ for all (π, n). By **A3** the participation constraint is satisfied with an inequality implying $\beta(\pi) > \Psi_1(\pi)$, and hence $\Psi'_{0,n}(\pi)\left[\beta(\pi) - \Psi_1(\pi)\right] \leq 0$. The third expression in (6) is strictly negative because $\Psi_{0,n}(\pi) > 0$, by **A4** $\beta'(\pi) \leq 0$, and:

$$\begin{aligned}\Psi'_1\left(\pi\right) &= \int \psi'\left(q\left[l\left(s\right), \pi\right]\right)\frac{\partial q\left[l\left(s\right), \pi\right]}{\partial\pi}\left[1-l\left(s\right)\right]\overline{f}\left(s\right)ds \\ &= \int (1-\pi)\frac{\partial q\left[l\left(s\right), \pi\right]}{\partial\pi}\left[\frac{1-l\left(s\right)}{1-\pi l\left(s\right)}\right]\overline{f}\left(s\right)ds \\ &= \int \frac{(\pi-1)\left[1-l\left(s\right)\right]^2}{\psi''\left(q\left[l\left(s\right), \pi\right]\right)\left[1-\pi l\left(s\right)\right]^3}\overline{f}\left(s\right)ds > 0.\end{aligned}$$

Appealing to **A3**, the second equality uses (4) to substitute out $\psi'\left[q\left(s, \pi\right)\right]$, the third equality uses (5) to substitute out $\partial q\left(l, \pi\right)/\partial\pi$, and the inequality follows from $\psi''\left(q\right) < 0$ and the assumption of an interior solution. Therefore $\underline{p}'_n(\pi) < 0$. □

2. Proof of Lemma 5.2

Lemma 5.2 $f_{\pi|c,n,v}\left(\pi\,|c, n, v\right)$ is identified.

Proof. Rewriting (4) to solve for π, we obtain:

$$\pi = \left[1 - \psi'(q)\right] / \left[1 - \psi'(q)\, l\right] . \tag{7}$$

Since $\psi(q)$ is identified, the π corresponding to each variable contract $(\overline{p}, q, l)$ is identified from (7). Hence $f_{\pi|c,n,v}(\pi\, |c, n, 1)$ is identified. Define the odds ratio related to contract types conditional on (c, n) as:

$$\varphi_{c,n} \equiv \Pr(v = 1|c, n) / \Pr(v = 0|c, n) .$$

and note that $\varphi_{c,n}$ is identified. The joint probability that the contract type is fixed and $\pi \leq \pi^*$ can be expressed as:

$$\begin{aligned} \Pr\{\pi \leq \pi^*, v = 0\, |c, n\} &= F_{\pi|c,n,v}(\pi^*\, |c, n, 0) \Pr(v = 0\, |c, n) \\ &= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}(\pi\, |c, n) \left[1 - (1 - \pi)^n\right] d\pi. \end{aligned} \tag{8}$$

Taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}(\pi^*\, |c, n, 0) \Pr(v = 0\, |c, n) = f_{\pi|c,n}(\pi^*\, |c, n) \left[1 - (1 - \pi^*)^n\right] . \tag{9}$$

Similarly:

$$\begin{aligned} \Pr\{\pi \leq \pi^*, v = 1\, |c, n\} &= F_{\pi|c,n,v}(\pi^*\, |n, v = 1) \Pr(v = 1\, |c, n) \\ &= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}(\pi\, |c, n) (1 - \pi)^n d\pi, \end{aligned}$$

and taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}(\pi^*\, |c, n, 1) \Pr(v = 1\, |c, n) = f_{\pi|c,n}(\pi^*\, |c, n) (1 - \pi^*)^n . \tag{10}$$

