CEMMAP Lectures on Bidding Frictions & Market Microstructure

3. Auction Dynamics

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- It is useful to distinguish between:
  - **sealed bid auctions,** examples of *simultaneous move Bayesian games.*
  - and **dynamic auctions**, *dynamic games of incomplete information*, where bidders have multiple opportunities to move.
- Dynamic auctions help close the gap between auctions and (*real time*) markets.
- This lecture analyzes a dynamic ascending auction:
  - Certificates of Deposit (CDs) auctioned by Texas to local banks.
  - where banks compete on interest rate.
- Bid data retrospectively collected by the platform providers displays evidence of bidding frictions:
  - that can be interpreted as **rational inattention**,
  - or **opportunity costs** for different time segments.
  - lie at the heart of the auction dynamics.

- The mechanism is an ascending auction lasting 30 minutes:
    - A reservation interest rate and an upper bound on total available funds is set prior to bidding, usually $80 million.
    - During the auction banks can bid on up to 5 **separate parcels**.
    - Parcels are between $100,000 and $7 million (in $100,000 lots).
    - The first bid (for a parcel) is a *quantity* and an *interest rate*.
    - Subsequent bids on a parcel must **increase** the interest rate.
    - Funds are allocated to banks offering the highest interest rates.
    - Winning banks pay their (highest) **last interest rate bid**.
    - Losing banks pay nothing.
    - Partial order-filling is possible.
- The *on-the-money* (ONM) rate is:
    - defined at each instant during the auction.
    - the current lowest winning bid if no more bids are tendered.
- Banks only know whether their most recent bid is below ONM or not.

Related Literature

First published work on structural estimation of dynamic auctions

- *Applied Work on Ascending Auctions*
  - Ackerberg, Hirano & Shahriar (2011)
  - Bajari & Hortacsu (2003)
  - Cho, Paarsch & Rust (2014)
  - Daniel & Hirschleifer (1998)
- *Identification of Bidder Valuations*
  - Guerre, Perrigne & Vuong (2000)
  - Haile & Tamer (2003)
- *Unobserved Heterogeneity in Auctions*
  - Krasnokutskaya (2011)
  - Decarolis (2017)
  - Freyberger and Larsen (2017)
- *Auction Markets in the Financial Crisis*
  - Cassola, Hortacsu, & Kastl (2013)

Data

Descriptive summary measures

- Our data set contains 78 auctions from 2006-2010. We split the sample into two periods:
  - pre-2008 (*before the financial crash*)
  - post-2008 (*after the financial crash*)
- Overall there is a pool of 73 potential banks with an average of 24.5 banks entering:
  - Only one bank closed following the crash
  - Averaging across auctions, 72% of banks win.
  - Money left on the table (MLT) is the dollar difference in interest payments for a winning submission and the highest losing bid.
  - MLT is $624 (pre-2008) and $1372 (post-2008) per winning bid.
  - Average national CD rate in the post-2008 period (earliest FDIC data we have) 0.79% per annum.
  - The average reserve rate between 2008 and 2010 in these auctions at 0.71% is slightly less.

Data

Summary statistics (Table 1 of Barkley, Groeger and Miller 2021)

Summary statistics on auctions.

	Pre	Post	Pre	Post	Pre	Post	Pre	Post
	Mean		Std. dev.		Min		Max	
(I) Number of banks per auction	27.5	22.8	5.39	6.50	16.0	9.00	41.0	38.0
(II) Number of bids per parcel	13.8	12.2	20.8	23.0	1.00	1.00	186	276
(III) Proportion of bids in the money (INM)	0.67	0.69	0.13	0.12	0.31	0.44	0.87	0.87
(IV) Proportion of bids out of the money (OUT)	0.17	0.14	0.15	0.14	0.00	0.00	0.59	0.41
(V) Proportion of bids on the money (ONM)	0.17	0.17	0.07	0.06	0.08	0.08	0.42	0.32
(VI) Proportion of last bids OUT at submission	0.12	0.09	0.33	0.29	0.00	0.00	1.00	1.00
(VII) Proportion of winning bids ONM	0.08	0.07	0.07	0.05	0.00	0.01	0.27	0.19
(VIII) Size of parcels ($, in millions)	1.63	1.49	0.40	0.44	1.10	0.84	2.55	2.53
(IX) Number of parcels	1.75	1.50	1.13	0.93	1.00	1.00	5.00	5.00
(X) Proportion of banks who win	0.70	0.74	0.21	0.22	0.30	0.31	1.00	1.00
(XI) Annual reserve coupon rate	4.83	0.71	0.45	0.80	3.25	0.06	5.30	3.45
(XII) Award amount to winning bank ($, in millions)	3.88	4.51	0.48	0.57	3.20	3.57	5.08	6.31
(XIII) Jump bids ($\geq$0.01 above ONM)	0.33	0.25	0.27	0.27	0.01	0.01	3.02	1.90
(XIV) Money left on table ($):	624	1372	2116	3,607	0.00	0.00	37,660	65,380
(XV) Proportion of winning bids that were first bids	0.46	0.44	0.23	0.18	0.09	0.14	0.97	0.84
(XVI) Proportion of auctions not completely filled	0.04	0.12						

