Applying CCP: Durable Goods and Human Capital

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Discrete Choice 4

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Another Renewal Problem (Rust,1987)

Bus engines

- The one occupation economy we analyzed in the second lecture was treated as a renewal problem.
- Replacing bus engines can be modeled that way too.
- Mr. Zurcher decides whether to replace the existing engine ($d_{1t} = 1$), or keep it for at least one more period ($d_{2t} = 1$).
- Bus mileage advances 1 unit ($x_{t+1} = x_t + 1$) if Zurcher keeps the engine ($d_{2t} = 1$) and is set to zero otherwise ($x_{t+1} = 0$ if $d_{1t} = 1$).
- Transitory iid choice-specific shocks, ϵ_{jt} are Type 1 Extreme value.
- Zurcher sequentially maximizes expected discounted sum of payoffs:

$$E\left\{\sum_{t=1}^{\infty}\beta^{t-1}\left[d_{2t}(\theta_1 x_t + \theta_2 s + \epsilon_{2t}) + d_{1t}\epsilon_{1t}\right]\right\}$$

Another Renewal Problem

Value functions and replacement CCP

- Let $V(x_t, s)$ denote the ex-ante value function at the beginning of period t, the discounted sum of current and future payoffs just before ϵ_t is realized and before the decision at t is made.
- We also define the conditional value function for each choice as:

$$v_j(x,s) = \begin{cases} \beta V(0,s) & \text{if } j = 1 \\ \theta_1 x + \theta_2 s + \beta V(x+1,s) & \text{if } j = 2 \end{cases}$$

- Let $p_1(x,s)$ denote the conditional choice probability (CCP) of replacing the engine given x and s.
- The parametric assumptions about the transitory shocks imply:

$$p_1(x,s) = \frac{1}{1 + \exp\left[v_2(x,s) - v_1(x,s)\right]}$$

- An ML estimator could be formed off this equation following the steps In Lecture 3.

Another Renewal Problem

Exploiting the renewal property

- One can show that when ϵ_{jt} is Type 1 Extreme value, then for all (x, s, j):

$$V(x,s) = v_j(x,s) - \log\left[p_j(x,s)\right] + 0.57\ldots$$

- Therefore the conditional valuation function of not replacing the engine is:

$$\begin{aligned} v_2(x,s) &= \theta_1 x + \theta_2 s + \beta V(x+1,s) \\ &= \theta_1 x + \theta_2 s + \beta\left\{v_1(x+1,s) - \log\left[p_1(x+1,s)\right] + 0.57\ldots\right\} \end{aligned}$$

- Similarly:

$$v_1(x,s) = \beta V(0,s) = \beta\left\{v_1(0,s) - \log\left[p_1(0,s)\right] + 0.57\ldots\right\}$$

- Because the miles on a bus engine does not affect the value of the bus the engine is replaced:

$$v_1(0,s) = v_1(x+1,s)$$

- Hence:

$$v_2(x,s) - v_1(x,s) = \theta_1 x + \theta_2 s + \beta \ln\left[p_1(0,s)\right] - \beta \ln\left[p_1(x+1,s)\right]$$

- Therefore:

$$\begin{aligned} p_1(x,s) &= \frac{1}{1+\exp\left[v_2(x,s) - v_1(x,s)\right]} \\ &= \frac{1}{1+\exp\left\{\theta_1 x + \theta_2 s + \beta \log\left[\frac{p_1(0,s)}{p_1(x+1,s)}\right]\right\}} \end{aligned}$$

- Intuitively the CCP for current replacement is the CCP for a static model with an offset term.
- The offset term accounts for differences in continuation values using future CCPs that characterize optimal future replacements.

- Consider the following CCP estimator.
- Form first stage estimate for $p_1(x, s)$, called $\hat{p}_1(x, s)$ from the relative frequencies:

$$\hat{p}_1(x,s) = \frac{\sum_{n=1}^{N} d_{1nt} I(x_{nt} = x) I(s_n = s)}{\sum_{n=1}^{N} I(x_{nt} = x) I(s_n = s)}$$

