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# Innovation and Reputation

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This paper analyzes a monopolist that markets successive generations of new and improving nondurable products. Prices, research intensity, and product innovations are derived as sequential equilibrium outcomes to a dynamic game with incomplete information. Asymmetric information is an important feature of the model. The monopolist is fully aware of the current product's quality, as are consumers who have tried it. However, the beliefs of other people are characterized by a probability distribution that depends on the monopolist's marketing strategy and the product's popularity. The analysis illustrates a new context in which price signaling might serve as a mechanism for ensuring that only high-quality products are marketed. More important, it shows how product life cycles are generated in the absence of signaling and how a reputation for producing high-quality goods becomes established in such cases.

## I. Introduction

When a new product is introduced to the market, the producer typically knows more about its attributes than most consumers. To maximize discounted profits, the firm should widely publicize those attributes that consumers find most appealing but be less frank about the product's undesirable ones, provided warranties are too costly to enforce and there are no reputational effects that might reduce its sales of other goods. Even so, uninformed people presumably account for such bias when deciding whether to purchase the product or not.

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Later on in the product's market life, these issues become much less important, for over time consumers learn all about the salient characteristics through their personal experiences as well as from indicators of the product's overall popularity.

This paper investigates the diffusion process described above, viewing it as the equilibrium outcome from a dynamic game with incomplete information. Section II lays out a model in which a monopolist markets a succession of new and improving nondurable products. The event sequence that occurs during a typical period of the game proceeds as follows. Suppose that a given product was sold last period. The monopolist must now decide whether to withdraw it and undertake more research in order to discover a new product or, if not, what price should be charged for the retained product. Once the price of a marketed product has been set, consumers (some of whom may be more informed than others) decide whether to buy it. At the time each new product is introduced, all consumers are less informed than the monopolist about its characteristics, but they can learn about them by buying the product or, alternatively, by later making inferences from those who have.

The equilibrium in this model yields the rate at which research is undertaken, the birth and death times of successive products, the diffusion of information about each product's characteristics throughout the population, and the monopolist's pricing policy. Section III establishes the existence of a sequential equilibrium satisfying two additional refinements and then characterizes the equilibrium outcomes of such equilibria.

Essentially one of three scenarios applies, thus determining whether and when a lower-quality product will be introduced and withdrawn from the market, how long it takes for the reputation of a higher-quality product to be established, and what the price policy is. A reason for withdrawing a lower-quality product from the market is that its customer base would vanish if people who had not tried out the product learned from the experience of those who had. In this setup uninformed buyers eventually deduce quality from retrospectively observing the aggregate quantities sold. However, similar results would emerge if neighbors could inspect each other's retail purchases. Suppose momentarily that such public information is the only deterrent to marketing lower-quality products, the first scenario. Then all products are introduced irrespective of quality, and the price of each subsequently declines over time. The decline reflects the diminishing value to consumers of acquiring private information about product quality as the date approaches when it will be made publicly available. At that point a lower-quality product would be invariably replaced by a new product, whereas a higher-quality product would be publicly revealed as such.

Even if the reason given above is unimportant, lower-quality products are not necessarily marketed indefinitely, as the second scenario shows. Granted, research is a costly activity, so there may be a positive probability of marketing defective products when this is private information to the supplier. Furthermore, the benefits of immediately undertaking more research are temporarily reduced when any product is introduced because the findings of such research might render the current product obsolete. However, as the total number of people who have tried out a defective product increases, the uninformed proportion of the population declines, shrinking demand. Consequently research is increased. In turn, more research activity raises the probability of superseding the current product. Hence the price of any surviving product increases because the longer a lower-quality product has been on the market, the more likely it would have been withdrawn previously. In fact, there comes a time when a lower-quality product is withdrawn for sure because even if it was priced as a higher-quality product and the remaining uninformed people (mistakenly) had no doubt that it was a higher-quality product, sales revenue would still not compensate the net benefits from introducing a new product to the whole population.

The two scenarios mentioned above illuminate the important role of consumer uncertainty about quality in generating intertemporal competition between product generations supplied by the same firm. In some circumstances this can induce dynamic signaling. Suppose that the monopolist would introduce a higher-quality product at a very low introductory price, anticipating high demand for it in subsequent periods. Then consumers might refuse to buy any product introduced at a higher price. If it is unprofitable for the monopolist to introduce a lower-quality product at the same price because future demand for it is less, this (third) scenario could represent equilibrium behavior.

Previous published works in several different areas are related to the analysis undertaken here. There is a distinction, commonly made in the marketing literature, between innovative consumers, who exhibit greater willingness to experiment with products of lower expected quality and pay a premium price for being first, and more cautious, imitating consumers, who buy the product only if its price falls or after its reputation has become established (see, e.g., the characterization of adopters given by Rogers [1983, pp. 241–70]). These differences in consumer behavior have been attributed to tastes and motivate economic models of intertemporal price discrimination such as Stokey's (1979). However, this analysis shows that such behavior may be observed in a homogeneous population as well. Before the product's characteristics are well known, the price an uninformed person is willing to pay accounts for future opportunities he might

have to exploit the private information gained through consumption. Because this information is rendered valueless when the characteristics become publicly revealed, the earlier it is acquired the more valuable it is. Thus the reservation price of a product whose characteristics come from a given probability distribution falls over time, or if the price rises, the probability that a product will be unsatisfactory must fall.

This paper also contributes to the theory of consumer behavior toward experience goods, as formulated by Nelson (1970), Grossman, Kihlstrom, and Mirman (1977), Hey and McKenna (1981), Wilde (1981), and others. First, it demonstrates how optimization problems such as theirs may arise from an equilibrium setting. Here one finds that nondurables produced by the same supplier at different times compete with each other to some extent because consumers are not fully informed. Also, the probability distribution describing a product's characteristics is itself endogenized through the monopolist's research and marketing strategies. This second remark also suggests the possibility that, in equilibrium, the benefits of information are offset not by higher prices but by products of lower expected quality.

Third, the equilibrium concept invoked is essentially a further refinement of those developed by Selten (1975) and Kreps and Wilson (1982*b*). Similar in spirit to Cho and Kreps (1987) and Banks and Sobel (1987), but most closely related to Milgrom and Roberts (1986), the analysis focuses on those sequential equilibria that the monopolist would prefer to play should its attempts to innovate prove successful.

Associated with the concept of equilibrium in environments with incomplete information is the process of acquiring a reputation. In this respect the analysis undertaken here is more similar to Kreps and Wilson's (1982*a*) game-theoretic treatment of the chain store paradox than Shapiro's (1982) decision-theoretic approach to reputation building. For as in the former, but in contrast to the latter, the beliefs of uninformed consumers are modeled as probability distributions that are updated using Bayes's rule as new information (itself endogenously determined in equilibrium) arrives.

Finally, how to maintain a reputation is the subject of articles by Dybvig and Spatt (1980), Klein and Leffler (1981), and Shapiro (1983). These authors extend Friedman's (1971) development of trigger strategy equilibria for supergames to situations in which there is asymmetric information about product quality. As mentioned earlier, high quality is assured in the third scenario. However, the enforcement mechanism analyzed here does not involve consumers collectively punishing the deviating monopolist that introduces a low-quality product by, say, paying less for all future new products (although this kind of sequential equilibrium also exists in some re-

gions of the parameter space). Rather, high quality is guaranteed by low introductory offers in a manner analogous to Spence's (1973) pioneering study of signaling. Reputations are thus identified with particular products rather than the firm itself, a feature that seems more appropriate the larger the touted innovation and the less diversified the product lines.

## II. The Model

### *An Overview*

The game is very simple. The product's characteristics space is summarized by one variable, quality, which can take only two values. Moreover, consumers do not demand low-quality products, or "fakes," at a positive price. High-quality products are called "cures." Apart from price, many other factors influence the demand for a product with a given set of characteristics; in this paper they are modeled as a Bernoulli random variable, which is independently and identically distributed across the population and over time. "Sickness" and "health" represent the two possible outcomes.

Play proceeds as follows. Each period a proportion of the population catch a disease that lasts one period. There is a sole supplier of drugs: after paying an initial fixed cost for its research laboratories, this monopolist faces a fixed probability of discovering a cure at date 0. If found, the cure could be marketed in the first period and forever after. But even if it is unsuccessful, the firm may choose to sell a fake to consumers for one or more periods before withdrawing it from the market and conducting another experiment. Alternatively, the firm might not enter with a fake in the first period, opting instead for another experiment in the hope of introducing a cure in period 2. The profitability of deceptive practice is attributable to the existence of asymmetric information. Although consumers know when an experiment is conducted, they are not automatically privy to the outcome. Hence sick consumers must decide whether or not to buy a prescription on the basis of incomplete information. Those who do buy are immediately cured if and only if the drug works, in the process acquiring full information about its quality.

### *Preferences*

How individuals react to the introduction of new drugs depends, among other things, on the nature of product demand, how reliable their personal experience is in evaluating product quality, whether they can infer anything from their friends' experiences, what market aggregates are published (either officially or as advertisements), and

what the decision rules of suppliers are. In this framework, a continuum of consumers, distributed uniformly on the  $[0, 1]$  interval, have identical preferences, caring about the goods  $x$  they consume and also their health  $z$ . Each person's tastes may be represented by a time-additive utility function  $\sum_{t=0}^{\infty} \beta^t U(x_t, z_t)$ , where  $U(x, z)$  is concave increasing in both arguments and  $\beta \in (0, 1)$  is the discount rate. There are only two states of health  $z \in \{0, 1\}$ , and  $\alpha$  is the probability that  $z = 0$  (falling sick); for convenience draws are distributed independently across periods and people.

