# Uncertainty and Learning in Pharmaceutical Demand
# Crawford and Shum
# (Econometrica, 2005)

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October 16, 2017

# Overview

- Uncertainty and heterogeneity in patient illness make any drug treatment model a complex matching process
- Risk aversion leads to persistence in drug use, but new trying new drugs allows learning to take place
- Dynamic discrete choice model with Bayesian updating to solve the doctor-patient optimization problem
- Drug-patient match can vary in two dimensions - symptomatic and curative match parameters
- Attempt to quantify the importance of uncertainty and learning in drug market

# Empirical Regularities

TABLE II

SWITCHING PROBABILITIES OVER THE COURSE OF TREATMENT[a]

| Prescription Number | Total Treatment Length | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 14.3 | 13.6 | 10.9 | 10.0 | 7.8 | 9.2 |
| 3 | 11.6 | 11.6 | 6.3 | 8.8 | 7.8 | 6.6 |
| 4 | 8.9 | 5.6 | 5.4 | 3.1 | 7.8 | 3.9 |
| 5 | 13.4 | 7.9 | 10.0 | 8.8 | 4.9 | 5.3 |
| 6 | | 11.3 | 6.3 | 5.7 | 2.9 | 5.3 |
| 7 | | | 9.5 | 10.0 | 7.8 | 11.8 |
| 8 | | | | 8.1 | 4.9 | 11.8 |
| 9 | | | | | 7.8 | 5.3 |
| 10 | | | | | | 11.8 |

[a]The $(i, j)$th entry is the percentage of treatment sequences of length $j$ in which a switch was observed during the $i$th $(i \leq j)$ prescription.

Figure: Switching Probabilities

# Timeline

- Patient contracts ulcer, visits doctor
- Doctor assigns severity type k $\in$ (1,..K). Using prior information about drug options and severity type, doctor chooses drug n.
- Match quality parameters drawn from distribution, unknown to patient. Noisy signals drawn each period to give patient information about true match quality.
  - Symptomatic parameter and curative parameter
- If the patient is not cured at the end of period, patient/doctor decide to take the same drug again or switch to a new drug.
- This process continues until the patient recovers.

# Model Basics

- Doctor/patient j seeks to maximize expected discounted utility by choosing drug sequence D:

$$\max_{D \equiv \left\{ \{d_{jnt}\}_{n=1}^{N} \right\}_{t=1}^{\infty}} \mathbb{E} \sum_{t=1}^{\infty} \sum_{n=1}^{\infty} \beta^t (1 - w_{j,t-1}) d_{jnt} u_{jnt}$$

- CARA preferences:

$$u(x_{jnt}, p_n, \epsilon_{jnt}) = -exp(-r * x_{jnt}) - \alpha * p_n + \epsilon_{jnt}$$

- Vector of state variables:

$$S_t = (\mu_{j1}^t, ..., \mu_{j5}^t, \nu_{j1}^t, ...\nu_{j5}^t, l_{j1}^t, ...l_{j5}^t, h_{jt}, \epsilon_{j1t}, ...\epsilon_{j5t})$$

- Bellman equation:

$$W(S_t) = \max_n \mathbb{E}(u(x_{jnt}, p_n, \epsilon_{jnt}) + \beta(1 - h_{jt}) \mathbb{E}(W(S_{t+1}) \mid x_{jnt}, y_{jnt}, n) \mid S_t)$$

# Symptomatic Match Parameter

- Indicates match quality for side effects
- True symptomatic match parameter $\mu_{jn}$ drawn from distribution N($\mu_{\_n}$, ${\sigma_{\_n}}^2$), unknown to patient
- Patient draws signal $x_{jnt}$ from N($\mu_{jn}$, $\sigma_n^2$) each period, enters utility function
- Patient j posterior beliefs:

$$\mu_{jn}^{t+1} = \begin{cases} \dfrac{\frac{\mu_{jn}^t}{V_{jn}^t} + \frac{x_{jnt+1}}{\sigma_n^2}}{\frac{1}{V_{jn}^t} + \frac{1}{\sigma_n^2}} & \text{if drug n taken in period t+1} \\ \mu_{jn}^t & \text{otherwise} \end{cases}$$

$$V_{jn}^{t+1} = \begin{cases} \dfrac{1}{\frac{1}{\sigma_{\_n}^2} + \frac{l_{jn}^{t+1}}{\sigma_n^2}} & \text{if drug n taken in period t+1} \\ V_{jn}^t & \text{otherwise} \end{cases}$$