- In second stage substitute $\hat{p}_1(x, s)$ into the likelihood as incidental parameters and estimate θ_1 and θ_2 with a logit:

$$\frac{d_{1nt} + d_{2nt} \exp(\theta_1 x_{nt} + \theta_2 s_n + \beta \ln[\hat{p}_1(0, s_n)] - \beta \ln[\hat{p}_1(x_{nt}+1, s_n)]}{1 + \exp(\theta_1 x_{nt} + \theta_2 s_n + \beta \ln[\hat{p}_1(0, s_n)] - \beta \ln[\hat{p}_1(x_{nt}+1, s_n)])}$$

Monte Carlo Study (Arcidiacono and Miller, 2011)

Modifying the bus engine problem

- Suppose bus type $s \in \{0, 1\}$ is equally weighted.
- There are two other state variables
  1. total accumulated mileage:

$$x_{1t+1} = \begin{cases} \Delta_t \text{ if } d_{1t} = 1 \\ x_{1t} + \Delta_t \text{ if } d_{2t} = 1 \end{cases}$$

  2. permanent route characteristic for the bus, x_2, that systematically affects miles added each period.
- We assume $\Delta_t \in \{0, 0.125, \ldots, 24.875, 25\}$ is drawn from a truncated exponential distribution:

$$f(\Delta_t | x_2) = \exp\left[-x_2(\Delta_t - 25)\right] - \exp\left[-x_2(\Delta_t - 24.875)\right]$$

and x_2 is a multiple 0.01 drawn from a discrete equi-probability distribution between 0.25 and 1.25.

Monte Carlo Study

Including aggregate shocks in panel estimation

- Let θ_{0t} denote an aggregate shock (denoting fully anticipated cost fluctuations). Then the difference in current payoff from retaining versus replacing the engine is:

$$u_{2t}(x_{1t}, s) - u_{1t}(x_{1t}, s) \equiv \theta_{0t} + \theta_1 \min\{x_{1t}, 25\} + \theta_2 s$$

- Denoting $x_t \equiv (x_{1t}, x_2)$, this implies:

$$\begin{aligned} v_{2t}(x_t, s) - v_{1t}(x_t, s) &= \theta_{0t} + \theta_1 \min\{x_{1t}, 25\} + \theta_2 s \\ &\quad + \beta \sum_{\Delta_t \in \Lambda} \left\{ \ln \left[\frac{p_{1t}(0, s)}{p_{1t}(x_{1t} + \Delta_t, s)} \right] \right\} f(\Delta_t | x_2) \end{aligned}$$

- In the first three columns of the next table each sample is on 1000 buses for 20 periods, while in the fourth column we assume 2000 buses are observed for 10 periods.
- The mean and standard deviations are compiled from 50 simulations.

Monte Carlo Study

Extract from Table 1 of Arcidiacono and Miller (2011)

Monte Carlo for Optimal Stopping Problem[+]

	DGP (1)	FIML (2)	CCP (3)	Time effects CCP (4)
θ_0 (Intercept)	2	2.0100	1.9911	
		(0.0405)	(0.0399)	
θ_1 (Mileage)	-0.15	-0.1488	-0.1441	-0.1440
		(0.0074)	(0.0098)	(0.0121)
θ_2 (Type)	1	0.9945	0.9726	0.9683
		(0.0611)	(0.0668)	(0.0636)
β (Discount Factor)	0.9	0.9102	0.9099	0.9172
		(0.0411)	(0.0554)	(0.0639)
Time (Minutes)		130.29	0.078	0.079
		(19.73)	(0.0041)	(0.0047)

[+] Mean and standard deviations for fifty simulations. For columns (2) and (3), the observed data consist of 1000 buses for 20 periods. For column (4), the intercept (θ_0) is allowed to vary over time and the data consist of 2000 buses for 10 periods.

Durable Goods and Human Capital

Trends in home ownership, fertility, marriage, labor supply and education

- The average age of a first-time home buyer was about 28 years old in the 1970s, about 30 in the 1990's, and is now about 32.5.
- This increase coincided with postponing marriage and fertility; the average age of mother at first birth rose from 22 forty years ago to 24 two decades ago, and is currently about 26.spea
- In contrast female labor-force participation rose from 46 percent in 1975, peaked at 59 percent in 1995 and subside to 56 percent in 2015, hours worked following a similar pattern.
- The median age of marriage and first birth practically coincide at each of the four census points (1970,. . . ,2000)
    - but age at first home purchase is several years older
    - and the gap between first birth and first home purchase widened a little and then stabilized.

