# Ascending Auctions

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Structural Econometrics

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# Introduction

- We study ascending auctions for government-issued financial products.
- These are Certificates of Deposit (CDs) issued by the state of Texas to local banks.
- The auction format used is an ascending English auction where banks compete on interest rate.
- Bid data displays evidence of bidding frictions.
- We formulate a model of bidding in ascending auctions with bidding frictions and estimate the model from the submitted bid data.

# Introduction

- We specify a model with stochastic arrival of bidding opportunities.
- Bidders are assumed to play undominated strategies.
- The timing of bidder activity within an auction is used to identify the overall distribution of valuations.
- The distribution of valuations is point-identified, but not the valuations of individual bidders.
- Pairs of bids within an auction identify auction-specific unobserved heterogeneity.

# Introduction

- Results show that banks private valuation for deposit funds has both higher mean and variance prior to the 2008 financial crisis.
- There is an increase in the monitoring rate in the post-2008 period.
- Frictions are costly in terms of both revenue and allocative efficiency:
    - Auction revenue would increase by 19.6% (pre-2008) and 6.5% (post-2008) without frictions.
    - The expected valuation of winning bidders also increases in a frictionless environment by up to 0.1653 percentage points pre-2008 and 0.0501 percentage points post-2008.

# Related Literature

- *Applied Work on Ascending Auctions*
  - Ackerberg, Hirano & Shahriar (2011)
  - Bajari & Hortacsu (2003)
  - Cho, Paarsch & Rust (2014)
  - Daniel & Hirschleifer (1998)
- *Identification of Bidder Valuations*
  - Guerre, Perrigne & Vuong (2000)
  - Haile & Tamer (2003)
- *Unobserved Heterogeneity in Auctions*
  - Krasnokutskaya (2011)
  - Decarolis (2017)
  - Freyberger and Larsen (2017)
- *Auction Markets in the Financial Crisis*
  - Cassola, Hortacsu, & Kastl (2013)

# Auction Description

- The mechanism is an ascending auction lasting 30 minutes.
- A reservation interest rate and an upper bound on total available funds is set prior to bidding, usually $80 million.
- During the 30 minute period banks can bid on up to 5 separate parcels by announcing a quantity and an interest rate.
- The minimum quantity is $100,000, the maximum $7 million (with increments of $100,000).
- Each bid is binding and can only be increased throughout the auction.
- Funds are allocated to banks offering the most attractive interest rates at the end.
- Winning banks pay the interest rate they bid. Losing banks pay nothing.
- Partial order-filling is possible.

# Auction Description

- Our data set contains 78 auctions from 2006-2010.
- There is a pool of 73 potential banks with an average of 24.5 banks entering.
- Averaging across auctions, 72% of banks win.
- Money left on the table (MLT) is the dollar difference in interest payments for a winning submission and the highest losing bid.
- MLT is \$624 (pre) and \$1372 (post) per winning bid.
- Average national CD rate in the post-2008 period (earliest FDIC data we have) 0.79% per annum.
- The average reserve rate between 2008 and 2010 in these auctions at 0.71% is slightly less.

