

# Human Capital and Nonseparable Preferences

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# Nonseparabilities over time

## Competitive equilibrium

- Panel data reject the joint null hypothesis that:
  - 1 consumer workers maximize expected utility.
  - 2 they have rational expectations.
  - 3 the disturbance structure is *iid*.
  - 4 current utility is CRRA.
  - 5 current utility is additively separable within and between periods.
  - 6 goods and labor productivity are additively separable over time.
  - 7 there are complete markets.
- There is less consensus on which of these factors is instrumental:
  - Some economists believe 7 fails.
  - Others believe 3 through 7 cause the rejections.
  - There are those who believe consumers violate 1 and/or 2.
- The remainder of the course focuses on the failure of complete markets and the incorporation of nonseparable preferences.
- In both cases we justify the extensions with evidence from the data.

# Female Wages

## Value of marginal product of labor

- Female labor supply receives much more attention than male labor supply from labor economists.
- There are two main reasons. Female labor supply:
  - ① exhibits much more variation than male labor supply.
  - ② is significantly correlated with variables typically observed in panel data (such as household composition and education).
- One noticeable feature of female wage data is that past labor supply decisions significantly and quantitatively affect their current wages.
- This form of human capital accumulation induces nonseparabilities over time within the production function.

# Female Wages

Wage equation (Altug and Miller, 1998)

- Suppose:

$$\tilde{w}_{nt} \equiv w_{nt} \exp(\varepsilon_{3nt})$$

and:

$$w_{nt} \equiv w_t v_n \exp \left[ x_{nt} B_3 + \sum_{s=1}^{\rho} (\gamma_{1s} l_{n,t-s} + \gamma_{2s} d_{n,t-s}) \right]$$

where:

- $\tilde{w}_{nt}$  measured wage rate of  $n$  at  $t$
- $w_{nt}$  wage rate of  $n$  at  $t$
- $\varepsilon_{3nt}$  *iid* measurement error
- $w_t$  wage rate for standardized unit of labor
- $v_n$  household specific fixed effect
- $x_{nt}$  time varying socioeconomic factors
- $l_{nt} \in [0, 1]$  normalized labor time
- $d_{nt} = \begin{cases} 0 & \text{if } l_{nt} = 0 \\ 1 & \text{if } l_{nt} > 0 \end{cases}$

# Female Wages

Estimates from the PSID (Table III, Altug and Miller, 1998)

$$\ln(w_{it}) = \ln(\omega_i) + \ln(v_{it}) + \sum_{s=1}^p (\gamma_{1s} I_{it-s} + \gamma_{2s} d_{it-s}) + X_{it}' B_3$$

