Equilibrium Tuition, Applications, Admissions, and Enrollment in the College Market
by Chao Fu (2014)

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11/20/2017

Introduction

- Interested in counter-factual college attendance and tuition levels.
- Related literature: Manski and Wise (1983); Arcidiacono(2005); Epple, Romano, and Sieg (2006).
- Model college tuition, applications, admissions and enrollment as joint outcomes from a subgame perfect Nash equilibrium (SPNE).
  - ⋄ Costly application, heterogeneous student ability and preferences
  - ⋄ Noisy signals of student ability
- Timing:
  - ⋄ Stage 1: colleges simultaneously announce tuition (committed)
  - ⋄ Stage 2: Students apply. Colleges simultaneously admit.
  - ⋄ Stage 3: Students learn admission and aid decisions, then enroll.
- Estimation following Moro (2003) and using NLSY97.

Setup: Colleges

- Student i can apply to J four-year colleges indexed by j, each with fixed capacity κ_j. $\kappa_j > 0\ \forall j$ and $\sum_{j=1}^{J} \kappa_j < 1$. j divided into four groups (g_j).
- One two-year community college indexed by $j = J + 1$ with no application needed.
- Focus on symmetric equilibrium (same college policies within g_j)
- Private college j has payoff:

$$W_j = \int (\omega_{a_i} + m_{1j}\pi_{ji}) dF_j^*(i) + m_{2j}\frac{\Pi_j^2}{N_j}, \text{ where } \pi_{ji} = t_{ji} - f_{ji} \quad (1)$$

- Public college j has payoff:

$$W_j = \sum_{\iota}^{1} [\int (\omega_{a_i} + m_{1j\iota}\pi_{ji\iota}) dF_{j\iota}^*(i) + m_{2j\iota}\frac{\Pi_{j\iota}^2}{N_{j\iota}}], \text{ where } \iota \equiv I(l_i = l_j) \quad (2)$$

Setup: Students

- Students divided into unobserved types $K(A, z)$ with ability $A \in \{1, 2, 3\}$ and public/private college preference $z \in \{1, 2\}$. Distributed according to $P(K|SAT, B)$.

- Student i obtains general financial aid f_{0i} and college-specific aid f_{ji}

$$f_{ij} = max\{f_j(B_i, SAT_i) + \eta_{ji}, 0\}, \; \eta_{ij} \sim N(0, \Omega_\eta) \tag{3}$$

- A type K student i's preferences for colleges j is

$$u_{ji} = \bar{u}_{g_j K} + \epsilon_{1 g_j i} + \epsilon_{2ji}, \; \epsilon_{1 g_j i} \sim N(0, \sigma^2_{\epsilon_{1 g_j i}}), \; \epsilon_{2ji} \sim N(0, \sigma^2_{\epsilon_{2ji}}) \tag{4}$$

- Given tuition profile $t\{\{t_{jl}\}_l\}_j$ and distaste for studying out-state $\xi_i \sim N(\bar{\xi}_K, \sigma^2_\xi)$, the ex post value of attending j for i is:

$$U_{ji}(t) = (-t_{ji} + f_{0i} + f_{ji}) + u_{ji} - I(l_j \neq l_i)\xi_i \tag{5}$$

Decision: Student's Problem

- Information: Student has private info $X_i = (K_i, B_i, \epsilon_i)$
- Enrollment: Strategy $d(O_i, X_i, \eta_i|t)$ solves:

$$v(O_i, X_i, \eta_i|t) = max\{U_{oi}, \{U_{ji}(t)\}_{j \in O_i}\} \tag{6}$$

- Application: Given admission probability $p_j(A_i, SAT_i|t)$, value of application portfolio Y is

$$V(Y, X_i, SAT_i|t) = \sum_{O \subseteq \{Y, J+1\}} Pr(O|A_i, SAT_i, t) E[v(O, X_i, \eta_i|t)] - C(|Y|) \tag{7}$$

and strategy $Y(X_i, SAT_i|t)$ solves $max_{Y \in \{1,\ldots,J\}}\{V(Y, X_i, SAT_i|t)\}$

Decision: College's Problem

- Information: Colleges observes SAT and signal $s \in \{1, 2, 3\}$ of applicant's ability A. $P(s|A)$ is public knowledge. Also observes $l_i \in B_i$ if practicing origin-based discrimination.