Note: The full sample consists of 24,488 bids from 78 auctions, of which 19,429 bids and 38 auctions are from pre-2008 and 5,059 bids and 40 auctions are from post-2008. We define the post-2008 period as starting September 2008 to coincide with the closure of Lehman Brothers on September 15, 2008. The total number of banks in the sample is 86, with 76 competing pre-2008 auctions and 68 competing post-2008.

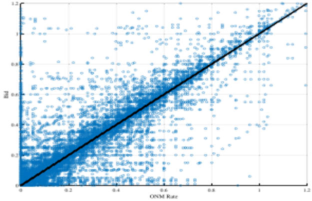
Data

How bids increase

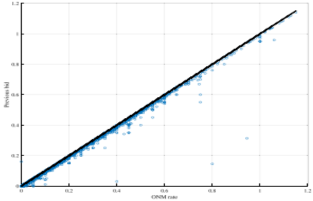
- The next slide illustrates key features of the bidding process:
  (a) Bids are submitted notably above the ONM rate.
  (b) Bids at the ONM rate are preceded by a bid close to the ONM rate.
  (c) Provisionally winning in-the-money (INM) bids are often large jumps above the bidder's most recent bid.
- Then we show all bids in a single auction to display:
  - First ONM rate is stable (as orders fill at reserve rate)
  - Then ONM steadily rises (as lowest bids become stale).
  - Jump bids (often to an INM rate)
  - Creeping bids (to the ONM rate)

Data

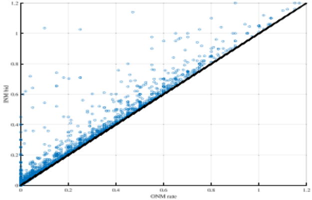
Figure 2 BGM 2021



(a) All bids



(b) Bids preceding an ONM bid



(c) INM bids made after reaching ONM

g. 2. Bidding patterns. These figures display bids in terms of percentage points above the reserve rate for the Texas CD auctions. Fig. 2(a) shows l bids plotted against the ONM rate at the time the bid was placed. Fig. 2(b) shows the bids placed just before an ONM bid was placed. Fig. 2(c) lots INM that were made after reaching or exceeding the ONM rate on a previous bid and represent jumps made after reaching the ONM rate.

Data

Figure 1 of BGM 2021

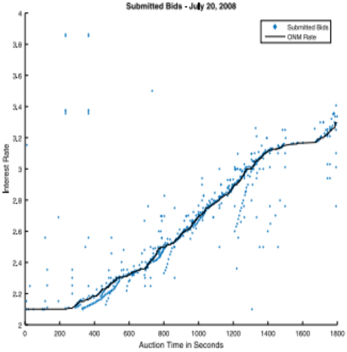
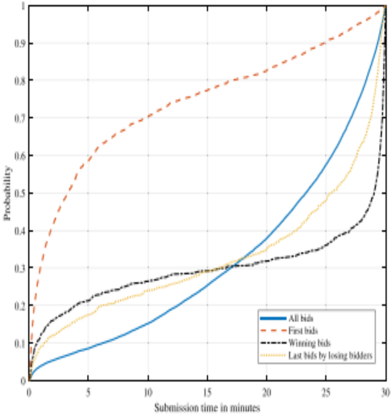


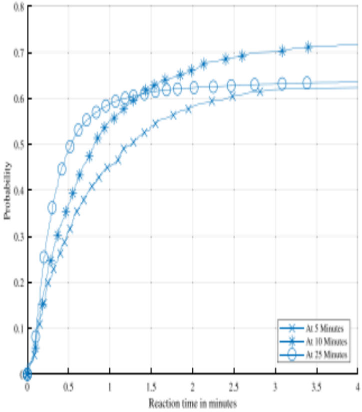
Fig. 1. Bid data from sample auction. The figure shows each submitted bid within one auction. Each point represents a single bid, and the solid black line is the ONM rate. The ONM rate starts at the auction reserve rate of 2.100 and increases over the course of the auction as more INM bids are placed.