*The wage equation†*

Variable	Parameter	(i) Estimate	(ii) Estimate	Variable	(i) Estimate	(ii) Estimate
<b>Lags of hours worked</b>				<b>Aggregate wages</b>		
$\Delta I_{it-1}$	$\gamma_{11}$	0-0002 (0-00001)	0-0002 (0-5E-5)	$\Delta \ln(\omega_7)$	—	0-045 (0-016)
$\Delta I_{it-2}$	$\gamma_{12}$	0-0001 (0-00001)	0-0001 (0-000006)	$\Delta \ln(\omega_8)$	—	-0-003 (0-021)
$\Delta I_{it-3}$	$\gamma_{13}$	0-00008 (0-00001)	0-00006 (0-000006)	$\Delta \ln(\omega_9)$	-0-013 (0-022)	-0-038 (0-011)
$\Delta I_{it-4}$	$\gamma_{14}$	0-00006 (0-00001)	0-00005 (0-000005)	$\Delta \ln(\omega_{10})$	-0-04 (0-02)	0-02 (0-01)
$\Delta I_{it-5}$	$\gamma_{15}$	0-00002 (0-00001)	—	$\Delta \ln(\omega_{11})$	0-11 (0-02)	0-074 (0-01)
$\Delta I_{it-6}$	$\gamma_{16}$	0-00002 (0-00001)	—	$\Delta \ln(\omega_{12})$	0-05 (0-02)	0-03 (0-02)
<b>Lags of participation</b>				$\Delta \ln(\omega_{13})$	0-05 (0-02)	0-02 (0-01)
$\Delta d_{it-1}$	$\gamma_{21}$	-0-11 (0-021)	-0-09 (0-01)	$\Delta \ln(\omega_{14})$	-0-01 (0-02)	-0-03 (0-01)
$\Delta d_{it-2}$	$\gamma_{22}$	-0-10 (0-021)	-0-07 (0-01)	$\Delta \ln(\omega_{15})$	0-05 (0-02)	0-02 (0-01)
$\Delta d_{it-3}$	$\gamma_{23}$	-0-10 (0-02)	-0-09 (0-01)	$\Delta \ln(\omega_{16})$	-0-02 (0-02)	-0-01 (0-01)
$\Delta I_{it-4}$	$\gamma_{24}$	-0-10 (0-02)	-0-07 (0-01)	$\Delta \ln(\omega_{17})$	0-07 (0-02)	0-04 (0-01)
$\Delta d_{it-5}$	$\gamma_{25}$	-0-05 (0-02)	—	$\Delta \ln(\omega_{18})$	-0-01 (0-02)	-0-02 (0-01)
$\Delta d_{it-6}$	$\gamma_{26}$	-0-03 (0-02)	—	$\Delta \ln(\omega_{19})$	0-082 (0-02)	0-05 (0-02)
<b>Socioeconomic variables</b>						
$\Delta \text{AGE}_{it}^2$	$B_{31}$	-0-0005 (0-0001)	-0-0002 (0-00005)			
$\Delta(\text{AGE}_{it} \times \text{EDU}_{it})$	$B_{32}$	0-0001 (0-0004)	0-0002 (0-0001)			

† Standard errors in parentheses.

# A Social Planner's Problem

## Preferences and resource constraint

- Suppose individual  $n$  is uniformly distributed on  $[0, 1]$  with current consumption  $c_{nt}$  and current utility:

$$U_{nt} = (1 - d_{nt})(x_{nt}B_0 + \varepsilon_{0nt}) + d_{nt}\varepsilon_{1nt} \\ + \alpha^{-1}c_{nt}^\alpha \exp[x_{nt}B_2 + \varepsilon_{2nt}] + \left[x_{nt}B_1 + \sum_{s=0}^{\rho} \delta_s l_{n,t-s}\right] l_{nt}$$

- Aggregate consumption cannot exceed aggregate income, comprising aggregate wages and aggregate nonlabor income,  $e_t$ :

$$\int_0^1 c_{nt} dn \leq e_t + \int_0^1 w_{nt} l_{nt} dn$$

- The planner's preferences are given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \int_0^1 \eta_n^{-1} U_{nt} dn \right] \right\}$$

where  $\beta$  is a subjective discount factor, and  $\eta_n$  is the marginal utility of wealth for  $n$  in the corresponding competitive equilibrium.

# A Social Planner's Problem

## First order conditions

- Let  $\lambda_t$  denote the Lagrange multiplier associated with the aggregate resource constraint.
- The log of the FOC for consumption is then:

$$(\alpha - 1) \ln c_{nt} + (x_{nt} B_2 + \varepsilon_{2nt}) = \ln \eta_n + \ln \lambda_t$$

- Similarly the interior FOC for leisure can be expressed as:

$$\begin{aligned} 0 = & x_{nt} B_1 + 2\delta_0 l_{nt}^o + \sum_{s=1}^{\rho} \delta_s l_{n,t-s} + \eta_n \lambda_t w_{nt} \\ & + \sum_{s=1}^{\rho} E_t [(\delta_s + \gamma_{1s} \eta_n \lambda_{t+s} w_{n,t+s}) l_{n,t+s}^o | l_{nt}^o] \end{aligned}$$

where we draw on the parameterization of utility and productivity, which imply:

$$\frac{\partial}{\partial l_{nt}} U_{n,t+s} = \delta_s l_{n,t+s}$$

$$\frac{\partial}{\partial l_{nt}} \eta_n \lambda_{t+s} w_{n,t+s} l_{n,t+s} = \gamma_{1s} \eta_n \lambda_{t+s} w_{n,t+s} l_{n,t+s}$$