Durable Goods and Human Capital

Figure 1 from Khorunzhina and Miller (2017)

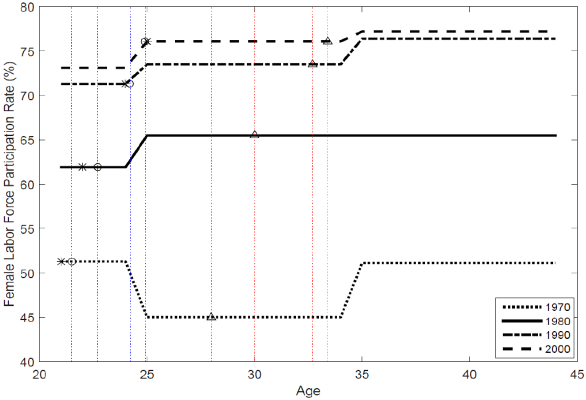


Figure 1: Labor force participation rate by age for 1970 - 2000. "Star" denotes median age at first marriage, "circle" denotes average age at first birth, "triangle" denotes average age at first homeownership

Durable Goods and Human Capital

More detail on time trends

- Tracking over the decades, the mean age at:
  - first home purchase, first birth and second birth have all increased.
  - second birth roughly tracks mean age at first home purchase.
- The trend to postpone buying the first house is matched by a trend to purchase a larger one:
  - Loosely speaking there is a quantity/quality trade-off, larger homes are now owned, but older people purchase their first home.
- There is no evidence for a trend in selling houses and reverting to renting:
  - In other words declining home ownership is explained by the trend of purchasing the first home later in life.

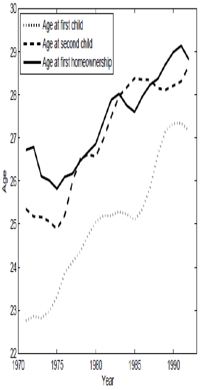


Figure 4: Timing of children and first ownership

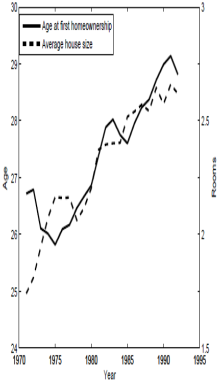


Figure 5: First homeownership and average home size

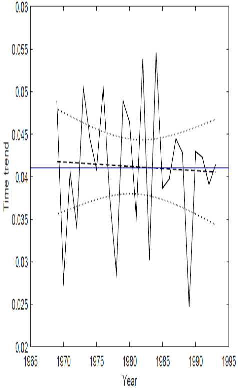


Figure 8: Transition from ownership to renting

Durable Goods and Human Capital

Cross sectional differences between owners and renters

- Compared to tenants, home owners:
  - are older.
  - are more educated.
  - are more likely to be married.
  - have more children.
  - live in larger dwellings.
  - are less likely to be employed.
  - work fewer hours if they are employed.
  - earn more income if they are employed.

Durable Goods and Human Capital

Table 1 from Khorunzhina and Miller (2017)

	Full sample	Owners	Renters
Age	32.4 (6.6)	33.9 (6.3)	29.7 (6.1)
Education	13.0 (2.2)	13.0 (2.1)	12.9 (2.3)
Married	0.82 (0.38)	0.92 (0.26)	0.64 (0.48)
White	0.89 (0.31)	0.93 (0.26)	0.82 (0.38)
Number of children	1.53 (1.30)	1.67 (1.24)	1.28 (1.36)
Home ownership rate	0.64 (0.47)		
House value for home owners		66,381 (42,859)	
Annual rent for renters			2,956 (2,102)
Move to owned house	0.087 (0.282)		
own-to-own**		0.062 (0.241)	
rent-to-own***		0.064 (0.244)	
Move to rental house	0.126 (0.331)		
rent-to-rent***			0.329 (0.470)
own-to-rent***			0.041 (0.198)
Number of rooms in dwelling	5.8 (1.7)	6.4 (1.5)	4.7 (1.5)
Labor force participation	0.753 (0.430)	0.736 (0.440)	0.783 (0.411)
Hours worked*	1,497 (742)	1,479 (741)	1,527 (743)
Labor income*	11,070 (8,850)	11,504 (9,374)	10,341 (7,842)
Number of observations	43,504	27,871	15,633

- At any given age home ownership ranks from highest to lowest by roughly tracking aggregate household weight:
  1. married with children
  2. married with no children
  3. single with children
  4. single with no children.