It is helpful to define  $u(p, \theta)$ , the expected utility a sick person attains in a period from buying a prescription at price  $p$  that works with probability  $\theta$  (given income  $y$ , which bounds their expenditure each period and is henceforth suppressed).

DEFINITION 1.  $u(p, \theta) = \theta U(y - p, 1) + (1 - \theta)U(y - p, 0)$ .

Notice that  $u(0, 1)$  is the utility of a healthy person, which without loss of generality may be normalized to zero, and  $u(0, 0)$  is the utility of a sick person who does not take medication. Substitution and differentiation show  $u_1(p, \theta) < 0$ ,  $u_2(p, \theta) > 0$ ,  $u_{11}(p, \theta) < 0$ , and  $u_{22}(p, \theta) = 0$  (where the subscript  $i \in \{1, 2\}$  indicates partial differentiation with respect to the  $i$ th argument).

In all the equilibria analyzed here, if he had the choice, an informed sick person would buy a cure provided its price is less than or equal to  $\bar{p}$ , defined below.

DEFINITION 2.  $u(\bar{p}, 1) = u(0, 0)$ .

To avoid cluttering the exposition, this behavior is imposed as part of the environment. Accordingly, let  $\delta(p_t)$  denote a sick person's demand for a known cure. From definition 2, if  $p_t \leq \bar{p}$ , then  $\delta(p_t) = 1$ , whereas if  $p_t > \bar{p}$ , then  $\delta(p_t) = 0$ . Also let  $c_t$  represent the quality of the drug most recently introduced by the monopolist:

$$c_t = \begin{cases} 1 & \text{the firm produces cures in period } t \\ 0 & \text{it produces fakes in period } t. \end{cases} \quad (1)$$

Then demand for the drug by an informed sick person at time  $t$  is simply  $c_t \delta(p_t)$ .

### *Information and Technology*

There are constant returns to scale. Irrespective of quality, drugs cost  $w$  each to produce. Incomplete information arises in the model because only the monopolist undertaking research directly observes the outcomes of the experiments it conducts. The introduction of a new brand in the  $t$ th period is denoted by

$$b_t = \begin{cases} 1 & \text{a new brand is developed in period } t \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The probability of conducting a successful experiment is  $\theta_0$ .

Within the class of equilibria considered here, if it had the choice, the monopolist would never withdraw a cure from the market. So to reduce the notational burden, the analysis simply imposes this behavioral restriction at the outset. Taken together, these remarks imply  $(b_0, c_0) = (1, 0)$  and

$$\Pr\{c_{t+1} = 1 | c_t, b_t\} = c_t + \theta_0 b_t (1 - c_t). \tag{3}$$

As mentioned above, uninformed sick people can become informed about a particular brand by trying it out when they are sick. Let  $\Lambda_t$  be the number of people (or, equivalently, the population proportion) who are informed at time  $t$ ; also let  $q_t$  denote the number of people who buy the drug then. If a new brand is introduced in period  $t$ , the number of informed people drops to zero (i.e.,  $b_t = 1$  implies  $\Lambda_t = 0$ ). Alternatively, when the current brand is retained (i.e., when  $b_t = 0$ ), the proportion of people who are informed increases by the number who buy the drug,  $q_t$ , less those who are repeat purchasers,  $\alpha \Lambda_t c_t \delta(p_t)$ . (Notice that  $\alpha \Lambda_t$  is the number of informed sick people, while  $c_t \delta(p_t)$  indicates whether they purchase the drug or not.) Therefore, the law of motion for  $\Lambda_t$  is

$$\Lambda_{t+1} = (1 - b_{t+1})[\Lambda_t + q_t - \alpha \Lambda_t c_t \delta(p_t)]. \tag{4}$$

The public record of a game's history,  $h^t \in H^t$ , comprises a vector sequence of new brands introduced  $b_t$ , prices posted  $p_t$ , and quantities traded  $q_t$ . In symbols,  $h_t = (b_t, p_t, q_t)$  and  $h^t = \{h_s\}_{s=1}^t$ . Sick consumers decide whether to purchase medication (this action being denoted by  $\gamma_t = 1$ ) or not (denoted  $\gamma_t = 0$ ); they rely on public records of the game history  $h^t$  plus current prices  $p_t$  to determine their subjective probability  $\theta_t \in [0, 1]$  that the most recently introduced brand works and to make their choice  $\gamma_t \in \{0, 1\}$ . Again, to avoid complications that occur off the equilibrium path, it is assumed that all sick uninformed people behave the same way. Consequently,  $q_t \in \{0, \alpha \Lambda_t, \alpha(1 - \Lambda_t), \alpha\}$ .

If the monopolist does not conduct an experiment, it announces a price  $p_t$  at which it supplies all customers. This price is a positive real number that potentially depends on the monopolist's private information  $c_t$  and the game's history  $h^t$  to date. Alternatively, the monopolist temporarily withdraws from the market to conduct another experiment. Let  $\psi_t$ , a function of  $h^t$ , denote the probability that an experiment will be conducted at time  $t$ . Then  $b_0 = 1$  and

$$\Pr\{b_t = 1 | c_t, \psi_t\} = (1 - c_t)\psi_t. \tag{5}$$



By preventing the monopolist from marketing a fake while simultaneously trying to discover a cure, the model crudely captures some variable costs of conducting research. (For if people observed the firm undertaking research, they would deduce that it had not discovered the cure yet.)

To summarize, an assessment, denoted by  $A$ , fully characterizes the beliefs and actions of the players in the game. This game is concerned with the beliefs of the uninformed sick  $\theta(h', p_t)$ , their choice whether to take medication  $\gamma(h', p_t)$  or not, the price of drugs not withdrawn from the market  $p(h', c_t)$ , and the probability of withdrawing a fake from the market  $\psi(h')$ . Thus the assessment  $A$  is defined, for each  $h' \in H'$  and  $c_t \in \{0, 1\}$ , as the fourtuple  $A(h', c_t) = (\theta_t, \gamma_t, p_t, \psi_t)$ .

### *Opportunity Costs and Reservation Prices*

The interesting aspects of incomplete information in the model arise because the monopolist is unable to commit itself to truthfully disclosing product quality. This creates a tension between its objectives before a cure is discovered (when it wants to mislead consumers) and those afterward (when its aim is to fully reveal the product's characteristics). The tension is reflected in the value of owning the monopoly at different stages in the game. In particular, given an assessment  $\bar{A}$ , the initial value of the game, denoted to the monopolist  $V(\bar{A})$  and abbreviated by  $\bar{V}$ , is

$$V(\bar{A}) = E_0 \left\{ \sum_{t=0}^{\infty} (1 - b_t) \alpha \beta^t (\tilde{p}_t - w) [\Lambda_t c_t \delta(\tilde{p}_t) + (1 - \Lambda_t) \tilde{\gamma}_t] \right\}. \quad (6)$$

Here, the tilde on a variable indicates its dependence on  $\bar{A}$ . To interpret (6), observe that the expectation  $E_0$  is taken over the sequence of random variables  $\{b_t, c_t\}_{t=0}^{\infty}$ , determined by a probability distribution parameterized by  $\theta$  and  $\{\tilde{\psi}_t\}_{t=0}^{\infty}$ . If a new brand is being developed in period  $t$ , then  $b_t = 1$  and sales are zero. Otherwise the net return per unit sold discounted back to zero is  $\beta^t(\tilde{p}_t - w)$ . Aggregate demand by the informed is  $\alpha \Lambda_t c_t \delta(\tilde{p}_t)$ , while demand by the uninformed is  $\alpha(1 - \Lambda_t) \tilde{\gamma}_t$ . When a cure is discovered, the value of the monopoly becomes  $V'(\bar{A})$ , abbreviated by  $\bar{V}'$ , defined as

$$V'(\bar{A}) = \sum_{t=0}^{\infty} \alpha \beta^t (\tilde{p}_t - w) [\Lambda_t \delta(\tilde{p}_t) + (1 - \Lambda_t) \tilde{\gamma}_t]. \quad (7)$$

In contrast to (6), equation (7) does not depend on the sequence of random variables  $\{b_t, c_t\}_{t=0}^{\infty}$ . Brand turnover stops once a cure is found.

In view of the last sentence, some time after a cure has been introduced, uninformed consumers might come to believe that the

product surely works and be willing to pay  $\bar{p}$  for it; this is when the reputation of a cure becomes established. More formally, for a complete history of the game  $\{h_s\}_{s=0}^\infty$  and given an assessment  $\bar{A}$ , let  $\bar{\tau}(h')$  be the first time that  $\bar{\gamma}(h', \bar{p}) = 1$ . Because  $h_t$  is a stochastic process induced by the monopolist's research and marketing strategies,  $\bar{\tau}$  is random.

DEFINITION 3.  $\bar{\tau}(h') = \min\{t: \bar{\gamma}(h', \bar{p}) = 1 | h'\}$ .

As explained below, the analysis focuses on assessments with a recursive form (see condition 1 in Sec. III). In these assessments, an unsuccessful monopolist incurs an opportunity cost by postponing its research program in order to market fakes. Given  $\bar{A}$ , let  $w(\Lambda, \bar{V})$  denote the price at which an unsuccessful monopolist is indifferent between selling all the uninformed a fake in the current period and withdrawing it next period versus withdrawing it from the market immediately, when  $\Lambda$  people are informed. The second option is worth  $\bar{V}$  (the initial value of the game to the monopolist), while the current value of the first option to the monopolist is  $\alpha(1 - \Lambda)[w(\Lambda, \bar{V}) - w] + \beta\bar{V}$ . Notice that  $\alpha(1 - \Lambda)[w(\Lambda, \bar{V}) - w]$  is the net revenue from current sales and  $\beta\bar{V}$  is the value of the second option discounted back one period. Definition 4 follows from making  $w(\Lambda, \bar{V})$  the subject of the expression that equates these two quantities.