# A Social Planner's Problem

## Partitioning the social planner's problem

- Noting the preferences for consumption and leisure are additively separable, the planner:

- chooses  $c_{nt}$  for all  $(n, t)$  to maximize:

$$E_0 \left\{ \alpha^{-1} \sum_{t=0}^{\infty} \beta^t \left[ \int_0^1 \eta_n^{-1} c_{nt}^{\alpha} \exp [x_{nt} B_2 + \varepsilon_{2nt}] dn \right] \right\}$$

subject to the budget constraints for each  $t$ :

$$\int_0^1 c_{nt} dn \leq e'_t \equiv e_t + \int_0^1 w_{nt} l_{nt}^o dn$$

- chooses  $l_{nt}$  for all  $(n, t)$  to maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ (1 - d_{nt}) (x_{nt} B_0 + \varepsilon_{0nt}) + d_{nt} \varepsilon_{1nt} + x_{nt} B_1 l_{nt} + \sum_{s=0}^{\rho} \delta_s l_{n,t-s} l_{nt} + \eta_n \lambda_t w_{nt} l_{nt} \right] \right\}$$

where  $\lambda_t$  is the Lagrange multiplier associated with the aggregate resource constraint for the consumption allocation problem.



# A Social Planner's Problem

## Optimal participation

- The ex ante value function for the labor supply problem of  $n$  at  $t$  is:

$$\begin{aligned} & V(l_{n,t-s}, \dots, l_{n,t-1}, x_{nt}, \lambda_t w_t, \eta_n) \\ = & E_t \left\{ \sum_{r=t}^{\infty} \beta^{r-t} \left[ (1 - d_{nr}) \varepsilon_{0nr} + d_{nr} (x_{nr} B_0 + \varepsilon_{1nr}) \right. \right. \\ & \left. \left. + x_{nr} B_1 l_{nr}^o + \sum_{s=0}^{\rho} \delta_s l_{n,r-s}^o + \eta_n \lambda_t w_{nr} l_{nr}^o \right] \right\} \end{aligned}$$

and denote the associated conditional value functions by:

$$v_{0nt} = x_{nt} B_0 + \int \left[ V(l_{n,t-s+1}, \dots, 0, x_{n,t+1}, \lambda_{t+1} w_{t+1}, \eta_n) \times dF(x_{n,t+1}, \lambda_{t+1} w_{t+1} | x_{nt}, \lambda_t w_t) \right]$$

$$\begin{aligned} v_{1nt} = & \left[ x_{nt} B_1 + \eta_n \lambda_t w_{nt} + \sum_{s=0}^{\rho} \delta_s l_{n,t-s} \right] l_{nt}^o \\ & + \int \left[ V(l_{n,t-s+1}, \dots, l_{nt}^o, x_{n,t+1}, \lambda_{t+1} w_{t+1}, \eta_n) \times dF(x_{n,t+1}, \lambda_{t+1} w_{t+1} | x_{nt}, \lambda_t w_t) \right] \end{aligned}$$

- Then optimal participation is given by:

$$d_{nt} = \begin{cases} 0 & \text{if } v_{1nt} + \varepsilon_{1nt} \leq v_{0nt} + \varepsilon_{0nt} \\ 1 & \text{if } v_{1nt} + \varepsilon_{1nt} > v_{0nt} + \varepsilon_{0nt} \end{cases}$$

# A Representation of the Value Function

Telescoping forwards using the nonparticipation choice

- Denote the conditional choice probability of participation by:

$$p_{nt} = E_t [d_{nt} | I_{n,t-s}, \dots, I_{n,t-1}, x_{nt}, \eta_n \lambda_t w_t]$$