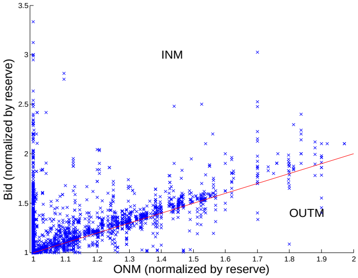
# Summary Statistics

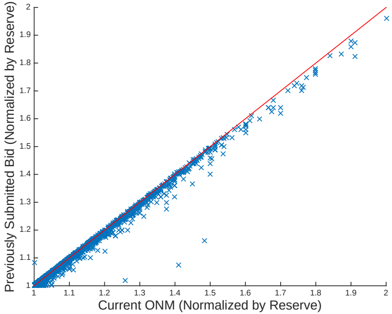
Table: Summary Statistics on Auctions

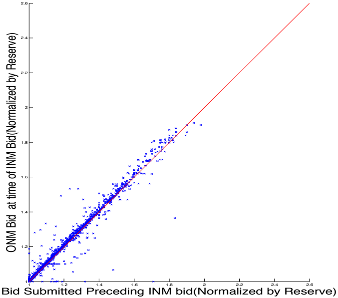
| | Pre | | Post | |
|---|---|---|---|---|
| | Mean | *(Std. Dev.)* | Mean | *(Std. Dev.)* |
| Number of Banks per Auction | 27.50 | *(5.39)* | 22.82 | *(6.50)* |
| Number of Bids Per Parcel | 13.76 | *(20.78)* | 12.18 | *(23.04)* |
| Proportion of Bids In The Money (INM) | 0.67 | *(0.13)* | 0.69 | *(0.12)* |
| Proportion of Bids Out of The Money (OUTM) | 0.17 | *(0.15)* | 0.14 | *(0.14)* |
| Proportion of Bids On The Money (ONM) | 0.17 | *(0.07)* | 0.17 | *(0.06)* |
| Size of Parcels (millions) | 1.63 | *(0.40)* | 1.49 | *(0.44)* |
| Number of Parcels | 1.75 | *(1.13)* | 1.50 | *(0.93)* |
| Proportion of Banks who win | 0.70 | *(0.21)* | 0.74 | *(0.22)* |
| Annual Reserve Coupon Rate | 4.83 | *(0.45)* | 0.71 | *(0.80)* |
| Award Amount to Winning Bank (millions) | 3.88 | *(0.48)* | 4.51 | *(0.57)* |
| MLT: (Winning Bid-Highest Losing Bid) $\times$(Size of parcel in $) | 624.30 | *(2117)* | 1372.15 | *(3606)* |

# Bids

- We now present figures describing key features of the bidding process.
- The first figure shows that bids are submitted in excess of the lowest provisionally winning bid, the on-the-money (ONM) rate.
- The second figure indicates that a bid on the ONM rate is preceded by a bid close to the ONM rate.
- The third figure show that who banks who submit at provisionally winning in-the-money (INM) rate, preceded with a bid that is on the ONM rate (but losing due to time priority).


INM
OUTM
Bid (normalized by reserve)
ONM (normalized by reserve)
3.5
3
2.5
2
1.5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

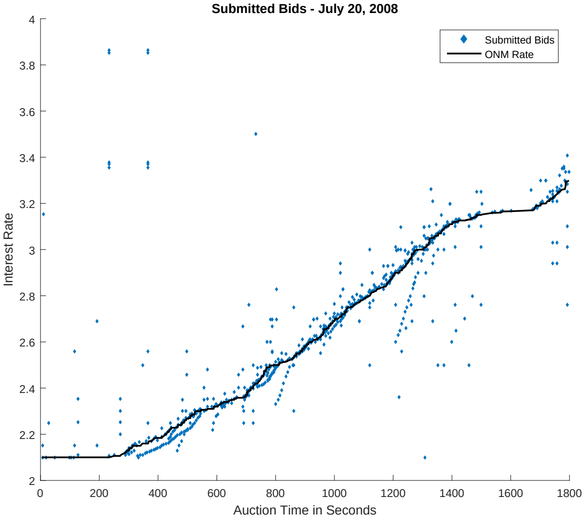

Previously Submitted Bid (Normalized by Reserve)
Current ONM (Normalized by Reserve)
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1


ONM Bid at time of INM Bid(Normalized by Reserve)
Bid Submitted Preceding INM bid(Normalized by Reserve)

# Banks' Information

- Banks do not know the current ONM rate when submitting bids.
- However, they can use a "creeping" strategy of submitting successive bids to learn the ONM rate.
- A large number of bidders submit ITM bids immediately after reaching the ONM rate.
- Other bidders jump directly ITM without creeping.
- The following slide shows all bids from a single auction. The bid data displays:
  - Jump bids
  - Creeping used by bidders
  - ONM rate that rises steadily throughout the auction.

# Bids

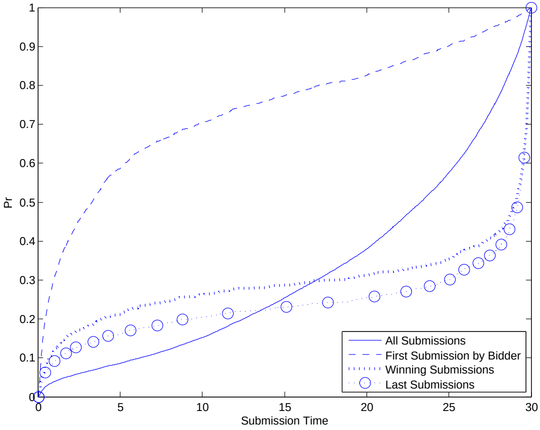

Submitted Bids - July 20, 2008
Submitted Bids
ONM Rate
Interest Rate
Auction Time in Seconds
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
0
200
400
600
800
1000
1200
1400
1600
1800

# Submission Times

- The next figure is the empirical distributions of initial bid submission times, submission times of all bids and winning bid submission times.
- The middle of the auction has very little bidding activity, with more activity at both ends.
- Many winning bids are submitted prior to the final minutes of the auction.
- That winning bids are sometimes submitted in the early stages of the auctions is further evidence that banks are not incrementally increasing their bids.