Rearranging the quotient of (9) and (10) to make $f_{\pi|c,n,v}(\pi^*\, |c, n, v = 0)$ the subject of the resulting equation, we obtain:

$$f_{\pi|c,n,v}(\pi\, |c, n, 0) = \varphi_{c,n} \frac{\left[1 - (1 - \pi)^n\right]}{(1 - \pi)^n} f_{\pi|c,n,v}(\pi\, |c, n, 1) ,$$

after dropping the superscripts asterisks. Because $\varphi_{c,n}$ and $f_{\pi|c,n,v}(\pi\, |c, n, 1)$are identified, so is $f_{\pi|c,n,v}(\pi\, |c, n, 0)$. □

3. Proof of Theorem 5.1

Theorem 5.1 $\psi(q)$, $\alpha(\pi)$ and $\beta(\pi)$ are identified, and for $n \in \{2, 3, \ldots\}$:

$$\begin{aligned} \alpha(\pi) &= \frac{1-(1-\pi)^n}{1-(1-\pi)^{n-1}} \underline{p}_n^*(\pi, c) - \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^{n-1}} \underline{p}_1^*(\pi, c), \\ \beta(\pi) &= \overline{p}(\pi) + \int \psi\left(h\left[\frac{1-\pi}{1-\pi l(t)}\right]\right) \overline{f}(t)\, dt - \alpha(\pi). \end{aligned} \tag{11}$$

Proof. To prove the first equation in (11) set $n = 1$ in (1) to obtain:

$$\underline{p}_1 - \alpha = \beta - \int \psi[q(s)]\,[1 - l(s)]\,\overline{f}(s) ds.$$

Substitute the right hand side of this expression back into (1) yielding:

$$\underline{p}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\underline{p}_1 - \alpha\right).$$

Make α the subject of the equation, and explicitly recognize the dependence of $\underline{p}_n$, $\underline{p}_1$ and α on π. To prove the second equation in (11), rearrange (7) using the definition of $h(\cdot)$ to obtain q as a function of s, specifically $q(s) = h((1-\pi)/[1-\pi l(s)])$, substitute the expression into (2), make β the subject, and the dependence of α, β and $\overline{p}$ on π explicit.

The proof that $\psi(q)$ is identified is given in the text. Proving $\alpha(\pi)$ and $\beta(\pi)$ are identified follows from noting that their representations given in (11) are identified by Lemma 5.2 and the arguments in the surrounding text. □

4. Optimal Competition under the Parametric Specification

The specification on $\kappa(\pi, n)$ is

$$\kappa(n, \pi) = (\kappa_1 + \kappa_2\pi)(n-1) + (\kappa_3 + \kappa_4\pi)(n-1)^2.$$

Given this specification, the expected total cost of competed procurement with effort λ, denoted by $U(\pi, \lambda)$, is:

$$\begin{aligned} U(\pi, \lambda) &= \alpha(\pi) + \exp^{-\lambda\pi}[\beta(\pi) + \Gamma(\pi)] + \mathbb{E}[\kappa(\pi, j+1)|\lambda] + \eta \\ &= \alpha(\pi) + \exp^{-\lambda\pi}[\beta(\pi) + \Gamma(\pi)] + \tilde{\kappa}_1(\pi)\lambda + \tilde{\kappa}_2(\pi)\lambda(1+\lambda) + \eta, \end{aligned}$$

where $\tilde{\kappa}_1(\pi) \equiv \kappa_1 + \kappa_2\pi$ and $\tilde{\kappa}_2(\pi) \equiv \kappa_3 + \kappa_4\pi$.

Taking the first order condition:

$$\pi \exp^{-\lambda\pi}[\beta(\pi) + \Gamma(\pi)] = \tilde{\kappa}_1(\pi) + \tilde{\kappa}_2(\pi)(1 + 2\lambda). \tag{12}$$

Because the left hand side is decreasing in λ while the right hand side is increasing in λ, there exists a unique solution to the above equation for any given π, denoted by $\tilde{\lambda}(\pi)$. Because $\lambda \geq 0$, $\lambda^*(\pi) = \max\{\tilde{\lambda}(\pi), 0\}$. In our estimation, we numerically solve for $\lambda^*(\pi)$ for each π.