- Suppressing the arguments  $(I_{n,t-s}, \dots, I_{n,t-1}, x_{nt}, \eta_n \lambda_t w_t)$ , previous lectures on discrete choice proved:

$$\begin{aligned} V_{nt} &= \psi_0(p_{nt}) + v_{0nt} \\ &= \psi_0(p_{nt}) + x_{nt} B_1 + E_t [V_{n,t+1} | d_{nt} = 0] \end{aligned}$$

- Telescoping forwards:

$$\begin{aligned} V_{nt} &= \psi_0(p_{nt}) + x_{nt} B_1 + E_t [\psi_0(p_{n,t+1}) + x_{n,t+1} B_1 | d_{nt} = 0] \\ &\quad + E_t [V_{n,t+2} | d_{nt} = 0, d_{n,t+1} = 0] \\ &= \sum_{s=0}^{\rho} E_t [\psi_0(p_{n,t+s}) + x_{n,t+s} B_1 | d_{nt} = 0, \dots, d_{n,t+s} = 0] \\ &\quad + E_t [V_{n,t+\rho+1} | d_{n,t+1} = 0, \dots, d_{n,t+\rho} = 0] \end{aligned}$$

# A Representation of the Value Function

## Finite dependence

- Since:

$$\psi_0(p_{nt}) - \psi_1(p_{nt}) = v_{1nt} - v_{0nt}$$

it now follows that:

$$\begin{aligned} & \psi_0(p_{nt}) - \psi_1(p_{nt}) & (1) \\ = & \left[ x_{nt} B_1 + \eta_n \lambda_t w_{nt} + \sum_{s=0}^{\rho} \delta_s l_{n,t-s} \right] l_{nt}^o - x_{nt} B_0 \\ & + E_t [V_{n,t+1} | d_{nt} = 0] - E_t [V_{n,t+1} | l_{nt} = l_{nt}^o] \\ = & \left[ x_{nt} B_1 + \eta_n \lambda_t w_{nt} + \sum_{s=0}^{\rho} \delta_s l_{n,t-s} \right] l_{nt}^o - x_{nt} B_0 \\ & + \sum_{s=0}^{\rho} E_t [\psi_0(p_{n,t+s}) | l_{nt} = l_{nt}^o, d_{n,t+1} = 0, \dots, d_{n,t+s} = 0] \\ & - \sum_{s=0}^{\rho} E_t [\psi_0(p_{n,t+s}) | d_{nt} = 0, \dots, d_{n,t+s} = 0] \end{aligned}$$

# Aggregate Shocks

## Aggregate shocks with intertemporal nonseparabilities

- The previous lecture shows how to estimate preferences for consumption from the FOC.
- But how should we treat terms like  $E_t [\lambda_{t+s} w_{n,t+s} l_{n,t+s} | I_{nt}]$  and  $E_t [l_{n,t+s} | I_{nt}]$ , which appear in the Euler equation determining labor supply for those who participate in the workforce?
- We cannot justify substituting:

$$l_{n,t+s} \text{ for } E_t [l_{n,t+s} | I_{nt}]$$

and:

$$\lambda_{t+s} w_{n,t+s} l_{n,t+s} \text{ for } E_t [\lambda_{t+s} w_{n,t+s} l_{n,t+s} | I_{nt}]$$

to equate to zero the sample moment:

$$\frac{1}{N} \sum_{n=1}^N z_{nt} \left[ \begin{array}{l} x_{nt} B_1 + 2\delta_0 l_{nt}^o + \eta_n \lambda_t w_{nt} \\ + \sum_{s=1}^{\rho} \delta_s (l_{n,t-s} + l_{n,t+s}^o) + \gamma_{1s} \eta_n \lambda_{t+s} w_{n,t+s} l_{n,t+s}^o \end{array} \right]$$

because  $\lambda_{t+s}$  and  $w_{t+s}$  do not average out over the population.