# Submission Times

Figure: Empirical Distribution of Order Submission Times

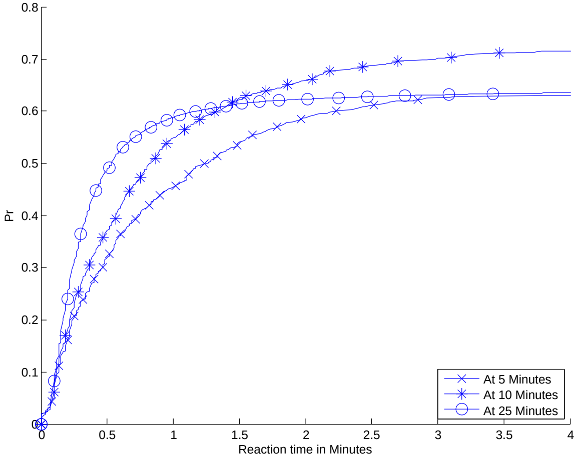


# Submission Times

- We then present empirical distributions on reaction times.
- Consider Banks who submit INM bids after being pushed OUTM.
- We look at the time to return INM.
- The second figure depicts the distribution of reaction times at five, 10, and 25 minutes mark.
- As the auction progresses monitoring becomes more intense.

# Submission Times

Figure: CDF Reaction Time



# Summary

1. The number of banks is uncertain until the auction ends.
2. Bidding activity is most intense at the beginning and end of the auction (like a limit order market).
3. Sniping is not universal, as many winning bids are submitted in the early stages of the auction.
    - This rules out observational equivalence to first-price sealed bid auctions, so we cannot use Guerre, Perrigne, and Vuong (2000) to identify bidder valuations.
4. The interest rate spread of winning bids is notable, in contrast to English auctions.
    - Most empirical papers on ascending auctions use a frictionless approach, including Paarsch (1997), Aradillas-Lopez et al (2013), and Freyberger and Larsen (2017).

# Frictionless Test

- We can also formally test the assumption of no bidding frictions.
- Empirically, a frictionless environment requires that all winning bidders pay the same price.
- Define $f_{\underline{W}}$ as the distribution of the lowest winning bid and $f_{\overline{W}}$ as the distribution of the highest winning bid.
- Under the null hypothesis of no frictions, $f_{\underline{W}} = f_{\overline{W}}$ a.e.
- We perform a nonparametric density comparison test using Li (1996).
- We reject the null hypothesis of equal distributions at the 1% confidence level.

# Relation to Haile and Tamer (2003)

- Another empirical approach to ascending auctions is the incomplete bidding model of Haile and Tamer (2003).
- They impose two restrictions on bidder behavior:
  1. Bidders never bid above their valuation.
  2. A bidder never lets another bidder win at a price they are willing to beat.
- These two rules allow for a large class of bidding strategies, many of which would be dominated strategies in our model.
- Consider the strategy where each bidder bids exactly their valuation.
  - This follows both of the Haile-Tamer rules.
  - However this is strictly dominated in a discriminatory auction.

# Relation to Haile and Tamer (2003)

- Our approach differs from Haile-Tamer in that we assume
  - $1^*$ Bidders don't play dominated strategies.
  - $2^*$ Frictions place restrictions on when bidders may place bids.
- Assumption $1^*$ can be seen as strengthening the assumption of bidder rationality relative to Haile-Tamer.
- Assumption $2^*$ relaxes the second Haile-Tamer assumption, as frictions may prevent the highest valuation bidder from winning the auction.

# Model of Bidding Frictions

- We formulate a model of bidding in ascending auctions with bidding frictions.
- Frictions are present through stochastic arrival of bidding opportunities.
- Because bidders may not have the opportunity to respond to other bids, there is no guarantee that the highest valuation bidders win the auction.
- Valuations have a private component and an auction-specific component common to all bidders.
- Bidding opportunity arrival rates are independent of bidder valuations.