- Figure 3a (next slide) shows the empirical distributions of:
  - initial bid submission times (*over 50% of first bids within 5 minutes*).
  - submission times of all bids (*least activity in middle of auction*).
  - winning bid submission times (*Many submitted prior to the final minutes of the auction*).
- If a bid is not INM or ONM then it is OUT (a losing bid).
- Figure 3b shows empirical distributions of **reaction times**:
  - the time to return to INM (*submitting an INM bids after being pushed OUT*).
  - conditional on being thrown OUT at the 5, 10, and 25 minutes mark.
  - noting many do not resubmit within five minutes (and may quit the auction).
  - As the auction progresses monitoring becomes more intense.

Figure 3 of BGM 2021



(a) Submission Times



(b) Reaction Times

Data

Frictions induce dynamics

1. The number of banks is uncertain until the auction ends.
2. Bidding activity is most intense at the beginning and end of the auction (like a *limit order market*).
3. **Sniping is not universal**:
   - many winning bids are submitted in the first few minutes.
   - This *rules out observational equivalence to FPSB auctions*.
4. **The interest rate spread of winning bids is remarkable**:
   - Let $f_{\underline{W}}$ denote the density of the lowest winning bid and $f_{\overline{W}}$ as the density of the highest.
   - The hypothesis of $f_{\underline{W}} = f_{\overline{W}}$ is rejected at the 1% confidence level. (See Lee, 1996 for the test.)
   - This *rules out observational equivalence with frictionless Japanese and English auctions*.

- Set of bidders given by $\mathcal{I} = \{1, ..., I\}$.
- Bidding occurs over a fixed time interval $[0, T]$.
- Auctions indexed by $k = 1, , , K$.
- Valuation for bidder i in auction k is given by:

$$\tilde{v}_{ik} \equiv x_{ik} y_k + r_k .$$

  - r_k is auction reserve rate.
  - x_{ik} is private value signal, an iid draw from distribution F_X.
  - y_k is auction specific component affecting all bidders, drawn from F_Y with $E[\ln Y] = 0$.

- X_{ik} and Y_k are *iid*.

- Bidders face frictions, and can only bid at random times:
  - *The monitoring mechanism is assumed to be an (exogenous) long term employment/time scheduling choice.*
- Let $G_t(z)$ with density $g_t(z)$ denote the probability that a bidder receives a bidding opportunity at or before time $z > t$:
  - his first opportunity if $t = 0$.
  - after being pushed OUT at time $t > 0$.
- Let t_j denote the time of a bidder's j^{th} bidding opportunity, and r_j the ONM rate at time t_j:
  1. The bidder first announces a number c_j.
  2. If $c_j \geq r_j$, the bidder learns the ONM rate and makes a bid $b_j \geq r_j$.
  3. If $c_j < r_j$ then i exits the auction.
- The partial history for the bidder is $h_j = \{\tau_s, r_s, b_s\}_{s=1}^{j-1}$.

- If $c_j \geq r_j$, then the bidder chooses $b \in [r_j, \infty)$ to solve:

$$V(h_j, r_j, v) = \max_b \left\{ \begin{array}{l} [\Pr(b \geq \overline{r}) \cdot (v - b)] \quad + \\ E\left[1\{b < \overline{r},\ j < J,\ v > r_{j+1}\}\ V(h_{j+1}, r_{j+1}, v) \middle| h_j, r_j, v\right] \end{array} \right\}$$

where:

  - J is the last bidding opportunity (a random variable).
  - $\overline{r}$ is the lowest winning bid at the end of the auction.
  - The first term in the sum corresponds to the case where there is not a future bidding opportunity.
  - The second term is the case in which the current bid b_{ij} is pushed OUT prior to the end of the auction and the bidder obtains another chance to bid after being pushed out.

Model

First order condition

- The first order condition of the bidder's problem is

$$
\begin{aligned}
0 &= \frac{\partial \Pr(b > \bar{r})}{\partial b}(v - b) - \Pr(b > \bar{r}) \\
&+ \frac{\partial}{\partial b} E\left[1\{b < \bar{r},\ j < J,\ v > r_{j+1}\}\ V(h_{j+1}, r_{j+1}, v) | h_j, r_j, v\right]
\end{aligned}
$$

- The first line is exactly the FPSB case.
- The second term arises from multiple bidding opportunities:
  - $\bar{r}$ and r_j depend on the strategies of all the other bidders
  - their strategies depend on previous bids (and the unobserved valuations).
- In principle, solving for the equilibrium gives the functional relationship between these terms.