Given $\lambda^*(\pi)$, it is optimal for the buyer to hold a competitive solicitation if and only if

$$U[\pi, \lambda^*(\pi)] \leq U_0(\pi).$$

The above inequality can be rewritten as:

$$\eta \leq (1 - e^{-\lambda^*(\pi)\pi}[\beta(\pi) + \Gamma(\pi)] - \tilde{\kappa}_1\lambda^*(\pi) - \tilde{\kappa}_2\lambda^*(\pi)[1 + \lambda^*(\pi)]. \tag{13}$$

Equations (1), (2), (3), (12), and (13) characterize the equilibrium contracts and competition.

5. Simulated GMM Estimator

Let us denote the vector of the parameters of the model by θ. Our estimator minimizes a weighted sum of squared distances:

$$g_n(\theta)' W g_n(\theta), \text{ with } g_n(\theta) = \frac{1}{n}\sum_{t=1}^{n} g(w_t; \theta),$$

where W is a symmetric positive-definite weighting matrix. The $g(w_t; \theta)$ vector is associated with 40 moment conditions: (i) 17 moment conditions on competition, contract type, and contract price for all projects, (ii) the same 17 moment conditions for non-military projects, and (iii) 6 moment conditions on the distribution of s, or the standardized delay.

The 17 moment conditions consist of $\Pr(c_i = 0)$, $\Pr(n_i = 1|c_i = 1)$, $\Pr(n_i \leq 2|c_i = 1)$, $\Pr(n_i \leq 5|c_i = 1)$, $\Pr(v_i = 0|c_i = 0)$, $\Pr(v_i = 0|c_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i \leq 2)$, $\Pr(v_i = 0|c_i = 1, n_i \leq 5)$, $\mathbb{E}[n_i]$, $\mathbb{E}[p_i|c_i = c, v_i = v]$ for $(c, v) \in \{0,1\} \times \{0,1\}$ where $p_i \equiv \underline{p}_i(1 - v_i) + (\overline{p}_i + q_i)v_i$, $\mathbb{E}[p_i|c_i = 1, n_i = 1]$, $\mathbb{E}[p_i|c_i = 1, n_i \leq 2]$, and $\mathbb{E}[p_i|c_i = 1, n_i \leq 5]$. The 6 moment conditions on the distribution of the standardized delay are $\Pr(s_i = 0, v_i = 0)$, $\Pr(s_i = 0, v_i = 1)$, $\Pr[s_i(1 - v_i)]$, $\Pr[s_i^2(1 - v_i)]$, $\Pr[s_i v_i]$, and $\Pr[s_i^2 v_i]$. Note that the moments as a function of θ are calculated using simulation. In our estimation, the simulation size is 5,000.

We use a two-step procedure to obtain the optimally weighted simulated GMM estimator. We start with a positive definite weighting matrix and obtain a first-step estimator, denoted by $\tilde{\theta}_n$. The asymptotic variance of $\sqrt{n}g_n(\theta_0)$, S, is estimated by:

$$\hat{S} \equiv \frac{1}{n}\sum_t g(w_t, , \tilde{\theta}_n) g(w_t; \tilde{\theta}_n)'.$$

Then we re-estimate the parameters using the optimal weighting matrix $\hat{S}^{-1}$ to obtain the optimally weighted simulated GMM estimator, which we denote by $\hat{\theta}_n$.

Under standard regularity conditions, this estimator is asymptotically normally distributed, and a consistent estimator of the asymptotic variance of $\sqrt{n}(\hat{\theta}_n - \theta_0)$ is:

$$\left(\frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'}' \hat{S}^{-1} \frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'} \right)^{-1} .$$

Since the moments are calculated by simulation, we use a numerical derivative of $g_n(\theta)$ to estimate the asymptotic variance of the estimator.