  - supply more labor to the market than , but both supply more than .
- With regards labor supply, the ordering from the most to the least is:
  1. single homeowners
  2. single tenants
  3. married females, whether they are homeowners or not

Durable Goods and Human Capital

Figures 2 and 3 from Khorunzhina and Miller (2017)

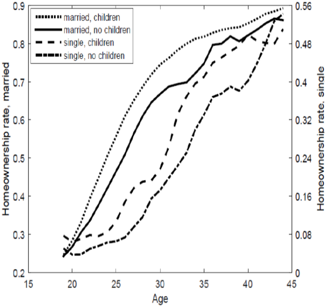


Figure 2: Average homeownership rate.

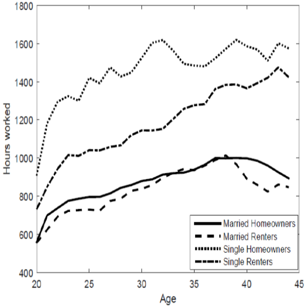


Figure 3: Average hours worked for females with children.

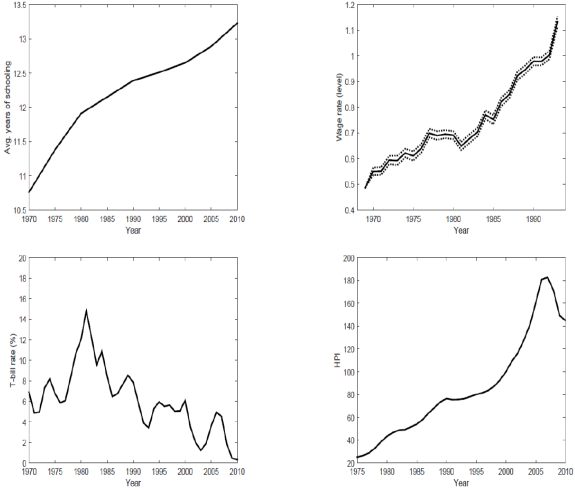
Durable Goods and Human Capital

Contributing Factors

- There are potentially four main economic factors driving these trends. Over these four decades:
  1. real wages rose.
  2. the real interest rate declined.
  3. housing prices rose and then fell.
  4. females became more educated.
- The challenge is to develop a dynamic life-cycle model linking these drivers to the joint behavioral outcomes to:
  1. estimate how educational attainment, wages, interest rates, housing prices determine fertility, labor supply and first home purchase decisions.
  2. quantify the importance of these four factors behind the secular decline in home ownership by conducting counterfactual exercises on the estimated model.

Durable Goods and Human Capital

Figure 6 from Khorunzhina and Miller (2017)



Model
Discrete choices

- $b_t \in \{0, 1\}$, where $b_t = 1$ if a child is born at time t.
- $f_t \in \{0, 1\}$, where $f_t = 1$ means female works at time t.
- $a_t \in \{0, 1\}$, where $a_t = 0$ means continuing to rent at t and $a_t = 1$ means first home is purchased.
- Consolidating the choices let $d_{kt} \in \{0, 1\}$ where $\sum_{k=0}^{7} d_{kt} = 1$ where $(a_t, b_t, f_t) = (0, 0, 0)$ by $d_{0t} = 1$ and otherwise set $d_{kt} = 1$ for:

$$
\begin{aligned}
k \equiv\; & (1 - a_t) b_t (1 - f_t) + 2 (1 - a_t)(1 - b_t) w_t + 3 (1 - a_t) b_t f_t \\
& + 4 a_t (1 - b_t)(1 - f_t) + 5 a_t b_t (1 - f_t) + 6 a_t (1 - b_t) f_t + \ldots
\end{aligned}
$$

- Purchasing the first house is a once-in-a-lifetime decision.
- If $a_t = 1$ then $a_s = 0$ for $s \in \{t + 1, \ldots, T\}$, and $\sum_{k=0}^{3} d_{kt} = 1$.
- Hence the model restricts homeowners to four discrete choices, tenants to eight.
- $c_t \in \mathcal{R}$ denotes nonhousing consumption, a continuous choice.