# Curative Match Parameter

- Indicates match quality for curing patient
- True curative match parameter $\nu_{jn}$ drawn from distribution N($\nu_{\underline{n}k}$, ${\tau_{\underline{n}}}^2$), unknown to patient
- Patient draws signal $y_{jnt}$ from N($\nu_{jn}$, $\tau_n^2$) each period, updates probability of recovery
- $h_{j0}$ recovery probability that patient j healed without any treatment

$$h_{jt}(h_{jt-1}, y_{jnt}) = \frac{\frac{h_{jt-1}}{1-h_{jt-1}} + d_{jnt}y_{jnt}}{1 + \frac{h_{jt-1}}{1-h_{jt-1}} + d_{jnt}y_{jnt}}$$

# Primitives and Data

- The primitives of the model
  - Drugs symptomatic effects: $\underline{\mu}_{nk}$, $\underline{\sigma}_n{}^2$, and $\sigma_n$
  - Utility function parameters: $r$ and $\alpha$
  - Drugs curative effects: $\underline{\nu}_{nk}$, $\underline{\tau}_n$, $\tau_n$, and $h_{0j}$
- Data: For each patient $j$, observations on
  - sequence of drug choices
  - the lower bound of treatment length ($T_j$)

# Identification

- The main identification restriction:
  - Drug's symptomatic effects only impact a patient utility
  - Drug's curative effects impact the recovery probabilities
- Utility Function and Symptomatic Match Parameters: $\underline{\mu}_{nk}$, ${\underline{\sigma}_n}^2$, $r$, $\sigma_n$, and $\alpha$

$$\begin{aligned}\mathbb{E}_{x_{jn1}}(u_{jn1}) &= -\mathbb{E}_{\mu}\mathbb{E}_{x|\mu}\Big(exp(-rx_{jn1})\Big) - \alpha p_n + \epsilon_{jn1} \\ &= -exp\Big(-r\underline{\mu}_{jn} + \frac{1}{2}r^2(\sigma_n^2 + \underline{\sigma}_n^2)\Big) - \alpha p_n + \epsilon_{jn1}\end{aligned}$$

$$V_{jn}^{t+1} = \begin{cases} \dfrac{1}{\frac{1}{\underline{\sigma}_n^2} + \frac{l_{jn}^{t+1}}{\sigma_n^2}} & \text{if drug n taken in period t+1} \\ V_{jn}^{t} & \text{otherwise} \end{cases}$$

# Identification

- Curative Match Parameters: $\nu_{\underline{n}k}$, $\tau_{\underline{n}}$, $\tau_n$, $h_{0j}$
  - ( jointly) variation in recovery frequencies conditional on different sequence of drug choices

$$\mathbb{E}[h_{jt}(h_{jt-1}, y_{jnt}) \mid \mathbb{S}] = \mathbb{E}_{v_{jn}} \mathbb{E}_{y_{njt}|v_{jn}} [\frac{\frac{h_{jt-1}}{1-h_{jt-1}} + d_{jnt} y_{jnt}}{1 + \frac{h_{jt-1}}{1-h_{jt-1}} + d_{jnt} y_{jnt}} \mid \mathbb{S}]$$

# Estimation Method

- Likelihood function for each patient j:

$$\sum_{k=1}^{K} p_k \mathbb{E}_{\vec{x}_{jnT_j,k}|h_{0j,k}} \left[ \prod_{t=1}^{T_j-1} \left( (1-h_{jt,k}) \prod_{n} \lambda_{jnt,k}^{d_{jnt}} \right) \right] h_{jnT_j,k} \prod_{n} \lambda_{jnT_j,k}^{d_{jnT_j}}$$