DEFINITION 4.  $w(\Lambda, \bar{V}) = w + [\alpha(1 - \Lambda)]^{-1}(1 - \beta)\bar{V}$ .

It will be shown below that if  $\bar{A}$  is an equilibrium assessment, then  $\bar{V} > 0$ ; hence  $w(\Lambda, \bar{V}) > w$ . (This follows from the fact that research has a positive net value in equilibrium and hence is costly to delay.) Observe that  $w(\Lambda, \bar{V})$  increases as the product's customer base for a fake erodes (i.e., in the proportion of the population who are informed), diverging to infinity. (To compensate the monopolist for delaying its research into superior products for a period, higher prices must offset lower quantities sold to the remaining uninformed.)

As the Introduction mentioned, demand for treatment by the uninformed depends on two factors, namely, prices (current and future) and expected quality. First, consider a drug whose reputation will be established next period. Whether an uninformed sick person buys this product depends purely on his current utility. For a drug that works  $\theta_0$  proportion of the time, let  $\omega$  be the price that equates the utility of an uninformed sick person from buying it with that attained from not being treated.

DEFINITION 5.  $u(\omega, \theta_0) = u(0, 0)$ .

Also, given  $\bar{A}$ , let  $\phi(\Lambda, \bar{V})$  be the minimum subjective probability an uninformed sick person would entertain and still buy treatment at the opportunity cost of producing fakes when the drug's reputation would be revealed next period.

DEFINITION 6.  $u[w(\Lambda, \bar{V}), \phi(\Lambda, \bar{V})] = u(0, 0)$ .

Because  $w(\Lambda, \bar{V})$  is increasing in  $\Lambda$  and  $\bar{V}$ , so is  $\phi(\Lambda, \bar{V})$ . The more people who have tried the product once, the greater the degree of confidence the others place in it. There is an economic rationale for this bandwagon effect. The higher is the informed proportion of the population, the lower is the customer base of an unsuccessful monopolist, so the more likely a fake is withdrawn from the market and, hence, the greater is the degree of confidence uninformed consumers place in retained products.

By theorem 2 in the next section, the reputation of a cure takes at most two periods to establish. So, in addition to  $\omega$  and  $\phi(\Lambda, \bar{V})$ , one must consider two-period demands. Denote by  $v(p)$  the reservation price of an uninformed sick person with subjective probability  $\theta_0$ , who acquires with his purchase the option to repeat purchase at price  $p$  next period.

DEFINITION 7.  $u[v(p), \theta_0] = (1 + \alpha\beta\theta_0)u(0, 0) - \alpha\beta\theta_0u(p, 1)$ .

When next period's price equals  $\bar{p}$ , the reservation value for a cure, there are no gains from acquiring private information. Consequently, an uninformed sick person is prepared to pay for benefits only accruing in the current period, that is, up to  $\omega$  (since the drug works with probability  $\theta_0$ ). Therefore,  $v(\bar{p}) = \omega$ . Lowering the price next period, or increasing the probability of falling ill and the weight attached to future utility, raises the value of acquiring private information. Similarly, the greater the chance of success to the firm, the higher are expected current benefits from taking treatment and the more likely the private information will be exploited. Thus  $v(p)$  is a decreasing function, while for each  $p \in (0, \bar{p})$ , the mapping is increasing in  $\theta_0$ ,  $\alpha$ , and  $\beta$ .

### III. Equilibrium

#### *Existence of Equilibrium*

Sequential equilibria are assessments in which the players in the game are sequentially rational and hold consistent beliefs (see Kreps and Wilson [1982*b*] for a theoretical analysis). This paper focuses on a subset of sequential equilibria exhibiting two distinctive features. First, the equilibrium beliefs and actions of players do not depend on events that occurred before the introduction of the current brand.

CONDITION 1. If  $\sum_{s=r}^t b_s > 0$ , then  $A(h^t, c_t)$  is independent of  $h^r$ .

Sequential equilibria that do not satisfy condition 1 include trigger strategies, which Dybvig and Spatt (1980), Klein and Leffler (1981), and Shapiro (1983) have studied in related modeling environments. While trigger strategy equilibria explain why the trademarks of repu-

table firms are conferred on the brands they produce, the question of how and indeed whether a new brand can acquire a reputation for high quality in the absence of support from a reputable parent firm still remains unanswered.

Among the sequential equilibria satisfying condition 1, only the most profitable one for a successful monopolist is examined. Loosely speaking, the rationale for this further refinement is that the monopolist should be able to communicate which equilibrium it expects everyone to play, through its pricing policy. (After all, the monopolist is the first mover each period, and everyone else observes its actions.) If so, once a cure is discovered, the monopolist picks the subgame equilibrium outcome that maximizes its discounted flow of net returns calculated at that time. Therefore, in the event that a research experiment is unsuccessful, the monopolist, when introducing a fake, is obliged to announce the prices everyone anticipates of a successful monopolist, to avoid immediate detection.

CONDITION 2. The assessment  $\bar{A}$  is chosen so that the sequence  $\{\bar{p}_t, \bar{\gamma}_t\}_{t=0}^\infty$  maximizes  $V'(\bar{A})$  subject to the constraint that it is a sequential equilibrium satisfying condition 1.

Aside from refining the set of Nash equilibria, the parameter space is also restricted. Once some fraction of the population becomes favorably informed, but before its reputation is established, a successful monopolist might optimally charge  $\bar{p}$  and sell prescriptions to informed people only rather than to uninformed sick people as well (at a lower price). It is straightforward to incorporate such behavior into the analysis. However, the main results are not affected by it, and extra notation is required. So to make the exposition more manageable, assumption 1 is imposed throughout. Under this assumption (which essentially bounds  $\theta_0$  from below and  $\beta$  from above), the behavior described above never occurs in equilibrium.

ASSUMPTION 1.  $\theta_0 > (1 - \beta)[\beta(2\beta + 1)(2\beta - 1)]^{-1}$ .

One way of establishing existence is to display a sequential equilibrium satisfying condition 1. Since the results derived below show that there are only a finite number of equilibrium outcome paths to consider, a maximum for the problem described in condition 2 exists.

THEOREM 1. Given assumption 1, a sequential equilibrium satisfying conditions 1 and 2 exists.

The Appendix contains all the proofs.

### *Equilibrium Outcomes*

Associated with the assessment  $\bar{A}$  are the outcomes it generates. Let  $\bar{H}^t$  denote the set of  $h^t$  partial histories that can arise from playing out  $\bar{A}$  for the first  $t - 1$  periods a brand is marketed, and from now on

suppose that  $\tilde{A}$  is any sequential equilibrium assessment that satisfies conditions 1 and 2. Describing the equilibrium outcomes for this game amounts to writing down the beliefs and actions of the various players, that is,  $\tilde{A}(h^t, c_t)$ , for each equilibrium history to date,  $h^t \in \tilde{H}^t$ , and drug type,  $c_t \in \{0, 1\}$ . The order in which the three scenarios were introduced is now reversed for expository purposes.

Suppose that the successful monopolist establishes its reputation after one period by making a low introductory price offer that would have been unprofitable had it not discovered a cure. Under these circumstances the monopolist's net revenue in the first period of marketing a cure is  $\alpha(\tilde{p}_1 - w)$ , and from then on it nets  $\alpha(\bar{p} - w)$  per period. Hence the value of the monopoly on discovery,  $V'_0$ , is

$$V'_0 = \alpha\beta\left[(\tilde{p}_1 - w) + \frac{\beta(\bar{p} - w)}{1 - \beta}\right]. \quad (8)$$

Since the probability of discovery is  $\theta_0$ , it follows that the initial value of the game to the monopolist is  $V_0 = \theta_0 V'_0 [1 + (1 - \theta_0)\beta + \dots]$ . Summing this infinite geometric series and substituting for  $V'_0$  from (8), one obtains

$$V_0 = \alpha\beta\theta_0\left[(\tilde{p}_1 - w) + \frac{\beta(\bar{p} - w)}{1 - \beta}\right](1 - \beta + \theta_0\beta)^{-1}. \quad (9)$$

There are two cases to consider. If  $w(\alpha, \tilde{V}) \geq \bar{p}$ , then an unsuccessful monopolist would invariably withdraw its fake after one period anyway; provided the successful monopolist sets  $\tilde{p}_1 \leq w(0, \tilde{V})$ , no fakes are introduced. Thus when the monopolist discovers a cure, its current value is  $\alpha\beta\{w(0, \tilde{V}) - w + [\beta(\bar{p} - w)/(1 - \beta)]\}$ . This occurs at date  $t$  with probability  $\theta(1 - \theta)^t$ . Multiplying the product of these expressions by  $\beta^t$  and summing over  $t$ , one obtains the present value of monopoly at date 0. Then, with definition 4 used to substitute for  $w(0, \tilde{V})$ , some straightforward manipulations yield the initial value of the game to the monopolist, which in this case is

$$V_1 = \alpha\beta^2\theta_0(\bar{p} - w)[(1 - \beta)(1 - \beta + \theta_0\beta - \alpha\beta\theta_0 + \alpha\beta^2\theta_0)]^{-1}. \quad (10)$$