- The household's lifetime utility is modeled as:

$$-\sum_{\tau=t}^{\infty}\sum_{j=0}^{7}\beta^{\tau-t}d_{jt}\exp(u_{\tau}^{h}+u_{\tau}^{b}+u_{j\tau}^{l}-\rho c_{\tau}-\varepsilon_{j\tau})$$

where:

- β denotes the subjective discount factor.
- u_{τ}^{h} indexes current utility payoff from housing.
- u_{τ}^{b} indexes the lifetime utility of giving birth and raising a child if this is the youngest.
- u_{τ}^{l} indexes the current utility of current leisure.
- ρ is the constant absolute risk aversion parameter.
- $\epsilon_{\tau}\equiv(\epsilon_{1\tau},\ldots,\epsilon_{J\tau})$ is revealed at the beginning of the period t, has continuous support and is *iid* with density function $g(\epsilon_{t})$.

- We parameterize the index functions as:

$$
\begin{aligned}
u_{jt}^{h} &\equiv a_t \left(x_t'\theta_0 + b_t\theta_1 + f_t\theta_2 + b_t f_t\theta_3\right) \\
&\quad s_t \left(x_t'\phi_0 + s_t\phi_1 + s_{t-1}\phi_2 + l_t\phi_3\right) \\
u_t^{b} &\equiv b_t \left(x_t'\gamma_0 + f_t\gamma_1 + s_t\gamma_2\right) \\
u_t^{l} &\equiv f_t x_t'\delta_0 + l_t \left(\delta_1 x_t' + \delta_2 l_t\right)
\end{aligned}
$$

where:

- x_t is a set of fixed or time varying attributes that characterize the decision maker (including age, education and marital status) along with previous fertility and labor market outcomes.
- s_t measures house size in period t.
- $l_t \in [0,1]$ is female labor supply in t where $l_t \in (0,1]$ implies $f_t = 1$.
- s_{t-1} is included in u_{jt}^{h} to all for adjustments associated with resale and future housing purchases.
- $x_t'\delta_0$ is the fixed cost of working, and lagged labor supply affects the marginal utility of current leisure.

Model

Budget constraint

- Assume prices, interest rates and hence aggregate fluctuations are known in advance.
- Denote by:
  - W_t household financial wealth at the beginning of period t.
  - y_t income from real wages paid to the female for work in period t
  - i_t the period t interest rate.
  - $R(s_t, q_t)$ rent by tenants.
  - $H(s_t, q_t)$ the house price, which depends on house size, quality and aggregate factors.
- The law of motion for disposable household wealth is:

$$\begin{aligned}(1+i_t)^{-1} W_{t+1} &\leq W_t + w_t - c_t \\ &\quad -R(s_t, q_t) \prod_{\tau=1}^{t} (1-a_\tau) - a_t H(s_t, q_t)\end{aligned}$$

Model

State variables

- The household directly controls some state variables, including:
  - W_t current financial wealth.
  - $(b_t, \ldots, b_{t-18})$ family composition.
  - $l_{t-1} \in [0, 1]$ female lagged labor supply.
  - A_t home ownership.
- The other state variables include:
  - s_t house size, a Markov process with transition density $f(s_t | s_{t-1})$ when the household rents, and when household owns, $s_t = s_{t-1}$.
  - q_t aggregate variables for housing prices.
  - Y_t wage rates.
  - B_t current price of a bond in t paying one consumption unit each period into perpetuity.
  - D fixed demographics for each woman including age and education.
- Summarizing the state variables are (z_t, W_t) where:

$$z_t \equiv (D, B_t, q_t, Y_t, b_t, \ldots, b_{t-18}, A_t, s_t, l_{t-1})$$

Model

Conditional choice probabilities (CCPs) and lifetime utility under optimization

- In period t the household observes (z_t, W_t), chooses c_t, then observes $\varepsilon_t \equiv (\varepsilon_{0t}, \ldots, \varepsilon_{7t})$, and finally chooses d_t.
- Denote by $d_t^o = (d_{1t}^o, \ldots, d_{7t}^o)$ the discrete choices that along with the optimal consumption choices, c_t^o, solve the household's problem.
- Let $p_{jt}(z_t) \equiv E\left[d_{jt}^o \,|z_t\right]$ denote the probability of optimal choice j at year t conditional on z_t.
- Denote by ε_{jt}^* the truncated variable that takes on the value of ε_{jt} when $d_{jt} = 1$ and is not defined when $d_{jt} = 0$.
- The theorem below implies the expected lifetime of the household may be expressed as

$$-\sum_{\tau=t}^{\infty} \beta^{\tau-t} E_t \left[\sum_{j=0}^{7} p_{j\tau}(z_\tau) \exp(u_\tau^h + u_\tau^b + u_{j\tau}^l - \rho c_\tau^o - \varepsilon_{j\tau}^*) \right]$$

- In other words $p_{jt}(z_t)$ are the CCPs and do not depend on W_t.