- There may be multiple equilibria, including mixed strategy equilibria.
- Solving for equilibria is computationally burdensome (infeasible with current technology).
- Instead, we utilize a condition that is:
  - more general than a best response.
  - consistent with any equilibrium outcome distribution.
  - robust to different equilibria played in different auctions.
- Specifically, we assume that:
  1. Bidders never submit a bid greater than their valuation.
  2. Bidders submit bids at every bidding opportunity until the reservation price climbs above their valuation.

Identification

Three steps

1. $G_t(z)$, the distribution of bidding opportunities:
   - Formally valuations are independent of the monitoring distribution
   - Hence $G_t(z)$ is identified from reaction times.
2. F_V, the distribution of valuations:
   - is identified by bidders submitting INM bids or not, as ONM rate increases.
   - Information in the last bid subsumes information in all previous bids.
   - If a bidder stops becomes inactive, it is because either
     1. the last bid is a winning bid.
     2. the ONM rate overtook the bidder's valuation.
     3. another opportunity to bid was never received.
   - Conditional on $G_t(z)$ the likelihood for F_V is concave.
3. The *individual* x and *auction-specific* y *valuation distributions* are identified off data on multiple bidders in an auction.

Estimation

The distribution of bidding opportunities and valuations

- The distribution of bid opportunities is estimated according to

$$\hat{G}_t(z) = \frac{\sum_{k=1}^{K} \sum_{i \in \mathcal{I}^k} \sum_{j=1}^{J_i} K\left(\frac{\tau_{ikj}^* - t}{h}\right) \mathbf{1}\{t_{ikj} - t_{ikj}^* < z\}}{\sum_{k=1}^{K} \sum_{i \in \mathcal{I}^k} \sum_{j=1}^{J_i} K\left(\frac{t_{ikj}^* - t}{h}\right)}$$

  where t_{ikj} is when bidder i re-enters after falling OUT at t_{ikj}^*.
- F_V is estimated by maximizing the likelihood across all bidders and auctions:

$$\hat{F}_V = \arg\max_F \prod_k \prod_i \mathcal{L}(F_V, \{\hat{G}_t\}; t_{ikJ}^*, t_{ikJ}, r_{ikJ}, \overline{r}_k, w_{ik})$$

  where k indexes auctions, $w_{ik} \in \{0, 1\}$ indicates whether bidder i is a winner in auction k or not.

Estimation

Likelihood components

- $\mathcal{L}(F_V, \{G_t\}; \{t_j^*, t_j, r_j\}_{j=1}^J)$ is formed from three events:
  1. The bidder submits a winning bid at time t_j:

$$g_{t_j^*}(t_j) \times \Pr(v > \bar{r} | v > r_{j-1}) = g_{t_j^*}(t_j) \left[\frac{1 - F_V(\bar{r})}{1 - F_V(r_{j-1})} \right]$$

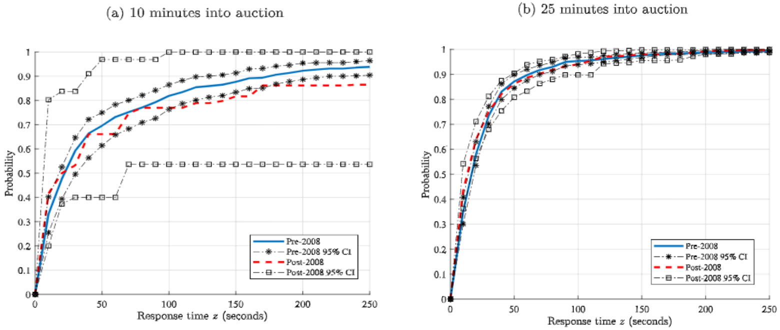
  2. The bidder submits another bid at time t_j, this bid is pushed OUT at t_{j+1}, another bidding chance is received and $c_j > r_j$:

$$g_{t_j^*}(t_j) \times \Pr(v > r_j | v > r_{j-1}) = g_{t_j^*}(t_j) \left[\frac{1 - F_V(r_j)}{1 - F_V(r_{j-1})} \right]$$