  - $\vec{x}_{jnTj,k} \equiv$ vector of experience signals until t
  - $\lambda_{\text{jnt,k}}^{d_{jnt}} \equiv \mathbb{E}(\mathbb{I}\{W_{jnt,k} > W_{jn't,k}, n' \neq n\})$
    $\epsilon_{jnt}$ are i.i.d. Type I extreme value, by Rust (1987)
    $\Rightarrow \lambda_{\text{jnt,k}}^{d_{jnt}} = \frac{\exp(W_{jnt,k})}{\sum_{n'=1}^{5} \exp(W_{jn't,k})}$
  - $h_{jnT_j,k}$: Only for uncensored observations

# Estimation Method

- Simulated maximum likelihood estimation: S draws of the unobservables $(k, \vec{x}_{jnT_j,k})$ for each patient

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{k=1}^{K} p_k \left[\prod_{t=1}^{T_j-1}\left((1-h^s_{jt,k})\prod_n(\lambda^s_{jnt,k})^{d_{jnt}}\right)\right] h^s_{jnT_j,k}\prod_n(\lambda^s_{jnT_j,k})^{d_{jnT_j}}$$

- Number of unobsevable types: start from 2 until negligible changes in model fit and qualitative conclusions
- Keane and Wolpin (1994) approximation method for computing value functions