The other case occurs when  $w(\alpha, \tilde{V}) < \bar{p}$ ; this inequality implies that an unsuccessful monopolist would continue marketing a fake it introduced in the previous period if the price was  $\bar{p}$ . Then entry by the unsuccessful monopolist into the market is deterred if  $V_0$  is not less than the value of introducing a fake and marketing it for two periods, namely,  $\alpha[(p_0 - w) + (1 - \alpha)\beta(\bar{p} - w)]$ , plus the value of the game discounted back two periods,  $\beta^2 V_0$ . (After two periods, uninformed people deduce that the drug was a fake from aggregate sales data. This is discussed later in more detail.) Equating the value of this

option with  $V_0$ , one solves for  $p_0$  to obtain

$$p_0 = w - \beta(1 + \alpha\beta - \alpha - \beta - \theta_0\alpha\beta - \theta_0\beta^2) \left( \frac{\bar{p} - w}{1 - \beta + \theta_0\beta^3} \right). \quad (11)$$

Are there any other price couplets  $(p_1, p_2)$  that make marketing fakes unprofitable? After all, provided  $\bar{p}_1 < w(0, \bar{V})$  and  $\bar{p}_1 + \beta\bar{p}_2 < w(0, \bar{V}) + \beta(1 - \alpha)w(\alpha, \bar{V})$ , only cures are introduced. Condition 2 effectively precludes this by allowing the successful monopolist to pick the most profitable price couplet that satisfies these two inequalities. In the second period an unsuccessful monopolist gains only  $\alpha(1 - \alpha)$  in sales revenue for price increases up to  $\bar{p}$  compared with  $\alpha$  by the successful monopolist, but in the first period their net returns are affected equally. Therefore, meeting the constraints imposed by his alter ego (the unsuccessful monopolist) at least cost requires the successful monopolist to choose  $(w(0, V_1), \bar{p})$  or  $(p_0, \bar{p})$  rather than some other price couplet.

LEMMA 1. If  $\psi_1 = 1$ , then  $\bar{\tau} = 2$ ; if in addition  $w(\alpha, \bar{V}) > \bar{p}$ , then  $(\bar{p}_1, \bar{\gamma}_1) = (w(0, V_1), 1)$ ; alternatively,  $w(\alpha, \bar{V}) < \bar{p}$  implies  $(\bar{p}_1, \bar{\gamma}_1) = (p_0, 1)$ . Finally, if  $\bar{\tau} > 2$ , then  $\psi_1 = 0$ .

The second sentence in lemma 1 asserts that if a cure must be marketed for more than one period to become established, then every failure preceding the cure was marketed for at least one period. One price path the lemma rules out involves introducing fakes at  $w(0, \bar{V})$  with less than unit probability, marketing a proportion of those introduced for one period only and the remainder for the second period as well, at  $w(\alpha, \bar{V})$ . For certain structural parameter values that imply  $v(\omega) < w(0, \bar{V}) < w(\alpha, \bar{V}) < \bar{p}$  (the middle inequality being a direct consequence of definition 4), this outcome is indeed a sequential equilibrium satisfying condition 1. However, the paragraph above implies  $p_0 + \beta\bar{p} > w(0, \bar{V}) + \beta w(\alpha, \bar{V})$ . Once successful, the monopolist prefers to signal rather than price the cure at the opportunity cost of marketing fakes. Consequently, if  $\bar{p}_1 = w(0, \bar{V})$  and  $\bar{p}_2 = w(\alpha, \bar{V})$ , then  $\bar{A}$  does not satisfy condition 2.

The characterization of the first two scenarios is developed in stages, drawing heavily on the opportunity cost and reservation price concepts defined in the previous section. As mentioned before, sick uninformed people may be willing to pay more than a price  $\hat{p}$  that equates their current utility from being sick  $u(0, 0)$  with their expected utility from taking the drug  $u[\hat{p}, \theta(h', \hat{p})]$  because there is value from acquiring information about its quality. Lemma 2 places an upper bound on their willingness to pay for this information; the reservation price of an uninformed person is certainly less than the reservation price for a known cure. Rather than pay more than  $\bar{p}$  now,

the individual attains a higher utility by waiting until the price falls below  $\bar{p}$  and buying the drug then (if he is sick again).

LEMMA 2. If  $p_t > \bar{p}$ , then  $\tilde{\gamma}(h^t, p_t) = 0$  for all  $h^t \in \tilde{H}^t$ .

The requirement of sequential equilibria that beliefs be consistent yields a close relationship between the rate at which fakes are withdrawn from the market and the subjective probability uninformed agents hold about brand quality. Once some people have become informed, the price of a cure remains above their reservation value  $\bar{p}$  until the last period of the component game; next period uninformed people infer from the quantity just sold whether the informed sick bought the product again. Because no one optimally repeats a purchase of a low-quality brand (for a positive price), its true quality is revealed to everyone else by their actions. In this way information about product quality inevitably seeps out, despite the fact that the opportunity cost of continuing to market fakes may be relatively low.

LEMMA 3. Suppose  $\tau(h^t) > t$  for some  $h^t \in H^t$ . If  $\Lambda_t > 0$  and  $p_t \leq \bar{p}$ , then  $\bar{\tau} = t + 1$ .

From lemma 3 (which applies to all histories, not just equilibrium paths), only uninformed people buy the product until the period before its reputation is established. Their demand reflects both the current period's expected utility from using a product of unknown quality and also the value of private information simultaneously acquired. This information is exploited only if the product has not been withdrawn from the market at date  $\bar{\tau}$ , and the person falls ill then.

Lemma 2 implies that if the price exceeds  $\bar{p}$ , sales will be zero. For this reason a successful monopolist sells drugs in successive periods at a price not exceeding  $\bar{p}$  as soon as they are invented. Hence the reputation of a cure is established within two periods. The reasoning runs as follows. Some sick people who buy the drug when it is introduced fall ill again the following period. This group buys the drug a second time if and only if it cures. Consequently, at the end of the second period in a drug's life, those people who have been healthy both periods infer from aggregate sales data whether the informed sick purchased the drug or not and hence its quality. Therefore, cures are priced at  $\bar{p}$  from then onward, while fakes are marketed for two periods at most.

THEOREM 2.  $\bar{\tau} \in \{2, 3\}$  and  $(\tilde{\gamma}_1, \tilde{\gamma}_2) = (1, 1)$ .

The theorem implies that from period 3 onward cures are priced at  $\bar{p}$ . The next lemma asserts that fakes are invariably marketed at the same price as cures. Otherwise uninformed consumers would be able to infer low quality merely by looking at the price of a fake; also their introductions and withdrawals are consistent with Bayesian expectations.

LEMMA 4. If  $h^t \in \tilde{H}^t$ , then  $\tilde{\Psi}_t = 1 - \tilde{\theta}_{t-1}(1 - \tilde{\theta}_{t-1})^{-1}(1 - \tilde{\theta}_t)\tilde{\theta}_t^{-1}$ . Also  $\tilde{p}(h^t, 0) = \tilde{p}(h^t, 1)$ .

Theorem 2 and lemma 4 imply that the equilibrium outcomes are fully characterized by the price path of a cure over the first two periods  $(\tilde{p}_1, \tilde{p}_2)$  and the beliefs of uninformed consumers  $(\tilde{\theta}_1, \tilde{\theta}_2)$ . If  $\tilde{p} < w(\alpha, \tilde{V})$ , it takes one period to establish a cure; after marketing a drug to  $\alpha$  proportion of the population, an unsuccessful monopolist finds that serving the remaining uninformed who fall sick next period is unprofitable. This is the second scenario the Introduction mentioned. There are two possibilities. Either  $\omega > w(0, \tilde{V})$  or vice versa. In both cases fakes are withdrawn after being marketed only one period at most because  $w(\alpha, \tilde{V}) > \tilde{p}$ , and, as lemma 2 shows, no uninformed person is willing to pay more than  $\tilde{p}$  for a drug. First, suppose  $\omega > w(0, \tilde{V})$ . Because there are no benefits from acquiring private information, the reservation price of consumers for a new product that cures with probability  $\theta_0$  is  $\omega$ . As that exceeds the opportunity cost of marketing fakes, every drug is introduced to the market. Second, let  $w(0, \tilde{V}) > \omega$ . That is, given beliefs of  $\theta_0$ , the opportunity cost of marketing fakes exceeds a sick person's reservation price. Accordingly, fakes are not always introduced to the market; the introductory price is  $w(0, \tilde{V})$ , people believing that the drug works with probability  $\phi(0, \tilde{V})$  at least.

LEMMA 5. Suppose  $\bar{\tau} = 2$  and  $\tilde{\Psi}_1 < 1$ . Then  $\tilde{p} \leq w(\alpha, \tilde{V})$ . In this case  $(\tilde{p}_1, \tilde{p}_2) = (w(0, \tilde{V}) \vee \omega, \tilde{p})$  and  $\tilde{\theta}_2 = 1$ . Moreover, if  $w(0, \tilde{V}) > \omega$ , then  $\tilde{\theta}_1 \geq \phi(0, \tilde{V})$ , but if  $\omega > w(0, \tilde{V})$ , then  $\tilde{\theta}_1 = \theta_0$ .