# Aggregate Shocks

## Assumptions on the price process

- To estimate the model we place assumptions on the  $\lambda_t w_t$  process.
- Since the  $\lambda_t w_t$  process is endogenous, we must first establish these assumptions are compatible with the social planner's problem. (See Altug and Miller 1998, Appendix.)
- Assume  $\lambda_{t+1} w_{t+1} = \Delta_t \lambda_t w_t$  where  $\Delta_t$ :
  - 1 is an *iid* random variable with distribution function  $F(\Delta, \sigma)$ .
  - 2 has bounded positive support of length  $\zeta \equiv F(\bar{\Delta}, \sigma) - F(\underline{\Delta}, \sigma)$  where  $F(\underline{\Delta}, \sigma) = 0$  and  $F(\bar{\Delta}, \sigma) = 1$ .
- The model has  $\rho$  period dependence; to verify this set  $l_{n,t+s} = 0$  for all initial choices  $l_{nt}$  and note that the labor experience of the state variable is  $(0, \dots, 0)$  after  $\rho$  periods.
- We also assume  $2\zeta\rho < \bar{\eta} - \underline{\eta}$  for some  $\underline{\eta}$  and  $\bar{\eta}$  within the interior of the support of  $\eta$ .

# Estimation

A nonparametric estimator of the CCPs

- Decompose any random variable  $y_n$  into its conditional expectation function on any  $x_n$  and a disturbance  $\varepsilon_n$ , such that:

$$\begin{aligned}y_n &= E[y_n | x_n] + \varepsilon_n \\ &\equiv E[y_n | x_n] + \varepsilon_n \text{ and hence } E[\varepsilon_n | x_n] = 0\end{aligned}$$

- Let  $J(\cdot)$  denote a multivariate continuous probability density function satisfying  $J(0) \neq 0$  and  $J(\infty) = J(-\infty) = 0$ .
- Suppose  $f(x_n)$  is estimated with a weighted average of the other sample  $y$  values:

$$\widehat{f}(x_n) \equiv \left[ \sum_{m=1, m \neq n}^N J\left(\frac{x_m - x_n}{\delta_N}\right) \right]^{-1} \sum_{m=1, m \neq n}^N y_m J\left(\frac{x_m - x_n}{\delta_N}\right)$$

- Then for all  $x$  one can show (see for example Prakasa Rao, 1983):

$$\text{If } \delta_N \rightarrow 0 \text{ and } (N\delta_N)^{-1} \rightarrow 0 \text{ then } \widehat{f}(x) \xrightarrow{p} f(x)$$

# Estimation

Semiparametric behavioral responses (Table V, Altug and Miller, 1998)

## Behavioural responses<sup>†</sup>

Variable	$\eta_n = \mu_\eta + \sigma_\eta; v_n = \mu_v$						$v_n = \mu_v + \sigma_v; \eta_n = \mu_\eta$					
	$z_1^{(1)}$	$z_1^{(2)}$	$z_1^{(3)}$	$z_0^{(1)}$	$z_0^{(2)}$	$z_0^{(3)}$	$z_1^{(1)}$	$z_1^{(2)}$	$z_1^{(3)}$	$z_0^{(1)}$	$z_0^{(2)}$	$z_0^{(3)}$
<i>p</i>	0.96	0.43	0.24	0.31	0.24	0.23	0.96	0.41	0.23	0.31	0.24	0.24
<i>l</i>	1513	1460	1394	1490	1394	1320	1504	1448	1382	1447	1382	1306
<i>w</i>	7.18	5.71	5.48	5.94	5.38	5.46	8.80	7.00	6.72	7.28	6.60	6.70
$\lambda w l$	10877	7803	6311	8666	7028	5950	13250	9493	7670	10543	8542	7224

  

Variable	$\eta_n = \mu_\eta + \sigma_\eta; v_n = \mu_v$						$v_n = \mu_v + \sigma_v; \eta_n = \mu_\eta$					
	$z_1^{(1)}$	$z_1^{(2)}$	$z_1^{(3)}$	$z_0^{(1)}$	$z_0^{(2)}$	$z_0^{(3)}$	$z_1^{(1)}$	$z_1^{(2)}$	$z_1^{(3)}$	$z_0^{(1)}$	$z_0^{(2)}$	$z_0^{(3)}$
<i>p</i>	0.96	0.39	0.22	0.29	0.23	0.23	0.96	0.39	0.23	0.30	0.23	0.23
<i>l</i>	1484	1425	1357	1425	1357	1282	1486	1427	1360	1427	1360	1285
<i>w</i>	7.18	5.71	5.48	5.94	5.38	5.46	5.56	4.42	4.24	4.60	4.17	4.23
$\lambda w l$	10668	7622	6145	8465	6843	5782	8370	5910	4766	6564	5307	4484

<sup>†</sup> Nonparametrically estimated individual effects.