# Model Fit

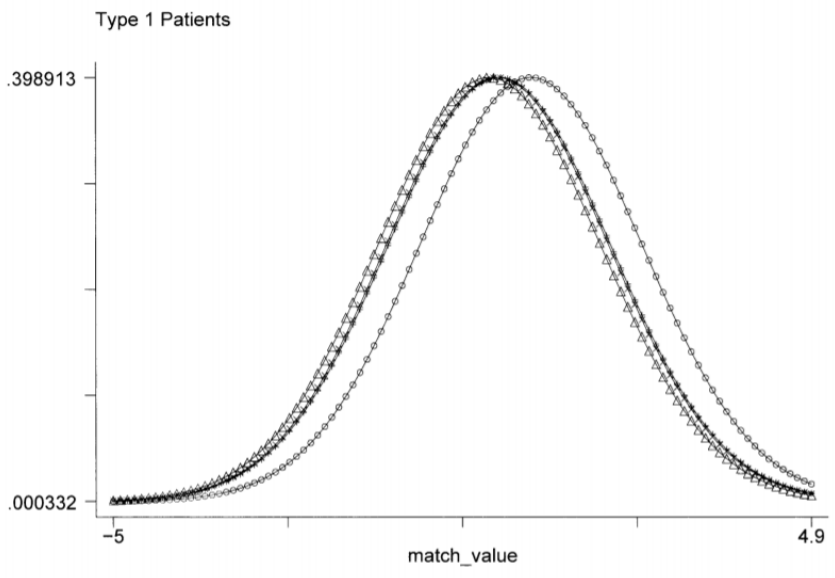
TABLE V

MODEL FIT: MARKET SHARES AND TREATMENT CHARACTERISTICS

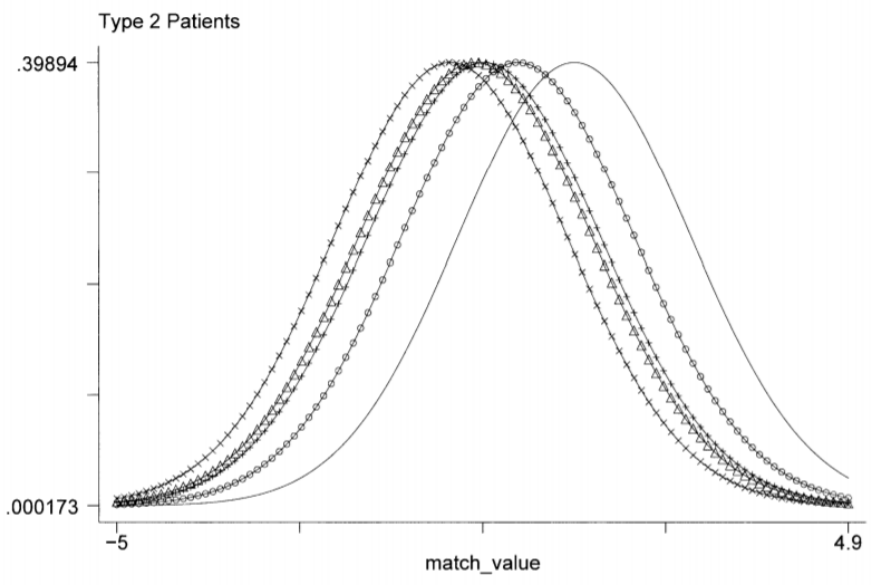
| | Overall | 1st Pres | 2nd Pres | 3rd Pres | 4th Pres |
|---|---|---|---|---|---|
| | | | Actual Data | | |
| Treat. length[a] | 2.8 | | | | |
| Treat. cost[b] | 147[c] | | | | |
| Market shares | | | | | |
| Drug 1 | 64.4 | 57.4 | 62.7 | 65.7 | 67.6 |
| Drug 2 | 11.0 | 9.5 | 11.6 | 12.4 | 12.1 |
| Drug 3 | 6.8 | 6.9 | 7.3 | 7.0 | 6.7 |
| Drug 4 | 3.3 | 3.5 | 3.2 | 3.1 | 3.1 |
| Drug 5 | 14.7 | 22.7 | 15.1 | 11.9 | 10.4 |
| | | | Simulated Data | | |
| Treat. length | 2.8 | | | | |
| Treat. cost | 146 | | | | |
| Market shares | | | | | |
| Drug 1 | 61.4 | 57.7 | 56.6 | 60.9 | 63.9 |
| Drug 2 | 14.2 | 13.8 | 15.7 | 14.8 | 14.4 |
| Drug 3 | 4.6 | 4.5 | 4.8 | 5.2 | 5.0 |
| Drug 4 | 2.6 | 3.1 | 2.6 | 2.8 | 2.6 |
| Drug 5 | 17.1 | 21.0 | 20.3 | 16.4 | 14.1 |

Figure: Model Fit

# Estimation Results of Primitives




(a) Type 1 (not-so-sick patients)




(b) Type 2 (sick patients)

Figure: Heterogeneity in symptomatic match values

# Uncertainty, Experimentation, and Its Consequences

- Learning occurs quickly - both symptomatic and curative impacts falls significantly after first prescription.
- Patients are risk averse - strong disincentive for switching
- Patients value reduced uncertainty at over 65% of their co-pay

# Counterfactuals

TABLE VI

RESULTS FROM COUNTERFACTUAL SIMULATIONS

| Baseline Specification[a] | |
|---|---|
| Avg. discounted utility | −28.7 |
| Avg. treatment length | 4.8 |
| Avg. treatment cost | 245 |
| Avg. number of different drugs | 1.4 |
| Market shares | |
| Drug 1 | 60.4 |
| Drug 2 | 14.1 |
| Drug 3 | 3.7 |
| Drug 4 | 2.5 |
| Drug 5 | 19.3 |
| Herfindahl index | 4,242 |

(a) Baseline Specification

| Counterfactual I: Complete Information[b] | |
|---|---|
| Avg. discounted utility | −26.4 |
| Avg. treatment length | 8.8 |
| Avg. treatment cost | 385 |
| Avg. number of different drugs | 1.9 |
| Market shares | |
| Drug 1 | 22.4 |
| Drug 2 | 12.9 |
| Drug 3 | 12.0 |
| Drug 4 | 10.9 |
| Drug 5 | 41.8 |
| Herfindahl index | 2,676 |
| **Counterfactual II: No Experimentation[c]** | |
| Avg. discounted utility | −30.6 |
| Avg. treatment length | 4.8 |
| Avg. treatment cost | 248 |

(b) Counterfactuals

Figure: Results from Counterfactual Simulations

# Conclusion

- Analysis reveals the importance of both experimentation and learning in drug choice
- Possible extensions
  - Allowing correlation between match parameters
  - Using doctor-level data to analyze learning by doctors across patients