Some algebra shows that if  $(\tilde{p}_1, \tilde{p}_2) = (\omega, \tilde{p})$  and  $(\tilde{\theta}_1, \tilde{\theta}_2) = (\theta_0, 1)$ , then the value of the game to the monopolist is  $V_2$ , defined as

$$V_2 = \alpha\beta\left\{(1 - \theta_0)(\omega - w) + \theta_0\left[\omega - w + \frac{\beta(\tilde{p} - w)}{1 - \beta}\right]\right\} \times (1 - \beta + \theta_0\beta^2)^{-1}. \tag{12}$$

The expression on the right-hand side of (12) may be interpreted as follows. Every even period the monopolist randomly draws a drug from a laboratory that invents cures and fakes in the proportions  $\theta_0$  and  $(1 - \theta_0)$ , respectively. If a fake is drawn, the monopolist receives  $\alpha\beta(\omega - w)$ , but the payoff from a cure is  $\alpha\beta\{\omega - w + [\beta(\tilde{p} - w)/(1 - \beta)]\}$ . This game ends once a cure is drawn. The numerator in (12) can be interpreted as the expected payment at the beginning of every even period the game is played, while  $(1 - \beta^2 + \theta_0\beta^2)^{-1}$  can be expressed as the infinite geometric sum  $[1 + \beta^2(1 - \theta_0) + \beta^4(1 - \theta_0)^2 + \dots]$ , which is the present value of receiving one unit of account every even period until the game ends. Differentiating  $V_2$



with respect to the structural parameters proves that the value of the monopoly is positively related to how widespread the disease is (as this affects  $\alpha$ ), the degree of its intensity (which raises the reservation prices  $\omega$  and  $\bar{p}$ ), and the probability of a discovery  $\theta_0$  but is negatively related to the production costs of drugs  $w$  as well as the interest rate  $\beta^{-1}(1 - \beta)$ .

Now suppose  $(\bar{p}_1, \bar{p}_2) = (w(0, \bar{V}), \bar{p})$  and  $(\bar{\theta}_1, \bar{\theta}_2) = (\phi(0, \bar{V}), 1)$ . In this case an unsuccessful monopolist is indifferent between introducing a fake and not doing so; consequently, the value of the game to the monopolist is unaffected by never introducing fakes. Thus  $\bar{V} = V_1$ , as defined by (10), and  $\bar{p}_1 = w(0, V_1)$ . When signaling does not confer a reputation on a cure more quickly, pricing by a successful monopolist that signals is identical to what would happen under the second scenario (where no signaling occurs).

The first scenario mentioned in the Introduction occurs when  $w(\alpha, \bar{V}) < \bar{p}$  and  $\bar{\tau} = 3$ . Again, there are two cases to consider. First, suppose  $w(\alpha, \bar{V}) < \omega$ . In the second (and final) period in which a fake is marketed, the uninformed sick are willing to pay up to  $\omega$  for it even if all fakes are marketed two periods. From the discussion following definition 7, their reservation price in the first period,  $\nu(\omega)$ , exceeds  $\omega$ . Hence, with  $w(0, \bar{V}) < w(\alpha, \bar{V}) < \omega < \nu(\omega)$ , it follows that a new drug is always introduced irrespective of quality and marketed for two periods, and at that time fakes are withdrawn. Given  $(\bar{\theta}_1, \bar{\theta}_2) = (\theta_0, \theta_0)$  and  $(\bar{p}_1, \bar{p}_2) = (\nu(\omega), \omega)$ , the value of owning the monopoly is

$$V_3 = \alpha\beta \left\{ (1 - \theta_0)[\nu(\omega) - w + \beta(1 - \alpha)(\omega - w)] \right. \\ \left. + \theta_0 \left[ \nu(\omega) - w + \beta(\omega - w) + \frac{\beta^2(\bar{p} - w)}{1 - \beta} \right] \right\} (1 - \beta^3 + \theta_0\beta^3)^{-1}. \quad (13)$$

To calculate  $V_3$ , observe that  $\alpha\beta(1 - \theta_0)[\nu(\omega) - w + \beta(1 - \alpha)(\omega - w)]$  is the probability of discovering a fake multiplied by its market value,  $\alpha\beta\theta_0[\nu(\omega) - w + \beta(\omega - w) + [\beta^2(\bar{p} - w)/(1 - \beta)]]$  is the probability of discovering a cure multiplied by its market value, while  $(1 - \beta^3 + \theta_0\beta^3)^{-1}$  is the infinite geometric sum  $[1 + (1 - \theta_0)\beta^3 + (1 - \theta_0)^2\beta^6 + \dots]$ . The interpretation is similar to that for  $V_2$ , except in this case the monopolist is sampling only every three periods.

If  $\omega < w(\alpha, \bar{V}) < \bar{p}$ , drugs marketed two periods or more are priced at  $w(\alpha, \bar{V})$  in the second. This case differs from the one above because  $\omega < w(\alpha, \bar{V})$  rather than vice versa. Consequently, fakes are withdrawn with strictly positive probability after one period. The chance that a fake is withdrawn from the market after one period induces uninformed consumers to revise upward their subjective beliefs about

the probability that a drug will work to at least  $\phi(\alpha, \bar{V})$ . Since signaling is unprofitable for the successful monopolist, the results of every experiment are introduced to the market regardless of the outcome, at consumer reservation price  $v[w(\alpha, \bar{V})]$ . The direction of the price change, from  $v[w(\alpha, \bar{V})]$  to  $w(\alpha, \bar{V})$ , is ambiguous. On the one hand, the opportunity cost of marketing a low-quality drug has risen; on the other hand, the reservation price falls for any given subjective probability because of the declining value of acquiring private information. Since  $v(p)$  is declining in  $p$ , there exists a unique number  $\xi$  such that  $\xi = v(\xi)$ ; since  $v(\bar{p}) = \omega < \bar{p}$ , it immediately follows that  $\omega < \xi < \bar{p}$ . One concludes that the price rises if and only if  $w(\alpha, \bar{V}) > \xi$ .

The Introduction mentioned that people might acquire information by purchasing lower-quality goods on average rather than through paying higher prices. The scenario above shows how this can happen even before the reputation of a brand has become established. For example, although the price of a new brand rises when  $\xi < w(\alpha, \bar{V}) < \bar{p}$  in the absence of signaling, so does the expected quality of brands retained because some fakes are randomly withdrawn after one period. (As  $\bar{\psi} > 0$ , it follows that  $\bar{\theta}_1 < \bar{\theta}_2$ .) Indeed, the low introductory price offer does not compensate consumers for expected lower quality in terms of its current benefits; that is,  $u(\bar{p}_1, \bar{\theta}_1) < u(0, 0)$ . To see this, first observe that  $\bar{\theta}_1 = \theta_0$ . Then, from definition 7, notice that  $u[v(\bar{p}_2), \theta_0] - u(0, 0) = \alpha\beta\bar{\theta}[u(0, 0) - u(\bar{p}_2, 1)]$ . Since  $\bar{p}_2 = w(\alpha, \bar{V}) < \bar{p}$ , it follows from definition 2 that  $u(0, 0) < u(\bar{p}_2, 1)$ . The claim is now established because  $v(p)$  is decreasing in  $p$ . Next period, however, uninformed consumers need not take the future into account to choose optimally:  $u(\bar{p}_2, \bar{\theta}_2) = u(0, 0)$ . (The equality follows directly from definition 6.)

In this case  $\bar{V}$  must solve

$$\begin{aligned}
 & [1 - \beta^2 - \theta_0\beta^3 - \beta^2(1 - \theta_0)(1 - \beta) - \theta_0\beta(1 - \alpha)^{-1}(1 - \beta)]\bar{V} \\
 & = \alpha\beta \left\{ v[w + \alpha^{-1}(1 - \alpha)^{-1}(1 - \beta)\bar{V}] - w + \frac{\beta^2\theta_0(\bar{p} - w)}{1 - \beta} \right\}.
 \end{aligned}
 \tag{14}$$

The left-hand side of (14) is proportional to  $\bar{V}$ , while the right-hand side is positive and declining in  $\bar{V}$ , thus guaranteeing the existence of a unique solution. (It does not admit a closed form.)

Lemma 6 summarizes the first scenario.

**LEMMA 6.** Suppose  $\bar{\tau} = 3$ . Then  $w(\alpha, \bar{V}) \leq \bar{p}$ . Also  $(\bar{p}_1, \bar{p}_2) = (v[w(\alpha, \bar{V}) \vee \omega], w(\alpha, \bar{V}) \vee \omega)$ . If  $\omega > w(\alpha, \bar{V})$ , then  $(\bar{\theta}_1, \bar{\theta}_2) = (\theta_0, \theta_0)$ . If  $\omega = w(\alpha, \bar{V})$ , then  $\bar{\theta}_1 = \theta_0$  and  $\bar{\theta}_2 \geq \phi(\alpha, \bar{V})$ .

*Discussion*

Two hallmarks of previous work on games with incomplete information are evident from this analysis, namely, signaling (as analyzed by Spence [1973] and others) and randomized revelation (see, e.g., Kreps and Wilson 1982*a*). Granted, neither phenomenon necessarily occurs as an equilibrium outcome in this model, in particular, if  $(\tilde{p}_1, \tilde{p}_2) = (\omega, \bar{p})$  or  $(\tilde{p}_1, \tilde{p}_2) = (v(\omega), \omega)$ . Nevertheless, if neither occurs, the resulting equilibrium outcome is indistinguishable from that generated by a similar model in which the monopolist has no private information.

The last assertion is established by briefly considering how the two outcomes alluded to above arise in environments in which the monopolist has no private information. First, suppose that nobody (including the monopolist) knows the quality of a new brand, but everybody retrospectively sees the effect of medication on others. Then, in equilibrium, every brand is introduced at price  $\omega$ , but only a cure is retained for more than one period; hence  $(\tilde{p}_1, \tilde{p}_2) = (\omega, \bar{p})$ . Second, suppose that nobody knows the quality of a new brand and that the only way the monopolist can determine its quality is via inference from aggregate sales figures; nevertheless, as in the original model, sick uninformed consumers taking medication simultaneously become informed. Then in equilibrium every brand is marketed for two periods, so  $(\tilde{p}_1, \tilde{p}_2) = (v(\omega), \omega)$ .