Corrected key to four panels

$$\begin{bmatrix} (\eta_n + \sigma_\eta, v_n + \sigma_v) & (\eta_n, v_n + \sigma_v) \\ (\eta_n + \sigma_\eta, v_n) & (\eta_n, v_n) \end{bmatrix}$$

# Estimation

A simulation estimator for the discrete choices

- To compute the CCPs required to form an orthogonality condition, we simulate for each  $(n, t)$  a future path of aggregate shocks that are specific to the individual, denoted by:

$$\lambda_{t+s+1}^{(n,t)} w_{t+s+1}^{(n,t)} = \lambda_{t+s}^{(n,t)} w_{t+s}^{(n,t)} \exp(\sigma\pi)$$

where  $\lambda_{t+s}^{(n,t)} w_{t+s}^{(n,t)}$  are estimates obtained from the consumption and wage equations,  $\pi$  is generated from a  $\mathcal{N}(0, 1)$  and  $\sigma$  is a volatility parameter to be estimated within the orthogonality conditions.



# Estimation

## Estimated CCPs for the simulation

- To account for the discreteness in participation, define:

$$p_{knt}^{(s)} = E_{\tau} \left[ d_{ns} \mid l_{n\tau} = kl_{nt}^o, l_{n,t+1} = 0 \dots, l_{n,s-1} = 0, x_{nt}^{(s)}, \eta_n \lambda_t w_t \right]$$

- To estimate  $p_{knt}^{(s)}$  define the indicator function and the weight for each  $(m, r) \in N \times T$ :

$$d_{mr}^{(s)} = [(1 - k) (1 - d_{m,r-s}) + kd_{m,r-s}] \prod_{r=1}^{s-1} (1 - d_{n,t-r})$$

$$J_{knt}^{(s)}(m, r) \equiv J_x \left( \frac{x_{mr} - x_{nt}^{(s)}}{\delta_N} \right) J \left( \frac{\eta_m \lambda_r w_r - \eta_n \lambda_t^{(n,s)} w_t^{(n,s)}}{\delta_N} \right) \\ \times d_{knt}^{(s)} J \left( \frac{l_{m,r-s} - l_{nt}}{\delta_N} \right)^k \prod_{\tau=s+1}^{\rho} J \left( \frac{l_{m,r-\tau} - l_{n,t+s-\tau}}{\delta_N} \right)$$

- Then the estimated CCPs are:

$$\widehat{p}_{knt}^{(s)} \equiv \left[ \sum_{(m,r)}^{N \times T} J_{knt}^{(s)}(m, r) \right]^{-1} \sum_{(m,r)}^{N \times T} d_{mr} J_{knt}^{(s)}(m, r)$$

# Estimation

A simulation estimator for the discrete choice optimality conditions

- Recall the Type 1 Extreme value assumption implies:

$$\psi_j(p_{nt}) = -\ln p_{jnt} \equiv -\ln p_j(l_{n,t-s}, \dots, l_{n,t-1}, x_{nt}, \eta_n \lambda_t w_t)$$

- Then from (1):

$$\begin{aligned} \ln p_{1nt} - \ln p_{0nt} &= \left( x_{nt} B_1 + \eta_n \lambda_t w_{nt} + \sum_{s=0}^{\rho} \delta_s l_{n,t-s} \right) l_{nt}^o - x_{nt} B_0 \\ &\quad + \sum_{s=1}^{\rho} E_t \left[ \ln \left( p_{0nt}^{(s)} \right) - \ln \left( p_{1nt}^{(s)} \right) \right] \end{aligned}$$