# Model of Bidding Frictions

- Set of bidders given by $\mathcal{I} = \{1, ..., I\}$.
- Auctions indexed by $k = 1, , , K$.
- Valuation for bidder $i$ in auction $k$ is given by

$$\tilde{v}_{ik} \equiv r_k + x_{ik} + y_k \,.$$

  - $r_k$ is auction reserve rate.
  - $x_{ik}$ is private value signal, an i.i.d. draw from distribution $F_X$.
  - $y_k$ is auction specific component affecting all bidders, drawn from $F_Y$ with $\mathbb{E}[Y] = 0$.
- $X_{ik}$ and $Y_k$ are assumed to be independent.

# Model of Bidding Frictions

- Bidding occurs over a fixed time interval $[0, T]$.
- Bidders face frictions in the form of random arrival of bidding opportunities.
- There are a set of probability distributions $\{G_\tau(t) : \tau \leq t\}$, with $g_\tau(t)$ the associated densities, that govern monitoring arrival times.
- These arrival processes are activated (i) at the start of the auction and (ii) whenever a bid is pushed OUTM.
- $G_\tau(t)$ is the probability that a bidder pushed OUTM at time $\tau$ receives another bidding opportunity at or before time $t$.

# Model of Bidding Frictions

- At each monitoring opportunity $j$, the action space of a bidder consists of a pair $(\overline{b}_{ij}, b_{ij}) \in \mathbb{R}^2_+$.
- Denoting the ONM rate at time $t_j$ by $r_{t_j}$, the bidder first announces a number $\overline{b}_{ij}$. If this number is higher than $r_{t_j}$, the bidder learns the ONM rate and proceeds to make a bid $b_{ij}$.
- We assume that if a bidder stops bidding then they exit the auction.
- This information structure mimics the "creeping" strategy bidders use prior to placing a "real" bid, with bidders learning the ONM rate due to time priority.
- The history for each bidder is the set $h_{ij} = \{\tau_{is}, \max\{b_{is}, r_{t_s}\}, b_{is}\}_{s=1}^{j}$, where $\tau_{is}$ denotes the time of bidder $i$'s $s$-th monitoring opportunity.

# Model of Bidding Frictions

- If a bidder's announcement $\overline{b}_{ij}$ is larger than $r_{t_j}$, then the bidder chooses a bid to solve

$$V(h_{ij}, r_{t_j}) =$$

$$\max_{b_{ij} \in [r_{t_j}, \infty)} \left\{ \begin{array}{l} \left[\Pr\left(b_{ij} \geq \overline{r}\right) \cdot \left(v_i - b_{ij}\right)\right] \quad + \\ \mathbb{E}\left[\ \mathbb{1}\{b_{ij} < \overline{r}\} \cdot \mathbb{1}\{j < J\} \cdot \mathbb{1}\{v_i > r_{t_{j+1}}\} \ \cdot \ V\left(h_{ij+1}, r_{t_{j+1}}\right) \middle| h_{ij} \ \right] \end{array} \right\}$$

where $J$ is the last monitoring opportunity (a random variable) and $\overline{r}$ is the lowest winning bid at the end of the auction.

- The first term in the sum corresponds to the case where there is not a future bidding opportunity.

- The second term is the case in which the current bid $b_{ij}$ is pushed OUTM prior to the end of the auction and the bidder obtains another chance to bid after being pushed out.

# Model

- The first order condition of the bidder's problem is

$$
\begin{aligned}
0 &= \frac{\partial \Pr(b > \bar{r})}{\partial b}(v - b) - \Pr(b > \bar{r}) \\
&+ \frac{\partial}{\partial b}\mathbb{E}\left[\ \mathbb{1}\{b_{ij} < \bar{r}\} \cdot \mathbb{1}\{j < J\} \cdot \mathbb{1}\{v_i > r_{t_{j+1}}\} \ \cdot\ V(h_{ij+1}, r_{t_{j+1}})\ \middle| h_{ij}\right]
\end{aligned}
$$

- The first line is exactly the first-price sealed bid case.
- The second term arises from the dynamics of multiple bidding opportunities.
- The main barrier is that $\bar{r}$ and $r_{t_{j+1}}$ depend on the strategies of all the other players, which in turn depends on previous bids (and therefore the unobserved valuation).
- We would need to solve for the equilibrium in order to know the functional relationship between these terms.