The notion that randomized revelation is important in games of incomplete information is further bolstered by considering a third alternative assumption about the structure of information. Let everyone be initially uninformed and suppose that the monopolist (but not healthy people) observes the effect of the drug on those who are treated. One can show that in equilibrium all drugs are introduced, but whether low-quality brands are withdrawn or not depends on the value of  $w(\alpha, \bar{V})$ ; thus a role for the opportunity cost of marketing fakes reappears in the analysis and with it the possibility of randomized withdrawals.

**IV. Conclusion**

This paper does provide a new context in which signaling may operate, namely, as a self-regulating device in a dynamic system. However, its main contribution is to suggest what the alternatives to signaling are and when they might arise. In particular, it explicitly models the production technology, individuals' preferences, and their information sets in order to investigate the intuitively appealing idea that knowledge may diffuse throughout the population over time. When signaling is not an important factor in marketing new products, peo-

ple try out a succession of them until a significant discovery is made, at which time the pace of research slows down substantially. In such cases the first group of buyers to try each new product pays more than later groups (per unit adjusted for expected quality) because they anticipate benefiting from private information acquired through consumption. The cycles that are generated look like faddish behavior. It would, however, be a mistake to conclude, after observing market aggregates in this environment, that people behaved whimsically or differed in their preferences over goods, their attitudes toward risk, or their ability to process information.

**Appendix**

*Proof of Theorem 1*

Because lemmas 1–6 show that there are only a finite number of outcome paths for sequential equilibrium that meet condition 1, it suffices to propose an assessment  $\hat{A}$  as a candidate and then verify that  $\hat{A}$  is a sequential equilibrium satisfying condition 1. Following notation in the text, let  $t$  denote the time the most recent experiment was undertaken; that is,  $b_0 = \sum_{s=0}^{t-1} b_s = 1$ . For expository purposes,  $\hat{A}$  is partitioned into four cases. When case *a* applies, quality is inferred from past prices and quantities traded. If *b* or *c* holds, the beliefs of the uninformed depend on the unsuccessful monopolist’s opportunity costs and the previous prices charged. The last case, *d*, deals with behavior when everyone is uninformed. To economize on notation, let  $\psi(\theta', \theta)$  be the probability of withdrawing a low-quality brand, which induces uninformed Bayesian agents to revise their beliefs upward from  $\theta'$  to  $\theta$ :

$$\psi(\theta', \theta) = \begin{cases} 1 - \theta'(1 - \theta')^{-1}(1 - \theta)\theta^{-1} & \text{if } \theta' < \theta \\ 0 & \text{if } \theta' \geq \theta. \end{cases} \tag{A1}$$

In this assessment,  $\hat{p}(h^t, c_t)$  does not depend on  $c_t$ , so  $\hat{A}(h^t, c_t)$  is expressed as  $\hat{A}(h^t)$  throughout, without creating ambiguities. Likewise, since periods are dated by the age of the current brand, the  $t$  subscript on  $c_t$  is redundant and is therefore dropped from now on. The four cases are now given as follows.

*a*) Suppose  $p_s \leq \bar{p}$  and  $\Lambda_s > 0$  for some  $s < t$ . With definition 3, it follows that  $q_s = \alpha\Lambda_s c + \alpha(1 - \Lambda_s)\gamma_s$ , and hence  $c = [q_s - (1 - \alpha)\Lambda_s\gamma_s]/\alpha\Lambda_s$ . Suppose  $c = 1$ ; then the game structure implies that the triplet  $(\theta_t, \gamma_t, p_t)$  constitutes  $A(h^t)$  for any assessment  $A$ . Accordingly, set  $\hat{A}(h^t) = (1, \delta(\bar{p}), \bar{p})$ . Suppose  $c = 0$ ; then set  $\hat{A}(h^t) = (0, 0, \bar{p}, 1)$ .

*b*) Suppose that  $\Lambda_t > 0$  and assume, for all  $s < t$ , that if  $\Lambda_s > 0$  then  $p_s > \bar{p}$ . Also assume that if  $\Lambda_s = 0$  and  $\Lambda_{s+1} > 0$  then  $p_s > p_0$ . There are three subcases. First, suppose that  $\bar{p} < w(\Lambda_t, \hat{V})$ ; then set  $A(h^t) = (1, \delta(p_t), \bar{p}, 1)$ . Second, suppose that  $\omega < w(\Lambda_t, \hat{V}) < \bar{p}$ ; then set  $(\phi(\Lambda_t, \hat{V}), 1, w(\Lambda_t, \hat{V}), \psi[\theta_{t-1}, \phi(\Lambda_t, \hat{V})])$  if  $p_t \leq w(\Lambda_t, \hat{V})$  and set  $\hat{A}(h^t) = (\phi(\Lambda_t, \hat{V}), 0, w(\Lambda_t, \hat{V}), \psi[\hat{\theta}_{t-1}, \phi(\Lambda_t, \hat{V})])$  if  $p_t > w(\Lambda_t, \hat{V})$ . Third, suppose that  $w(\Lambda_t, \hat{V}) < \omega$ ; then set  $\hat{A}(h^t) = (\theta_0, 1, \omega, 0)$  if  $p_t \leq \omega$ , and set  $\hat{A}(h^t) = (\theta_0, 0, \omega, 0)$  if  $p_t > \omega$ .

c) Suppose that  $\Lambda_t > 0$  and assume, for all  $s < t$ , that if  $\Lambda_s > 0$  then  $p_s > \bar{p}$ . Also assume that if  $\Lambda_s = 0$  and  $\Lambda_{s+1} > 0$  then  $p_s \leq \bar{p}_0$ . Then set  $\hat{A}(h^t) = (1, \delta(p_t), \bar{p}, 0)$  if  $w(\Lambda_t, \hat{V}) \leq \bar{p}$  and set  $\hat{A}(h^t) = (1, \delta(p_t), \bar{p}, 1)$  if  $w(\Lambda_t, \hat{V}) > \bar{p}$ .

d) Suppose  $\Lambda_t = 0$ . Set  $h_t = (1, 1, p_0 \wedge w(0, \hat{V}), 1)$  if  $p_t \leq p_0 \wedge w(0, \hat{V})$  and set  $A(h_t) = (0, 0, p_0 \wedge w(0, \hat{V}), 1)$  if  $p_t > p_0 \wedge w(0, \hat{V})$ .

The interested reader can verify that, under assumption 1,  $\hat{A}$  is a sequential equilibrium satisfying condition 1. Q.E.D.

### Proof of Lemma 1

Suppose  $\bar{\psi}_1 = 1$ . Define dates  $r$  and  $s$  such that  $r < s$ ,  $\bar{\gamma}_r = \bar{\gamma}_s = 1$ , and  $\Sigma_{i=1}^s \bar{\gamma}_i = 2$ . By lemma 2, proved independently,  $\bar{p}_s \leq \bar{p}$ . Therefore, since  $\bar{\gamma}_r = 1$  and consequently  $\bar{\Lambda}_s > 0$ , lemma 3, proved independently, implies  $\bar{\tau} \leq s + 1$ . Necessary and sufficient conditions for an unsuccessful monopolist not to defect by introducing a fake are

$$\alpha\beta^r(p_r - w) \leq (\beta^r - \beta^{r+1})\bar{V}, \quad (\text{A2})$$

$$\alpha\beta^r(p_r - w) + \alpha(1 - \alpha)\beta^s(p_s - w) \leq (\beta^r - \beta^{s+1})\bar{V}. \quad (\text{A3})$$

The first inequality, (A2), ensures that an unsuccessful monopolist does not withdraw his fake after one period of sales in  $r$ ; the second, (A3), ensures that it is not profitable to market a fake in both periods. Subject to (A2) and (A3), a successful monopolist chooses  $r, s, p_r$ , and  $p_s$ , where  $0 < r < s$  and  $p_r \leq \bar{p}$  and  $p_s \leq \bar{p}$ , to maximize

$$\alpha\beta^r(p_r - w) + \alpha\beta^s(p_s - w) + \frac{\beta^{s+1}(\bar{p} - w)}{1 - \beta}. \quad (\text{A4})$$

By inspection, the monopolist optimally sets  $r = 1$  and  $s = 2$ . There are two cases to consider, depending on whether (A2) or (A3) is binding. When (A2) is solved with equality,  $\bar{p}_1 = w(0, \bar{V})$  (see definition 4). Then if  $w(\alpha, \bar{V}) \geq \bar{p}$ , it follows that  $\alpha\beta[w(0, \bar{V}) - w] + \alpha(1 - \alpha)\beta^2(\bar{p} - w)$  is less than  $(\beta - \beta^2)\bar{V}$ , which implies that (A3) is met with  $\bar{p}_2 = \bar{p}$ . Alternatively, assume that  $w(\alpha, \bar{V}) \leq \bar{p}$ ; setting  $\bar{p}_1 = p_0$  and  $\bar{p}_2 = \bar{p}$  solves (A3) with equality while (A2) is automatically satisfied. Theorem 1 shows that this outcome can be supported as a sequential equilibrium satisfying condition 1; hence the first part of the lemma is proved.

Now suppose that  $\bar{\psi}_1 > 0$  and  $\bar{\tau} > 2$ . Define the dates  $r$  and  $s$  as above. Then inequalities (A2) and (A3) must be satisfied; otherwise it would be suboptimal for an unsuccessful monopolist not to introduce a fake. From the previous paragraph, given (A2), (A3), and  $r < s$ , the value of the game is maximized by the successful monopolist if it chooses  $(\bar{p}_1, \bar{p}_2) = (p_0, \bar{p})$ . Hence  $\bar{\tau} = 2$ . The second sentence in the lemma follows immediately. Q.E.D.