- Admission policy $e_j(s, SAT|t)$ maximizes

$$\sum_{s,SAT} e_j(s, SAT|t)\alpha_j(s, SAT|t, e_{-j}, Y, d)\mu_j(s, SAT|\cdot)\gamma_j(s, SAT|\cdot) \tag{8}$$

subject to

$$\sum_{s,SAT} e_j(s, SAT|t)\alpha_j(s, SAT|t, e_{-j}, Y, d)\mu_j(s, SAT|\cdot) \leq \kappa_j \tag{9}$$

$$e_j(s, SAT|t) \in [0, 1] \tag{10}$$

- Tuition policy $\tilde{t_{jl}}$ maximizes $E(W_j|AE(\tilde{t_j}, t_{-j}))$, with private college tuition the same across l. $AE(\cdot)$ is the equilibrium profile.

Subgame Perfect Nash Equilibrium

- Given tuition profile t, a symmetric application-admission equilibrium, denoted as AE(t), is $(d(\cdot|t), Y(\cdot|t), e(\cdot|t), p(\cdot|t))$ such that
  - ◇ $d(O, X, \eta|t)$ is an optimal enrollment decision for every (O, X, η)
  - ◇ given $p(\cdot|t)$, $Y(X, SAT|t)$ is an optimal college application portfolio for every (X, SAT)
  - ◇ for every j, given $(d(\cdot|t), Y(\cdot|t), p_j(\cdot|t))$, $e_j(\cdot|t)$ is an optimal admission policy, and $e_j(\cdot|t) = e_{j'}(\cdot|t)$ if $g_j = g_{j'}$
  - ◇ $p_j(A, Sat|t) = \sum_s P(s|A)e_j(s, SAT|t)$ (consistency)
- A symmetric subgame perfect Nash equilibrium for the college market is $(t*), d(\cdot|\cdot), Y(\cdot|\cdot), e(\cdot|\cdot), p(\cdot|\cdot))$ such that
  - ◇ for every t,$(d(\cdot|\cdot), Y(\cdot|\cdot), e(\cdot|\cdot), p(\cdot|\cdot))$ constitutes and AE(t)
  - ◇ for every j, given $t*_{-j}$, $t*_j$ solves tuition policy problem, and $t*_j = t*_{j'}$ if $g_j = g_{j'}$

Identification

Information asymmetry

- Both application and admission decisions depend on student type $K = (A, z)$, which is only observable to student.
- Colleges infer the types from private signal s and SAT.

Identification of primitives also requires identification of

- Type distribution $P(K|SAT, B)$ conditional on SAT and family background
- Signal distribution $P(s|A)$ conditional on ability type

Identification

Point identified: type distribution $P(K|SAT, y)$, systematic tastes u_g, application cost c, admission probability $p_j(A, SAT)$

- Assumption I1: The number of student type is finite; idiosyncratic tastes are i.i.d. noises drawn from single-mode distributions; tastes are independent of (SAT,y,K).
- Assumption I2: At least one variable in the financial aid function is excluded from the type distribution function; conditional on (SAT,y), this variable is independent of K.
- Proof is done in a 2-type 1-college environment.

Ability values ω to colleges are not point identified.

Empirical Strategy

The challenges are

- Potential multiple equilibria
- Equilibrium computation

The strategy is to

- Assume that agents play the same equilibrium
- Exploit subgame perfection – admission probabilities $\{p_j\}$ (CCP) uniquely determines the equilibrium.
  - ◇ Knowing $\{p_j\}$ is sufficient for student to choose application portfolio
  - ◇ Knowing $\{p_{-j}\}$ is sufficient for college to choose admission policy

3-step Estimation

- Treat $\{p_j\}$ as parameters and estimate them along with student-side parameters (SMLE)
  - ◇ Test the existence of origin-based admissions
    Can't reject the null of origin-independent admission
    Assume the irrelevance of origin
  - ◇ Assume that admission is independent of net tuition revenue
- Solve each college's decision problem and fit college-side parameters to $\{\hat{p}_j\}$ and capacity κ_j (SMDE)
- Recover college's taste for tuition by matching college's tuition levels (GMM)

Data

- NLSY97, 1646 observations
  - ◇ 54% not apply for 4Y; 23% attend 2Y; 42% attend 4Y
- Information: application, admission, financial aid, enrollment, SAT/ACT, family background
- Assume a student can apply for at most two colleges within each group
  - ◇ Reduce computation
  - ◇ Capture competition between colleges within a group
  - ◇ Consistent with most application behaviors