# Equilibrium Properties

- The model may have multiple equilibria, including mixed strategy equilibria.
- Solving for equilibria is computationally burdensome, and there is no way of knowing which equilbirium is being played by bidders.
- Instead, we utilize a condition that is weaker than best response but is consistent with everybody playing their equilibrium strategy.
- This has the advantage of being robust to any equilibrium played in the data – in fact, the presence of multiple equilibria can even aid in estimation.
- Specifically, in any equilibrium we will have that
    1. Bidders never submit a bid greater than their valuation.
    2. Bidders will submit a bid at every bidding opportunity.

# Identification

- Identification of $F_V$ is based on the fact that whenever a bidder submits an INM bid we know their valuation is at least as high as the current ONM rate.
- If a bidder stops being active, it is because either
    1. The ONM rate had passed the bidder's valuation at their next bidding opportunity
    2. Another opportunity to bid was never received.
- The last time a bidder is pushed out of the money contains the most information about their valuation, so the likelihood uses only these observations.
- Valutions are independent of the monitoring distribution, so we can identify the bid arrival distribution directly from reaction times to being pushed OUTM.
- Multiple bids within the same auction allow for identification of both the individual valuations and auction-specific heterogeneity.

# Identification

- Let $j = 1, \ldots, J$ index the events when a bidder is pushed OUTM.
- When a bidder is pushed out of the money for the $j$-th time at $t_j^o$, three things can happen:
    1. The bidder submits another bid at time $t_j^b$ which is a winning bid. This happens when another bidding chance is received and $\bar{b}_{ij} > \bar{r}$.
    2. The bidder submits another bid at time $t_j^b$ but this bid is also pushed OUTM (at $t_{j+1}^o$): another bidding chance is received and $\bar{b}_{ij} > r_{t_j^b}$.
    3. No other bids are submitted by the bidder: either another chance to bid is never obtained or $\bar{b}_{ij} < r_{t_j^b}$.

# Identification

- The likelihoods associated with these three events are
  1. Winning bid:

$$g_{t_j^o}\left(t_j^b\right) \times \Pr\left(v_i > \bar{r} \middle| v_i > r_{t_{j-1}^b}\right) = g_{t_j^o}\left(t_j^b\right)\left[\frac{1 - F_V\left(\bar{r}\right)}{1 - F_V\left(r_{t_{J-1}^b}\right)}\right]$$

  2. Non-winning bid:

$$g_{t_j^o}\left(t_j^b\right) \times \Pr\left(v_i > r_{t_j^b} \middle| v_i > r_{t_{j-1}^b}\right) = g_{t_j^o}\left(t_j^b\right)\left[\frac{1 - F_V\left(r_{t_j^b}\right)}{1 - F_V\left(r_{t_{j-1}^b}\right)}\right]$$

  3. No future bids:

$$\sum_{s=r_{t_{J-1}^o}}^{\rho} G_{t_J^o}\left(t_{s+1}\right)\left[\frac{F_V\left(r_{s+1}\right)}{1 - F_V\left(r_{t_{J-1}^b}\right)} - \frac{F_V\left(r_s\right)}{1 - F_V\left(r_{t_{J-1}^b}\right)}\right] + \left(1 - G_{t_J^o}\left(T\right)\right).$$

  where $s = 1, \ldots, \rho$ indexes increases to the ONM rate during the auction.

# Identification

- Taking the product across all times being pushed OUTM yields

$$\mathcal{L}(F_V, \{G_t\}; \{t_j^o, t_j^b\}_{j=1}^J)$$

$$= \prod_{j=1}^{J-1} g_{t_j^o}(t_j^b) \left[\frac{1 - F_V(r_{t_j^b})}{1 - F_V(r_{t_{j-1}^b})}\right] \left( \mathbb{1}\{i \text{ wins}\} \left( g_{t_J^o}(t_J^b) \left[\frac{1 - F_V(\bar{r})}{1 - F_V(r_{t_{J-1}^b})}\right]\right) + \right.$$

$$\left. \mathbb{1}\{i \text{ loses}\} \left\{ \sum_{s=r_{t^o_{J_{ik}}-1}}^{\rho} G_{t_J^o}(t_s) \left[\frac{F_V(r_{t_{s+1}}) - F_V(r_{t_s})}{1 - F_V(r_{t^b_{J_i-1}})}\right] + (1 - G_{t_j^o}(T)) \right\} \right)$$

$$= \prod_{j=1}^{J-1} g_{t_j^o}(t_j^b) \times \left( \mathbb{1}\{i \text{ wins}\} \left(g_{t_J^o}(t_J^b)\left[1 - F_V(\bar{r})\right]\right) + \right.$$

$$\left. \mathbb{1}\{i \text{ loses}\} \left\{ \sum_{s=r_{t^o_{J-1}}}^{\rho} G_{t_J^o}(t_s)\left[F_V(r_{t_{s+1}}) - F_V(r_{t_s})\right] + (1 - G_{t_j^o}(T)) \right\} \right)$$