Model

Theorem from Gayle, Golan and Miller (2015) for representing optimal discrete choices

- The optimal discrete choices d_t^o maximize:

$$\sum_{j=0}^{7} d_{jt} \left[\rho y_{jt} - u_\tau^h - u_\tau^b - u_{j\tau}^l - (B_t - 1) \ln A_{t+1}(z_{t+1}^{(j)}) + \varepsilon_{jt} \right]$$

  - where $A_t(z_t)$ is an index of household capital:

$$A_t(z_t) = \sum_{j=0}^{7} p_{jt}(z_t) \alpha_j(z_t)^{\frac{1}{B_t}} e^{-\rho y_{jt}/B_t} E_t \left[e^{-\varepsilon_{jt}^*/B_t} \right] A_{t+1} \left(z_{t+1}^{(j)} \right)^{1-\frac{1}{B_t}}$$

$$A_{T+1}(z_{T+1}) \equiv \prod\nolimits_{\tau=T+1}^{\infty} \alpha \left(z_\tau^{(0)} \right)^{1/B_{T+1}}$$

  - $\alpha_j(z_t) \equiv \exp \left[u_\tau^h(z_t) + u_\tau^b(z_t) + u_{j\tau}^l(z_t) \right]$
  - $z_{t+1}^{(j)}$ is the state vector value at $t+1$ following choice j applied to z_t.
- Note $A_t(z_t) > 0$ and lower values of $A_t(z_t)$ come from higher current income and lower rent, both captured within y_{jt}.

- Following (Arcidiacono and Miller, 2017) the parameters are identified given a *pdf* for $\varepsilon_t \equiv (\varepsilon_{0t}, \ldots, \varepsilon_{7t})$ and normalizing constants for each (t, z_t).
- We assume ε_{jt} is independently and identically distributed as a T1EV with location and scale parameters $(0, 1)$.
- It is well known that in this case:

$$\ln\left[\frac{p_{0t}(z_t)}{p_{jt}(z_t)}\right] = \rho\left(y_{0t} - y_{jt}\right) + \ln\left(\alpha_{jt}\right) + \left(B_t - 1\right)\ln\left[\frac{A_{t+1}\left(z_{t+1}^{(j)}\right)}{A_{t+1}\left(z_{t+1}^{(0)}\right)}\right]$$

- The model exhibits finite dependence: if any two choices j and k are both followed by the zero choice for as long it it takes for a child to grow up, the state variables are equalized, implying:

$$A_{t+1}\left(z_{t+18}^{(j)}\right) = A_{t+1}\left(z_{t+18}^{(0)}\right)$$

Inference

Finite dependence

- Estimation is based on successively telescoping $\ln\left[A_{t+1}\left(z_{t+1}^{(j)}\right)/A_{t+1}\left(z_{t+1}^{(0)}\right)\right]$ into the future out to $T = 18$.
- The following theorem provides the basis for our CCP estimator. For each $j \in \{1, \ldots, 7\}$ and $t \in \{1, \ldots, T\}$:

$$\ln\left[\frac{p_{jt}(z_t)}{p_{0t}(z_t)}\right] = \rho\left(y_{jt} - y_{0t}\right) - u_{jt}^{h} - u_{jt}^{b} - u_{jt}^{l}$$

$$+ \sum_{s=t+1}^{18} \prod_{r=t+1}^{s}\left(\frac{1}{1+i_r}\right)\left[\rho\left(y_s^{(j,t)} - y_s^{(0,t)}\right) + \ln\frac{p_{0s}\left(z_s^{(0)}\right)}{p_{0s}\left(z_s^{(j)}\right)}\right]$$

and $z_{t+s}^{(j)}$ is the value of the state vector at $t+s$ following the sequence of decisions $\left(d_{jt}, d_{0,t+1}, \ldots, d_{0,t+s}\right)$ applied to z_t, the value of the state vector in period t.