Model Fit

Model versus Data

	Data	Model
	A. Number of Applications (%)	
Size:		
0	54.2	54.5
1	28.0	27.8
2 or more	17.8	17.7
χ^2 statistic	.06	
	B. Application and Admission: Applicants (%)	
Application rate:		
(pri, elite)	9.7	9.4
(pub, elite)	31.8	29.0
(pri, non)	44.6	44.4
(pub, non)	71.5	67.6
Admission rate:		
(pri, elite)	53.4	58.5
(pub, elite)	83.0	90.1
(pri, non)	91.4	91.5
(pub, non)	94.0	95.9
	C. Final Allocation of Students (%)	
(pri, elite)	1.0	1.5
(pub, elite)	7.7	8.0
(pri, non)	11.5	10.9
(pub, non)	21.9	20.2
2-year college	22.7	22.9
Noncollege	35.2	36.5
χ^2 statistic		6.98
	D. Home Bias (%)	
Home-only applicants	65.6	67.5
Home-state attendees	76.2	78.0

The model fits well, but it's unclear which moments are not targeted.

Heterogeneous Preference for College Groups

- Preference for non-college is set to 0.
- The estimation uses 1-year tuition.

PREFERENCES FOR COLLEGES ($1,000)

	(pri, elite)	(pub, elite)	(pri, non)	(pub, non)	2-Year
$\bar{u}_g(A=1, z=1)$	−187.7	−183.2	−123.5	−188.6	−38.1
	(188.0)	(5.1)	(3.8)	(4.4)	(1.7)
$\bar{u}_g(A=2, z=1)$	−42.2	−37.2	31.0	56.8	36.1
	(66.5)	(4.6)	(1.4)	(2.1)	(1.4)
$\bar{u}_g(A=3, z=1)$	−52.8	127.3	8.2	73.2	9.8
	(21.4)	(.4)	(7.6)	(3.9)	(4.5)
$\bar{u}_g(A=2, z=2)$	−74.4	−115.7	96.6	19.4	−13.3
	(29.4)	(34.9)	(4.6)	(3.19)	(5.6)
$\bar{u}_g(A=3, z=2)$	139.9	30.4	35.6	−66.2	−12.7
	(14.3)	(14.5)	(19.5)	(16.4)	(33.2)
$\sigma^2_{\epsilon_{1g}}$ (college group)	49.9	24.9	42.3	57.4	61.4
	(8.4)	(3.0)	(1.0)	(1.8)	(1.2)
$\sigma^2_{\epsilon_2}$ (specific college)			61.5		
			(1.2)		

NOTE.—The restriction $\bar{u}_g(A=1, z=2) = \bar{u}_g(A=1, z=1)$ holds at a 10 percent significance level.

Counterfactual: Increasing Capacity of Public Non-elite

New 1: Lower bound for 4-year college tuition is set to \$2,744, the level of community college

New 2: Community college tuition and the same lower bound become 0.

INCREASING SUPPLY

	Baseline	New 1	New 2	All Open and Free
	A. Attendance (%)			
4-year	40.6	43.2	44.2	55.6
2-year	22.9	21.9	22.9	18.0
	B. Attendance by Ability (%)			
$A = 1$:				
4-year	1.0	3.5	4.3	18.9
2-year	27.0	26.5	29.2	26.5
$A = 2$:				
4-year	72.3	75.1	76.7	86.9
2-year	24.0	21.9	21.1	12.7
$A = 3$:				
4-year	93.3	94.1	94.4	97.8
2-year	5.8	5.3	5.1	2.2

Counterfactual: SAT-based Admission

- Elite colleges raise tuition to screen, nonelite colleges lower tuition to compete

Ignore Signals: Tuition ($)

		(pub, elite)			(pub, non)	
	(pri, elite)	In-State	Out-of-State	(pri, non)	In-State	Out-of-State
Baseline	27,530	5,090	13,892	16,891	3,451	10,540
New	30,028	5,131	14,079	14,800	3,083	9,426

- In equilibrium, enrollee ability drops in elite colleges and increases in nonelite colleges

Ignore Signals: High-Ability Students (%)

	(pri, top)	(pub, top)	(pri, non)	(pub, non)
Baseline	94.7	80.2	11.0	15.9
New	86.4	79.0	12.7	16.1

Conclusion

- This paper
  - ◇ Estimates a market equilibrium model of tuition setting, applications, admissions and enrollment.
  - ◇ Simultaneously models students heterogeneity, uncertainty and costs in application, noisy measures.
  - ◇ Exploits subgame perfection to facilitate estimations
- To do so, need to
  - ◇ Assume tuition agent and admission agent make decisions separately (without sequential rationality)
  - ◇ Capture forces outside the model by preference parameters, which might affect the counterfactuals.