- Given instruments  $z_{nt}$  we obtain an estimator from:

$$0 = \sum_{n=1}^N z_{nt} \left[ y_{0nt} - \left( x_{nt} B_1 + \sum_{s=0}^{\rho} \delta_s l_{n,t-s} \right) l_{nt}^o - x_{nt} B_0 \right]$$

where:

$$y_{0nt} \equiv \sum_{s=0}^{\rho} \left[ \ln \left( p_{0nt}^{(s)} \right) - \ln \left( p_{1nt}^{(s)} \right) \right] + \eta_n \lambda_t w_{nt} l_{nt}^o$$

# Estimation

A simulation estimator for the interior labor market choices

- We could exploit the interior FOC for leisure directly in estimation by simulating future values of labor supply off the price shocks that we draw for each person.
- Alternatively the interior FOC for leisure can be expressed as:

$$\begin{aligned} & x_{nt} B_1 + 2\delta_0 l_{nt}^o + \sum_{s=1}^{\rho} \delta_s l_{n,t-s} + \eta_n w_{nt} \\ = & \frac{-\partial}{\partial l_{nt}} E [V (l_{n,t-s+1}, \dots, l_{nt}^o, x_{n,t+1}, \lambda_{t+1}, \eta_n)] \\ = & \frac{-\partial}{\partial l_{nt}} \sum_{s=1}^{\rho} \beta^s E_t \left[ \psi_0 \left( p_{1n}^{(s)} \right) \mid l_{nt} = l_{nt}^o \right] \end{aligned}$$

where the third line exploits the facts that neither  $x_{n,t+s} B_1$  nor  $E_t [V_{n,t+\rho+1} \mid d_{n,t+1} = 0, \dots, d_{n,t+\rho} = 0]$  depend on  $l_{nt}$ .

# Estimation

A simulation estimator for the interior labor market choices

- Appealing to the Type 1 Extreme Value assumption and defining:

$$y_{nt} \equiv \frac{\partial}{\partial l_{nt}} \sum_{s=1}^{\rho} \beta^s \ln \left( p_{1n}^{(s)} \right) + \eta_n w_{nt}$$

gives us:

$$y_{nt} = x_{nt} B_1 + 2\delta_0 l_{nt}^o + \sum_{s=1}^{\rho} \delta_s l_{n,t-s} + \varepsilon_{nt}$$

where for all  $z_{nt}$  in the information set of  $n$  at  $t$ :

$$E[\varepsilon_{nt} | z_{nt}] = 0$$

- An estimator is now obtained from:

$$0 = \sum_{n=1}^N z_{nt} \left[ y_{nt} - x_{nt} B_1 - 2\delta_0 l_{nt}^o - \sum_{s=1}^{\rho} \delta_s l_{n,t-s} \right]$$

# Estimation

Current utility parameter estimates (Table VI, Altug and Miller, 1998)