### Proof of Lemma 2

Along the equilibrium path  $\bar{\theta}_t$  is nondecreasing in  $t$  since only fakes are withdrawn and beliefs are structurally consistent. Therefore, from definition 4,  $\bar{\theta}_t = 1$  for all  $t \geq \bar{\tau}$ . Hence  $\gamma(h^t, p_t) = 0$  if  $p_t > \bar{p}$  for  $t \geq \bar{\tau}$ .

Accordingly, suppose that  $h^t \in \bar{H}^t$  and  $\bar{p}_t > \bar{p}$  for some  $t < \bar{\tau}$ . Consider an uninformed sick person optimizing his discounted expected utility over the

duration of the current component game. Assume that his equilibrium action is to buy the product; consequently he becomes informed and hence  $\Lambda_s > 0$  for all  $s > t$ . If  $\tilde{p}_s > \bar{p}$ , then  $\delta(p_s) = 0$ . But if  $\tilde{p}_s \leq \bar{p}$  for some  $s > t$ , then by lemma 3 (which is proved independently),  $\tilde{\tau} \leq s + 1$ . Thus a sick uninformed person, whose equilibrium action is to take medication in period  $t$ , repeats his purchase at most once before a cure's reputation is established and then only if he falls ill on date  $\tilde{\tau} - 1$  and  $\tilde{p}_{\tilde{\tau}-1} \leq \bar{p}$ . Therefore, since  $u(0, 1) = 0$ , his expected utility at time  $t$  until the end of the current component game is

$$\begin{aligned} \beta^t u(\tilde{p}_t, \tilde{\theta}_t) + \alpha u(0, 0) & \sum_{s=t+1}^{\tilde{\tau}-1} \beta^s \left[ \tilde{\theta}_t + (1 - \tilde{\theta}_t) \prod_{r=t+1}^s (1 - \psi_r) \right] \\ & + \tilde{\theta}_t \alpha \beta^{\tilde{\tau}-1} [u(\tilde{p}_{\tilde{\tau}-1}, 1) - u(0, 0)]. \end{aligned} \tag{A5}$$

Now consider the following defection. The consumer does not buy the product until  $\tilde{\tau} - 1$  (provided it lasts that long), and then only if he falls ill. The expected utility, calculated at  $t$ , is

$$\begin{aligned} \beta^t u(0, 0) + \alpha u(0, 0) & \sum_{s=t+1}^{\tilde{\tau}-1} \beta^s \left[ \tilde{\theta}_t + (1 - \tilde{\theta}_t) \prod_{r=t+1}^s (1 - \tilde{\psi}_r) \right] \\ & + \alpha(1 - \tilde{\theta}_t) \prod_{r=t+1}^{\tilde{\tau}-1} (1 - \tilde{\psi}_r) \beta^{\tilde{\tau}-1} [u(\tilde{p}_{\tilde{\tau}-1}, 0) - u(0, 0)] \\ & + \alpha \tilde{\theta}_t \beta^{\tilde{\tau}-1} [u(\tilde{p}_{\tilde{\tau}-1}, 1) - u(0, 0)]. \end{aligned} \tag{A6}$$

Subtracting (A6) from (A5), one obtains

$$\begin{aligned} \beta^t [u(\tilde{p}_t, \tilde{\theta}_t) - u(0, 0)] + \alpha(1 - \tilde{\theta}_t) & \prod_{r=t+1}^{\tilde{\tau}-1} (1 - \tilde{\psi}_r) \beta^{\tilde{\tau}-1} [u(0, 0) - u(\tilde{p}_{\tilde{\tau}-1}, 0)] \\ = \tilde{\theta}_t \beta^t u(\tilde{p}_t, 1) + (1 - \tilde{\theta}_t) \beta^t u(\tilde{p}_t, 0) - \beta^t u(0, 0) \\ & + \alpha(1 - \tilde{\theta}_t) \prod_{r=t+1}^{\tilde{\tau}-1} (1 - \tilde{\psi}_r) \beta^{\tilde{\tau}-1} [u(0, 0) - u(\tilde{p}_{\tilde{\tau}-1}, 0)] \\ < (1 - \tilde{\theta}_t) \beta^t [u(\tilde{p}_t, 0) - u(0, 0)] \\ & + \alpha(1 - \tilde{\theta}_t) \prod_{r=t+1}^{\tilde{\tau}-1} (1 - \tilde{\psi}_r) \beta^{\tilde{\tau}-1} [u(0, 0) - u(\tilde{p}_{\tilde{\tau}-1}, 0)] \\ \leq (1 - \tilde{\theta}_t) [u(\tilde{p}_{\tilde{\tau}-1}, 0) - u(0, 0)] \\ & + \alpha(1 - \tilde{\theta}_t) \prod_{r=t+1}^{\tilde{\tau}-1} (1 - \tilde{\psi}_r) \beta^{\tilde{\tau}-1} [u(0, 0) - u(\tilde{p}_{\tilde{\tau}-1}, 0)] \\ = -(1 - \tilde{\theta}_t) \left[ 1 - \alpha \prod_{r=t+1}^{\tilde{\tau}-1} (1 - \tilde{\psi}_r) \beta^{\tilde{\tau}-1} \right] & [u(0, 0) - u(\tilde{p}_{\tilde{\tau}-1}, 0)]. \end{aligned} \tag{A7}$$

The first equality in (A7) expands  $u(\bar{p}_t, \bar{\theta}_t)$  using definition 1; the next line follows because  $u(\bar{p}_t, 1) < u(0, 0)$  (by hypothesis); the third is a consequence of the hypothesis that  $\bar{p}_{t-1} \leq \bar{p} < p_t$ . Collecting terms establishes the bottom equality, which, by inspection, is negative. So from (A7) one deduces that the hypothesized equilibrium path requires an uninformed consumer to move nonoptimally, implying that the contrary hypothesis is false. Q.E.D.

*Proof of Lemma 3*

Suppose  $\Lambda_t > 0$  and  $p_t \leq \bar{p}$ . Then  $q_t = \alpha[\tilde{\gamma}_t(1 - \Lambda_t) + c_t\Lambda_t]$  or  $c_t = \Lambda_t^{-1}[\alpha^{-1}q_t - \tilde{\gamma}_t(1 - \Lambda_t)]$ . If  $c_t = 0$ , then  $\tilde{\gamma}(h^s, p_s) = 0$  for all  $s > t$ . Therefore, the present value of the firm at date  $t + 1$  is

$$\bar{V} \sum_{s=t+1}^{\infty} \beta^s \bar{\Psi}_s \left[ \prod_{k=t+1}^{s-1} (1 - \bar{\Psi}_k) \right].$$

This is maximized by setting  $\bar{\Psi}_{t+1} = 1$ . If  $c_t = 1$ , then  $\bar{\theta}_{t+1} = 1$ . Hence  $\bar{p}_{t+1} = \bar{p}$  and  $\gamma(h^{t+1}, \bar{p}) = 1$ , which, from definition 3, yields the result. Q.E.D.

*Proof of Theorem 2*

This proceeds in four steps. The first step shows  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \leq 2$ . The second step shows  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \geq 1$ . Putting the two statements together, one obtains  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \in \{1, 2\}$ . For notational convenience define  $\tilde{\gamma}^{\tau} = (\tilde{\gamma}_1, \dots, \tilde{\gamma}_{\tau-1})$ . The third step shows that if  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i = 1$  then  $\tilde{\gamma}^{\tau} = (1)$ ; similarly the final step shows that if  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i = 2$  then  $\tilde{\gamma}^{\tau} = (1, 1)$ .

First, the theorem implies  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \leq 2$ . Consider the alternative hypothesis, that there exist dates  $r < s < t \leq \bar{\tau} - 1$  such that  $\tilde{\gamma}_r = \tilde{\gamma}_s = \tilde{\gamma}_t = 1$ . Then  $\Lambda_s > 0$  since  $\tilde{\gamma}_r = 1$ . Furthermore, lemma 2 implies  $\bar{p}_s \leq \bar{p}$  because  $\tilde{\gamma}_s = 1$ . Hence  $s = \bar{\tau} - 1$  by lemma 3. Consequently,  $\bar{\tau} - 1 < t$ . This inequality contradicts the alternative hypothesis. Therefore,  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \leq 2$ .

Second, the theorem also implies  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \geq 1$ . Clearly,  $\bar{\tau} > 1$ , otherwise an unsuccessful monopolist would invariably introduce its fakes at price  $\bar{p}$  and it would not be optimal for sick people to take treatment. Accordingly, consider the alternative hypothesis that  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i = 0$  for some  $\bar{\tau} > 1$ . Then the current value of an unsuccessful monopolist at date 1 is

$$\bar{V} \sum_{s=0}^{\infty} \beta^s \bar{\Psi}_{s+1} \left[ \prod_{k=1}^{s-1} (1 - \bar{\Psi}_k) \right].$$

To optimize,  $\bar{\Psi}_1 = 1$ . But lemma 1 shows that if  $\bar{\Psi}_1 = 1$  then  $\tilde{\gamma}_1 = 1$ . Therefore,  $\sum_{i=0}^{\bar{\tau}-1} \tilde{\gamma}_i > 0$ , contradicting the hypothesis.