# Identification

- Because bidders submit bids at every bidding opportunity, $G_t$ is identified directly from the response times of bidders to being pushed out of the money.
- The distribution $F_V$ is identified because the likelihood function is globally concave.
- In order to obtain the joint distribution $F_{V_1,V_2}$ we use the fact that multiple bids within the same auction are observed to condition the valuation of one bidder's valuation on a lower bound for the second bidder's valuation.
- Specifically, if bidder 1 in auction $k$ submits a bid $b_{1k}$, then bidder 2's valuation is a draw from $F_{V_2|V_1 \geq b_{1k}}$.
- Together, $F_V$ and $F_{V_2|V_1 \geq a}$ are used to construct the joint distribution $F_{V_1,V_2}$:

$$F_{V_1,V_2}(a_1, a_2) = F_V(a_1) - \Pr(V_2 > a_2)\Pr(V_1 < a_1 | V_2 > a_2)$$

# Identification

- Recall that $V_{ik}$ is additive in $X_{ik}$ and $Y_k$:

$$\begin{aligned} V_{1k} &= X_{1k} + Y_k \\ V_{2k} &= X_{2k} + Y_k \end{aligned}$$

- From the lemma of Kotlarksi, the characteristic functions are multiplicative:

$$\psi_{V_1,V_2}(t_1, t_2) = \psi_Y(t_1 + t_2)\psi_X(t_1)\psi_X(t_2)$$

- With the location of one of the variables fixed (we have assumed $\mathbb{E}[Y] = 0$) the characteristic functions of $Y$ and $X$ are identified from $\psi_{V_1,V_2}$.
- Since the characteristic functions are a one-to-one mapping to the distributions $F_X$ and $F_Y$ are identified.

# Estimation

- The distribution of bid arrival times is estimated according to

$$\hat{G}_t(z) = \frac{\sum_{k=1}^{K} \sum_{i \in \mathcal{I}^k} \sum_{j=1}^{J_i} K\left(\frac{\tau_{ij}^{*k} - t}{h}\right) \mathbf{1}\{\tau_{ij}^{k} - \tau_{ij}^{*k} < z\}}{\sum_{k=1}^{K} \sum_{i \in \mathcal{I}^k} \sum_{j=1}^{J_i} K\left(\frac{\tau_{ij}^{*k} - t}{h}\right)}$$

  where $\tau_{ij}^{k}$ is the time a bidder re-enters after being pushed OUTM at $\tau_{ij}^{*k}$.

- $F_V$ is estimated by maximizing the likelihood across all bidders and auctions:

$$\hat{F}_V = \arg\max_{F} \prod_{k} \prod_{i} \mathcal{L}(F, \{\hat{G}_t\}; \{t_{J_i k}^{o}, t_{J_i k}^{b}\})$$

- Similarly, the conditional distribution $F_{V_1|V_2>a}$ is estimated by maximizing the likelihood given another bidder in the auction has bid at least $a$:

$$\hat{F}_{V_1|V_2>a} = \arg\max_{F_{V_1|V_2}} \prod_{k \in K_a} \prod_{i \neq i_a} \mathcal{L}(F_{V_1|V_2}, \{\hat{G}_t\}; \{t_{J_i k}^{o}, t_{J_i k}^{b}\})$$

  where $K_a$ is the set of auctions in which a bidder $i_a$ submits a bid $b_{i_a} \geq a$.

# Identification and Estimation

- With the joint distribution we can generate the characteristic function $\psi_{V_1,V_2}$ and use deconvolution methods to obtain the characteristic functions for $X$ and $Y$, $\psi_X$ and $\psi_Y$.