## Hours and participation†

Variable	Parameter	1. $\beta=0.9, \sigma=0.05$			2. $\beta=0.9, \sigma=0.05$			3. $\beta=0.9, \sigma=0$			4. $\beta=0.9, \sigma=0$
		(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(c)
1	$B_{00}$	1257 (566)	—	-676 (230)	-14 (264)	—	-154 (115)	-44 (264)	—	-277 (113)	—
KIDS <sub>it</sub>	$B_{01}$	31 (222)	—	116 (110)	204 (132)	—	-43 (72)	173 (132)	—	120 (41)	—
KIDS <sub>it</sub> × MAR <sub>it</sub>	$B_{02}$	-184 (208)	—	5-62 (106)	-187 (129)	—	42 (71)	-154 (129)	—	-77 (25)	—
AGE <sub>it</sub>	$B_{03}$	-37 (33)	—	42.2 (16)	31 (13)	—	13 (6-6)	29 (13)	—	18 (6-5)	—
AGE <sub>it</sub> <sup>2</sup>	$B_{04}$	0-06 (0-03)	—	0-03 (0-02)	-0-03 (0-01)	—	-0-02 (0-01)	-0-03 (0-01)	—	-0-03 (0-01)	—
$I_{it}$	$B_{10}$	-11-6 (0-51)	-10-4 (0-04)	-11 (0-25)	-5-4 (0-24)	-5-6 (0-02)	-5-5 (0-12)	-4-8 (0-24)	-5-1 (0-02)	-4-5 (0-11)	-2-7 (0-12)
KIDS <sub>it</sub> × $I_{it}$	$B_{11}$	0-16 (0-17)	0-28 (0-02)	0-22 (0-10)	-0-55 (0-10)	-0-14 (0-01)	-0-40 (0-07)	-0-49 (0-10)	-0-14 (0-01)	0-3E-05 (0-1E-04)	-0-51 (0-15)
(KIDS <sub>it</sub> × MAR <sub>it</sub> ) $I_{it}$	$B_{12}$	0-18 (0-16)	-0-12 (0-02)	0-07 (0-10)	0-60 (0-10)	0-34 (0-01)	0-52 (0-07)	0-54 (0-10)	0-34 (0-01)	-0-05 (0-02)	0-90 (0-15)
AGE <sub>it</sub> × $I_{it}$	$B_{13}$	0-31 (0-49)	0-03 (0-003)	-0-60 (0-26)	-0-52 (0-17)	0-008 (0-001)	-0-40 (0-01)	-0-47 (0-17)	0-003 (0-001)	-0-24 (0-01)	0-19 (0-005)
AGE <sub>it</sub> <sup>2</sup> × $I_{it}$	$B_{14}$	0-002 (0-05E-03)	0-002 (0-5E-04)	0-002 (0-3E-03)	0-002 (0-2E-03)	0-002 (0-2E-04)	0-002 (0-1E-03)	0-002 (0-2E-03)	0-001 (0-2E-04)	0-002 (0-1E-03)	0-002 (0-2E-03)
$I_{it}^2$	$\delta_0$	0-002 (0-1E-03)	0-002 (0-2E-04)	0-002 (0-6E-04)	0-001 (0-6E-04)	0-001 (0-1E-05)	0-001 (0-3E-04)	0-001 (0-6E-04)	0-001 (0-8E-05)	0-001 (0-3E-04)	0-001 (0-1E-03)
$I_{it}I_{it-1}$	$\delta_1$	-0-001 (0-1E-03)	-0-001 (0-2E-04)	-0-001 (0-1E-04)	-0-6E-03 (0-5E-04)	-0-5E-03 (0-1E-05)	-0-6E-03 (0-4E-04)	-0-5E-03 (0-5E-04)	-0-5E-03 (0-1E-05)	-0-6E-03 (0-4E-04)	-0-7E-03 (0-1E-03)
$I_{it}I_{it-2}$	$\delta_2$	-0-7E-03 (0-1E-03)	-0-7E-03 (0-2E-04)	-0-7E-03 (0-1E-04)	-0-4E-03 (0-6E-04)	-0-4E-03 (0-1E-05)	-0-3E-03 (0-4E-04)	-0-3E-03 (0-6E-04)	-0-4E-03 (0-9E-05)	-0-3E-03 (0-5E-04)	-0-6E-03 (0-1E-03)
$I_{it}I_{it-3}$	$\delta_3$	-0-7E-03 (0-1E-03)	-0-8E-03 (0-2E-04)	-0-6E-03 (0-1E-04)	-0-3E-03 (0-5E-04)	-0-3E-03 (0-1E-05)	-0-3E-03 (0-4E-04)	-0-2E-03 (0-5E-05)	-0-3E-03 (0-9E-05)	-0-2E-03 (0-4E-04)	-0-5E-03 (0-1E-03)

† Standard errors in parentheses. a. Participation equation; b. Hours equation; c. Joint estimates.  
Column 1: Nonparametric fixed effects. Columns 2, 3 and 4: Time-averaged fixed effects.