Model Fit

One period forecasts

- To check the model fit we:
  1. solved the optimal decision rule model for the estimated parameters (by integrating over disturbance and using a backwards induction recursion to successively solve the CCPs).
  2. approximately replicated the PSID sample population with an artificial population.
  3. simulated the artificial population one period forward using the first step.
- The model fits the data quite well but:
  - predicts too much home ownership at later ages (83% versus 81% at 41 - 45)
  - home sizes are too large at later ages (6.4 rooms versus 6.3 at 36 - 40)
  - predicts too much labor force participation (by about 3%)
  - flattens the inverted U shape of hours (by up to 50 per year)
  - predicts too many children (up to 0.15 at the mean)

Model Fit

Table 4 of Khorunzhina and Miller (2017)

	21-25	26-30	31-35	36-40	41-45
Homeownership rate					
Data	0.37	0.57	0.72	0.77	0.81
Model	0.39	0.58	0.72	0.78	0.83
Home size					
Data	4.70	5.44	6.04	6.35	6.32
Model	4.70	5.43	6.09	6.44	6.41
Labor force participation					
Data	0.79	0.75	0.75	0.77	0.75
Model	0.82	0.78	0.75	0.76	0.80
Hours worked					
Data	1455	1496	1482	1528	1556
Model	1506	1532	1501	1500	1544
Children					
Data	0.86	1.34	1.84	1.77	1.25
Model	0.91	1.35	1.86	1.91	1.40

Structural Estimates

Table 2 of Khorunzhina and Miller (2017)

	Utility from:				
	Home Purchase	Home size	Offspring	Work	Work hours
	(1)	(2)	(3)	(4)	(5)
	$d_t^p \times$	$s_t \times$	$d_t^k \times$	$d_t^w \times$	$l_t \times$
Constant	1.124 (0.469)	0.623 (0.082)	2.137 (0.194)	−0.298 (0.168)	6.233 (0.916)
Work	0.333 (0.495)		−4.343 (0.251)		
Birth	−3.221 (0.491)				
Work*Birth	−23.027 (0.515)				
Demographic characteristics (x_t)					
Female age	−0.196 (0.014)	0.038 (0.002)	−0.415 (0.005)	−0.040 (0.005)	0.109 (0.025)
Female education	0.436 (0.029)	−0.067 (0.005)	0.142 (0.011)	0.117 (0.009)	−0.419 (0.050)
Husbands age	0.085 (0.012)	−0.013 (0.002)	−0.020 (0.005)	−0.010 (0.004)	0.052 (0.022)
Husbands education	−0.517 (0.026)	0.082 (0.004)	0.138 (0.010)	−0.074 (0.008)	0.258 (0.044)
Single	−10.429 (0.544)	0.439 (0.090)	−5.783 (0.215)	1.793 (0.201)	−12.261 (1.024)
Non-White	−8.804 (0.203)	0.459 (0.035)	−0.576 (0.080)	−1.307 (0.075)	9.424 (0.398)
Single*Non-White	−24.063 (0.505)	2.465 (0.091)	5.260 (0.165)	−1.145 (0.154)	13.634 (0.764)
Children at $t-1$	3.769 (0.135)	−0.159 (0.021)	4.281 (0.050)	−0.652 (0.042)	−3.396 (0.242)
Children sq. at $t-1$	−2.882 (0.043)	0.139 (0.007)	−2.464 (0.015)	0.066 (0.010)	−0.368 (0.063)
Age of last child	−0.352 (0.020)	−0.056 (0.003)	−1.482 (0.006)	0.120 (0.006)	−0.390 (0.030)
Homeowner at $t-1$			2.643 (0.055)	−0.651 (0.041)	5.180 (0.213)
Single*Homeowner at $t-1$			−16.352 (0.148)	−1.054 (0.165)	8.570 (0.735)
Current home size (s_t)		−0.048 (0.004)	0.015 (0.012)	−0.003 (0.006)	
Prior home size (s_{t-1})		0.009 (0.001)			
Employed at $t-1$ (d_{t-1}^w)	0.171 (0.037)			1.281 (0.029)	
Employed at $t-2$ (d_{t-2}^w)	0.087 (0.036)			0.594 (0.027)	
Work time (l_t)		−2.104 (0.025)			−128.566 (0.857)
Work time at $t-1$ (l_{t-1})		−0.295 (0.032)			99.939 (0.650)
Work time at $t-2$ (l_{t-2})		−0.055 (0.028)			−10.004 (0.597)