- Characteristic function for $\psi_{V_1,V_2}$ estimated by

$$\hat{\psi}_{V_1,V_2}(t_1, t_2) = \sum_{v_1 \in \mathbf{V}} \sum_{v_2 \in \mathbf{V}} e^{(it_1 v_1 + it_2 v_2)} \hat{p}_{V_1,V_2}(v_1, v_2)$$

  where $\hat{p}_{V_1,V_2}$ is the estimated pmf for $(V_1, V_2)$.

- Using the deconvolution results of Kotlarski (with this formulation due to Rao (1992)) yields

$$\hat{\psi}_Y(t) = \exp\left(\int_0^t \frac{\partial}{\partial u}\left[\frac{\hat{\psi}_{V_1,V_2}(u,v)}{\hat{\psi}_{V_1,V_2}(u,0)\hat{\psi}_{V_1,V_2}(0,v)}\right]_{u=0} dv\right) \quad (1)$$

$$\hat{\psi}_X(t) = \frac{\hat{\psi}_{V_1,V_2}(t,0)}{\hat{\psi}_Y(t)} \quad (2)$$

# Identification and Estimation

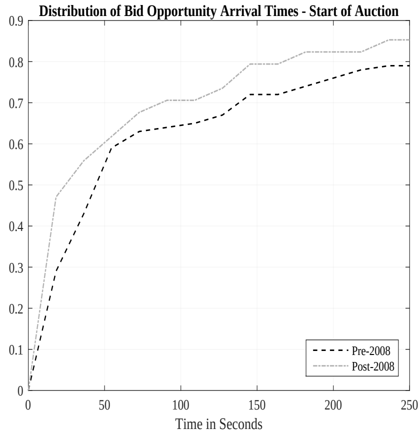
- We discretize the support of $X_{ik}$ and $Y_k$.
- Given the discretization, we map the estimated characteristic functions back into the probability mass functions according to the inverse Fourier transform:

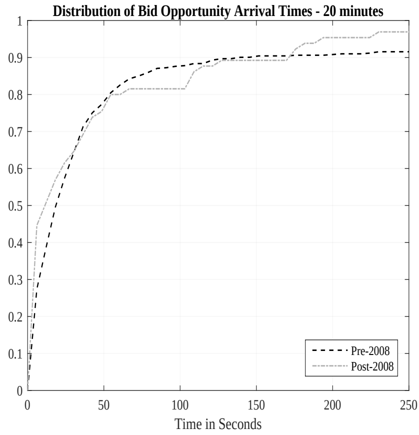
$$\hat{p}_X(kh_X) = \frac{1}{2\pi/h_X}\int_{-\pi/h_X}^{\pi/h_X} e^{itkh_X}\hat{\psi}_X(t)dt$$

$$\hat{p}_Y(kh_Y) = \frac{1}{2\pi/h_Y}\int_{-\pi/h_Y}^{\pi/h_Y} e^{itkh_Y}\hat{\psi}_Y(t)dt$$

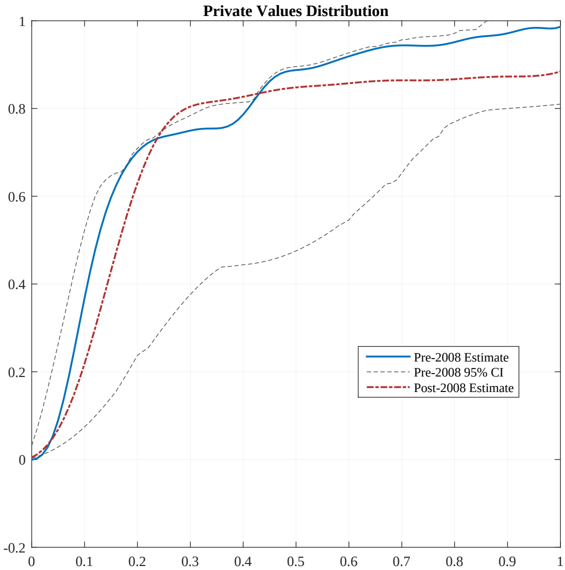
where $k$ takes integer value and each random variable takes value $kh$ for $h > 0$.