The two paragraphs above imply  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i \in \{1, 2\}$ , and these two possibilities are now considered in turn. Third, suppose  $\sum_{i=1}^{\bar{\tau}-1} \tilde{\gamma}_i = 1$ . Then  $\tilde{\gamma}_{\bar{\tau}} = (1)$  or  $\tilde{\gamma}^{\tau} = (0, \dots, 0, 1)$  or  $\tilde{\gamma}^{\tau} = (0, \dots, 0, 1, 0, \dots, 0)$ . Suppose  $\tilde{\gamma}_{\bar{\tau}} = (1, 0, \dots, 0)$  or  $\tilde{\gamma}^{\tau} = (0, \dots, 0, 1, 0, \dots, 0)$ , where  $\tilde{\gamma}_t = 1$  (i.e., the nonzero element occurs in the  $t$ th place in  $\tilde{\gamma}^{\tau}$ ). Again the value of the game to the unsuccessful monopo-

list at the beginning of the next period is maximized by setting  $\tilde{\psi}_{t+1} = 1$ ; hence  $\tilde{\theta}_{t+1} = 1$  and  $\tilde{\gamma}(h^{t+1}, \bar{p}) = 1$ . Therefore,  $t = \tilde{\tau} - 1$ , contradicting the hypotheses that  $\tilde{\gamma}^\tau = (0, \dots, 0, 1, 0, \dots, 0)$  and that  $\tilde{\gamma}^\tau = (1, 0, \dots, 0)$ .

Now consider the hypothesis that  $\tilde{\gamma}^\tau = (0, \dots, 0, 1)$ . By lemma 1,  $\tilde{\psi}_1 = 0$ . This implies  $\beta^{\tilde{\tau}-1}[\alpha(\tilde{p}_{\tilde{\tau}-1} - w) + \beta V(\tilde{A})] > V(\tilde{A})$ ; therefore,  $\beta^s[\alpha(\tilde{p}_{\tilde{\tau}-1} - w) + \beta V_0(\tilde{A})] > V(\tilde{A})$  for all  $s < \tilde{\tau} - 1$  (since  $\beta < 1$ ); consequently,  $\tilde{\psi}_s = 0$  for all  $1 \leq s \leq \tilde{\tau} - 1$ . Therefore,  $(\tilde{\theta}_{\tilde{\tau}-1}, \tilde{p}_{\tilde{\tau}-1}) = (\theta_0, \omega)$ . Also,  $w(\alpha, \tilde{V}) > \bar{p}$  (otherwise it would not be optimal for an unsuccessful monopolist to withdraw the fake in period  $\tilde{\tau}$ ). The third step is completed by showing that there exists another sequential equilibrium assessment, denoted by  $\hat{A}$ , that satisfies condition 1 and is more profitable to a successful monopolist than  $\tilde{A}$ . This proves that  $\tilde{A}$  does not satisfy condition 2 and hence, by a contradiction argument,  $\tilde{\gamma}^\tau \neq (0, \dots, 0, 1)$ . For convenience, let  $A(h^s, h^t)$  denote  $A(h^{s+t-1})$  if  $h^s$  comprises the first  $(s - 1)$  elements of  $h^{s+t-1}$  and  $h^t$  the final  $(t - 1)$  elements. Given  $h^{\tau-1} \in \tilde{H}^{\tau-1}$ , define the assessment  $\hat{A}$  by setting  $\hat{A}(h^t) = \tilde{A}(h^{\tau-1}, h^t)$  for all  $h^t \in H^t$  and all  $t$ . The key difference between the two assessments is that in  $\hat{A}$ , but not  $\tilde{A}$ , the drug is marketed as soon as it is developed. It is straightforward to check that since  $\tilde{A}$  is a sequential equilibrium satisfying condition 1, so is  $\hat{A}$ . But

$$\begin{aligned} \hat{V}' &= \alpha\beta[(\omega - w) + \beta(\bar{p} - w)(1 - \beta)^{-1}] \\ &> \alpha\beta^{\tilde{\tau}-1}[(\omega - w) + \beta(\bar{p} - w)(1 - \beta)^{-1}] \\ &= \tilde{V}'. \end{aligned} \tag{A8}$$

Thus a successful monopolist prefers  $\hat{A}$  to  $\tilde{A}$ . Hence  $\tilde{A}$  does not satisfy condition 2. Therefore,  $\sum_{i=1}^{\tilde{\tau}-1} \tilde{\gamma}_i = 1$  implies  $\tilde{\gamma}^\tau = (1)$ .

Fourth, suppose  $\sum_{i=1}^{\tilde{\tau}-1} \tilde{\gamma}_i = 2$ . By the same argument that was used in the third step,  $\tilde{\gamma}_1 = 1$ . Therefore,  $\tilde{\gamma}^\tau = (1, 1)$  or  $\tilde{\gamma}^\tau = (1, 0, \dots, 1)$ . Suppose that, contrary to the theorem,  $\tilde{\gamma}^\tau = (1, 0, \dots, 1)$ . Now if a successful monopolist charged  $\hat{p} \leq \bar{p}$  in period 2, the  $\alpha^2$  informed sick would purchase the cure, and by lemma 3, its reputation is established in period 3. So to prevent the monopolist from defecting from  $\tilde{A}$  after discovering the cure, its value at the beginning of period 2 under  $\tilde{A}$  must be at least  $\alpha\{\alpha(\bar{p} - w) + [\beta(\bar{p} - w)/(1 - \beta)]\}$ . Moreover, since  $\tilde{\tau} > 3$  by hypothesis, the argument above shows  $\tilde{p}_2 > \bar{p}$  and hence  $\tilde{q}_2 = 0$ . Therefore, an upper bound on the value of owning a successful monopoly under  $\tilde{A}$  is  $\alpha\beta(\bar{p} - w)/(1 - \beta)$ , contradicting the lower bound derived above. Hence  $\sum_{i=1}^{\tilde{\tau}-1} \tilde{\gamma}_i = 2$  implies  $\tilde{\gamma}^\tau = (1, 1)$ , as claimed. Q.E.D.

*Proof of Lemma 4*

The formula for  $\tilde{\psi}_t$  follows directly from the definition of structural consistency. Consider any sequential equilibrium  $\tilde{A}$  satisfying condition 1. Suppose that there exists some  $h^t \in H^t$  such that if  $c_t = 0$  then  $\tilde{p}_t = p'$ , but if  $c_t = 1$  then  $\tilde{p}_t = p''$ , where  $p' \neq p''$ . A necessary condition for beliefs to be consistent is that  $\tilde{\theta}_t = 0$  if  $\tilde{p}_t = p'$ . In this event  $\tilde{\gamma}_s = 0$  for all  $s \geq t$  satisfying  $\sum_{k=0}^s b_k = 0$ . The



value of an unsuccessful firm is thus maximized by setting  $\bar{\psi}_t = 1$ . Hence  $p'' = \bar{p}$  and  $\bar{\gamma}(h', \bar{p}) = 1$ . But this implies that it is profitable for the monopolist, if unsuccessful, to defect from  $\bar{A}$  by charging  $\bar{p}$  in period  $t$ , thus upsetting the equilibrium and hence contradicting the conjecture. Q.E.D.

*Proof of Lemma 5*

Suppose, to the contrary, that  $w(\alpha, \bar{V}) < \bar{p}$ . The value to an unsuccessful monopolist from defecting by not withdrawing the fake after one period is  $\alpha(1 - \alpha)(\bar{p} - w) + \beta\bar{V}$ , which is greater than  $\alpha(1 - \alpha)[w(\alpha, \bar{V}) - w] + \beta\bar{V}$ . But definition 4 implies  $\bar{V} = \alpha(1 - \alpha)[w(\alpha, \bar{V}) - w] + \beta\bar{V}$ . Hence it is not optimal to withdraw a fake after one period, contradicting the premise that  $\bar{\tau} = 2$ . Therefore,  $w(\alpha, \bar{V}) > \bar{p}$  as claimed. Since  $\bar{\theta}_1 \geq \theta_0$ , the firm can sell its drug to the sick for at least  $\omega$ . First, if  $\omega > w(0, \bar{V})$ , it follows that  $\bar{\psi}_1 = 0$ . Bayesian consistency requires  $\bar{\theta}_1 = \theta_0$ , and hence the reservation price of the uninformed  $\omega$  is charged, as claimed. Second, suppose  $w(0, \bar{V}) > \omega$ . Since  $\bar{\gamma}_1 = 1$ , it follows that  $u(\bar{p}_1, \bar{\theta}_1) \geq u(0, 0)$ . Also  $w(\alpha, \bar{V}) < \bar{p}$  implies  $\bar{p}_1 \geq w(0, \bar{V})$ ; since  $w(0, \bar{V}) > \omega$ , it follows that  $\bar{\theta}_1 > \theta_0$ . Consequently,  $1 > \bar{\psi}_1 > 0$ , which implies  $\bar{p}_1 = w(0, \bar{V})$ . Q.E.D.

*Proof of Lemma 6*

If  $w(\alpha, \bar{V}) > \bar{p}$ , then an unsuccessful monopolist would optimally withdraw its brand after one period, contradicting the premise that  $\bar{\tau} = 3$ . Therefore,  $w(\alpha, \bar{V}) \leq \bar{p}$ . Also lemma 1 implies  $\bar{\psi}_1 = 0$ ; hence consistency of beliefs requires  $\bar{\theta}_1 = \theta_0$ . Then  $\bar{p}_1 = v(\bar{p}_2)$  and, by arguments similar to those given in the proof to lemma 5,  $\bar{p}_2 = \bar{w}(\alpha) \vee \omega$ , and  $\omega > w(\alpha, \bar{V})$  implies  $\bar{\theta}_1 = \theta_0$ , while  $\omega < w(\alpha, \bar{V})$  implies  $\bar{\theta}_1 \geq \phi(\alpha, \bar{V})$ . Q.E.D.

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