Structural Estimates

Utility from first home purchase

- The utility of purchasing a first house:
  - is a rare event that declines with age.
  - declines with (female) employment but is increasing in lagged employment and education.
  - declines with births, increases with the number of children, but declines with the age of the youngest.
  - lower for nonwhites, declines with marriage, with notable exception of married nonwhites.
- Conditional on purchasing a home, utility from a larger home:
  - is concave increasing.
  - increases with age.
  - declines with education.
  - increase with the age but not the number of children.
  - declines with the size of previous home.

Structural Estimates

Lifetime utility from giving birth

- The benefits of having another child decline with
  - age.
  - age of the youngest child.
  - education.
  - being currently employed.
  - house size.
  - owning a home the previous period.
- The benefits of having another child are greater for:
  - white if married.
  - nonwhite if not married.

Structural Estimates

Fixed utility cost of participation and utility from leisure time

- The current utility of leisure is concave increasing in leisure, and also increasing in past leisure, consistent with previous studies.
- The other state variables affect current utility for leisure in a nonlinear way, captured by the fixed and linear components
- The estimates pertaining to the effects of race, marriage education and family composition are also in line with previous work:
  - increasing in marriage, especially for nonwhites.
  - declining in age.
  - increasing in education.
  - higher for nonwhites.
  - increasing in family size and declining in the age of the youngest child.
- Homeowning and house size increase the benefit from leisure.

Structural Estimates

Wage equation

- The estimates of the wage equation are similar to those found in the literature:
  - Wages are concave over the lifecyle.
  - They are increasing in hours worked one and two periods ago (above an hours threshold that captures nonlinear effects induced by the participation dummy variable).
  - Marriage magnifies the effects of lagged hours on the wage rate.
- Table 1 appeared to suggest that (female) homeowners earn higher wages than tenants.
- The second column of Table 3 shows the effects of being a homeowner on wages:
  - Only age squared is significant, and its quantitative effect is small.
  - Thus the implicit inequality in Table 1 arises from other factors (such as education and age).

Structural Estimates

Table 3 Khorunzhina and Miller (2017)

$$\ln(wage_{it}) = B_1 X_{it} + B_2(O_{it} X_{it}) + \mu_t + \eta_i + \epsilon_{it},$$
where O_{it} is a dummy for homeowner

X_t	B_1 (1)	B_2 (2)
ΔHours worked at $t-1$	0.130 (0.007)	0.003 (0.006)
ΔHours worked at $t-2$	0.050 (0.007)	−0.005 (0.006)
ΔWork at $t-1$	−0.061 (0.011)	0.018 (0.013)
ΔWork at $t-2$	−0.027 (0.011)	−0.001 (0.013)
Δ(Age×Education)	0.516 (0.126)	0.138 (0.072)
ΔAge^2	−0.284 (0.049)	−0.072 (0.025)
ΔMarital*Hours worked at $t-1$	0.038 (0.008)	
ΔMarital*Hours worked at $t-2$	0.008 (0.008)	

Counterfactual Simulations

Effects of changes in wages, interest rates, educational levels and housing prices

- We inoculated the counterfactual analysis against the unbalanced (PSID) sample and buffering from aggregate effects by:
  - generating an artificial population of 23 year olds that approximates the population distribution of that age group within the PSID.
  - successively applying the optimal rule for 25 years in a steady state economy.
- We are running counterfactuals that:
  1. reduce the interest rate by 1 percent.
  2. increase wages by 5 percent.
  3. increase housing prices by 10 percent.
  4. increase educational attainment of those with no college by a year.
- We intend to analyze the:
  1. long term effects (comparing steady states) of permanent shifts.
  2. short term (one period) effects of permanent shifts.
  3. short term effects of temporary shifts (that measure flexibility).

Counterfactual Simulations

Table 5 Khorunzhina and Miller (2017)

	Average age at		LFP (%)	
	first child	first own home	before age 30	after age 30
Benchmark	22.9	29.0	76	76
More education (to HS or less)	23.3	28.3	82	80
Increase wage by 5%	23.7	29.4	79	78
Reduce interest rate by 1%	22.5	27.9	77	78
Increase house prices by 10%	23.2	29.6	75	76