# Results: Bid Opportunity Arrival Times

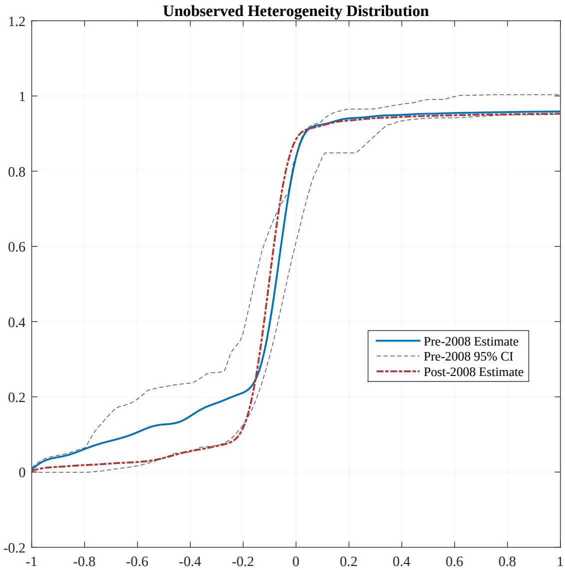


# Results: Private Values



# Results: Unobserved Heterogeneity



# Results

- Estimates indicate more monitoring post-2008, especially earlier in the auction.
  - Effect of frictions in determining auction outcomes is reduced.
- Private valuation component has higher variance post-2008 compared with pre-2008.
- Unobserved auction-specific term also sees lower dispersion post-2008

# Measurements

- Frictions affect auction outcomes through the inability of high-valuation bidders to respond to being pushed out.
- In order to assess how costly frictions are, we bound the expected valuation of winning bidders and compare this to a frictionless environment.
- Our comparison is to a uniform price ascending auction without frictions in which all winners pay the highest loser's valuation.
- We also compare realized auction revenue to that generated by the uniform price auction.

# Costs of Bidding Frictions

- Let $W$ denote the event of placing a winning bid and $\widetilde{W}$ its complement, losing.
- The law of iterated expectations, $E\left[v\right]$ can be expressed as a weighted sum of $E\left[v\left|W\right.\right]$ and $E\left[v\left|\widetilde{W}\right.\right]$. Upon rearrangement we obtain:

$$E\left[v\left|W\right.\right] = \left\{E\left[v\right] - \Pr\left[\widetilde{W}\right] E\left[v\left|\widetilde{W}\right.\right]\right\} / \Pr\left[W\right]$$

- Denote by $\left\{t_s\right\}_{s=1}^{\rho}$ the times at which the reservation price changes, and let $t_\eta$ denote the time its final bid becomes stale, that is when the reservation price changes to $r_\eta$. Denoting by $b_\eta$ its last bid, it follows that $r_{\eta-1} < b_\eta < r_\eta$.
- Since the bank would bid at its first opportunity after its bid falls OUTM if its valuation remains higher than the reservation price then:

$$E\left[v\left|\widetilde{W}\right.\right] < \left\{\sum_{s=\eta}^{\rho} \frac{G_{t_\eta}\left(t_s\right) - G_{t_\eta}\left(t_{s+1}\right)}{G_{t_\eta}\left(t_\rho\right)} \int_{r_\eta}^{r_s} \frac{vf\left(v\right)}{F\left(r_s\right) - F\left(r_\eta\right)} dv\right\}$$

- A lower bound is derived in a similar manner.

# Costs of Bidding Frictions

Table: Efficiency Measurements

| | Pre-2008 | Post-2008 |
|---|---|---|
| Lower Bound on $\mathbb{E}\left[V \middle| W\right]$ | 0.3336 | 0.2349 |
| Upper Bound on $\mathbb{E}\left[V \middle| W\right]$ | 0.3468 | 0.2533 |
| Expected Valuation of Winner, Uniform Price | 0.4989 | 0.2850 |
| % Increase in Revenue using Uniform Price | 19.58 % | 6.38% |

# Costs of Bidding Frictions

- The bounds on winners' expected valutions are tight for both the pre and post 2008 periods.
- The absence of frictions leads to significant improvement in allocative efficiency and revenue, especially prior to the financial crisis.
- Less dispersion in valuations and greater monitoring rates in the post-2008 auctions help explain the lower gains in allocative efficiency relative to pre-2008.

# Conclusion

- We study ascending price auctions for financial products in local markets and provide evidence of bidding frictions.
- We build a model of bidding in ascending auctions with frictions
  - The model may have many equilibria.
- Identification of the model is accomplished through a restriction to undominated strategies
  - The distributions of private values and auction-level heterogeneity are pointwise identified (but not individual valuations).
- Estimate bank valuations before and after the 2008 financial crisis.
  - Frictions are costly relative to the alternative of a frictionless uniform-price auction.
  - Bidder valuations appear to have higher variation pre-2008.