  3. No other bids are submitted by the bidder: either another chance to bid is never obtained or $c_j < r_j$:

$$\sum_{s=r_{t^o_{J_i - 1}}}^{\rho} \left[G_{t^*_{J_i}}(t_{s+1}) - G_{t^*_{J_i}}(t_s) \right] \frac{F_V(r_{s+1})}{1 - F_V(r_{J-1})} + (1 - G_{t_J^*}(T)).$$

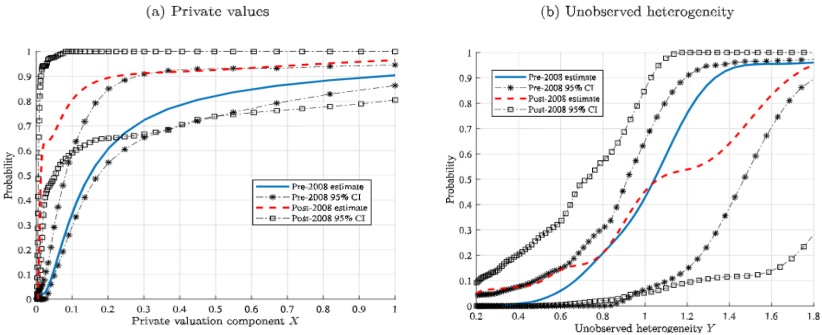
  where $s = 1, ..., \rho$ indexes ONM rate increases during the auction.



- The response time near the end of the auction is much faster than at the beginning.
- It is quite hard to distinguish the pre-2008 the monitoring rates from the post-2008 rate.

Empirical Results from the Model

The distribution of valuations (Figure 5 of BGM 2021)



- Private valuations have higher mean and variance pre-2008 than post-2008.
- Unobserved auction-specific terms (centered at zero by normalization) also have lower dispersion post-2008.
- Because valuations are more dispersed pre-2008, frictions distort the relationship between final bids and valuations to a greater extent.

Costs of Bidding Frictions

Two sources of loss from bidding frictions

- Frictions affect auction outcomes through the inability of high-valuation bidders to respond to falling OUT:
  - Lower valuation bidders win.
  - Less revenue is generated.
- We can bound the expected valuation of winning bidders to compare this mechanism with a uniform price ascending auction without frictions in which all winners pay the highest loser's valuation.

Costs of Bidding Frictions

Bounding a conditional expectation

- Let W denote the event of placing a winning bid and $\widetilde{W}$ its complement, losing. Then:

$$\begin{aligned} E\left[v\right] &= \Pr\left[W\right] E\left[v \left| W\right.\right] + \Pr\left[\widetilde{W}\right] E\left[v \left| \widetilde{W}\right.\right] \\ \Longrightarrow E\left[v \left| W\right.\right] &= \left\{ E\left[v\right] - \Pr\left[\widetilde{W}\right] E\left[v \left| \widetilde{W}\right.\right] \right\} / \Pr\left[W\right] \end{aligned}$$

- Denote by $\{t_s\}_{s=1}^{\rho}$ the times when the ONM rate changes.
- Let t_η denote the time its final bid becomes stale, that is when the ONM changes to r_η.
- Denoting by b_η its last bid, it follows that $r_{\eta-1} < b_\eta < r_\eta$.
- Since the bank would bid at its first opportunity after its bid falls OUT if its valuation remains higher than the reservation price then:

$$E\left[v \left| \widetilde{W}\right.\right] < \left\{ \sum_{s=\eta}^{\rho} \frac{G_{t_\eta}\left(t_s\right) - G_{t_\eta}\left(t_{s+1}\right)}{G_{t_\eta}\left(t_\rho\right)} \int_{r_\eta}^{r_s} \frac{vf\left(v\right)}{F\left(r_s\right) - F\left(r_\eta\right)} dv \right\}$$

Costs of Bidding Frictions

Estimates of the losses from frictions

	Pre-2008	Post-2008
Lower Bound on $\mathbb{E}[V\|W]$	0.57	0.33
Upper Bound on $\mathbb{E}[V\|W]$	0.59	0.36
Expected Valuation of Winner, Uniform Price	0.63	0.36
% Increase in Revenue using Uniform Price	8.54	2.63

- The bounds on expected valuations are quite informative.
- The absence of frictions leads to:
  - some improvement in allocative efficiency and revenue
  - especially prior to the financial crisis.
- Less dispersion in valuations and greater bidding opportunities in the post-2008 auctions help explain the lower gains in allocative efficiency relative to pre-2008.