

# Limit Order Markets

Robert A. Miller

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# Market Microstructure

## Data on limit order markets

- Empirical studies of auctions help us understand how markets work.
- However inferences from auction data are limited by the fact that a monopolist controls one side of the market.
- A limit order market (LOM) for financial securities offer an excellent laboratory analyzing trading mechanisms where there are many players on both sides of the market:
  - 1 The rules governing trading in limit order markets are transparent, and therefore easy to capture with a model (compared to labor markets and transactions in industrial organization).
  - 2 Different units of the securities are perfect substitutes and therefore comparable (in contrast to many real assets).
  - 3 The volume and value of traded securities is huge, inducing traders to perform as well as they can (unlike experimental settings).
  - 4 Reliable data can be obtained from several limit order exchanges because they form part of the contract to which parties agree on both sides (relative to say survey data or information small businesses provide to the government for taxation purposes)

# Market Microstructure

## Framing the issues

- The agenda for this lecture is based around eight questions.
- The first five questions have at least partial answers:
  - ① What are limit order markets?
  - ② How are they related to other trading mechanisms?
  - ③ Can LOM models be tested?
  - ④ Do LOM outcomes mimic competitive equilibrium?
  - ⑤ How much inefficiency do LOM outcomes generate from breaking the law of one price?
- The remaining three questions are essentially research topics:
  - ① How are LOM models related to portfolio theory in finance?
  - ② Can LOM models explain how markets form and dissolve?
  - ③ Can LOM models explain the diffusion of information?

# 1 What are Limit Order Markets?

## The order book

- The trading mechanism for a given security in a generic limit order market can be described by:
  - 1 the order book
  - 2 the rules and procedures for submitting and withdrawing orders.
- At any given instant during business hours, there is:
  - 1 a list of unfilled orders to buy the security
  - 2 another list of unfilled orders to sell the security
- Each limit order on each list consists of:
  - 1 a price
  - 2 a quantity
  - 3 a submission time
- Every order on the sell list is marked with a higher price than every order on the buy list.
- The difference between the lowest unfilled sell order (the ask) and the highest unfilled buy order (the bid) is called the spread.

# 1 What are Limit Order Markets?

## Orders

- An investor seeking to trade the security in this market can:
  - ① add to one of the lists by placing a buy (sell) order, which is lower than the offer (higher than the bid). This is called making a limit buy (sell) order.
  - ② execute a trade by accepting the ask (bid) on the other side of the market. This is called a market buy (sell) order.
- If two unfilled orders have the same price, then the order submitted earlier is executed first.
- Investors wishing to execute only a proportion of another investor's unfilled limit order with their own market order may do so.
- Investors wishing to withdraw their limit orders may do so at any time before a market order cancels them with a transaction.
- Summarizing limit order markets exhibit price/time precedence.

# 1 What are Limit Order Markets?

Trading window

Limit order book

Price	Quantity	Cumulati...	Player..	Playertype	
6000	4	0	2	Telecommunication ...	
5800	9	4	3	Telecommunication ...	
3800	1	0	0	Telecommunication ...	
200	4	1	1	Telecommunication ...	

## 2 Similarities with Other Trading Mechanisms

Differences in information and trading rules

- In a conventional limit order market paradigm, potential traders observe the book and can place limit or market orders.
- There are two important ways variations on the conventional limit order market mechanism varies. Some traders might lack:
  - ① information about the whole book when they place their orders, observing only
    - one side of the book.
    - the spread, defined as the bid (highest limit order buy price) and ask price (lowest limit order sell price).
    - transaction price history (so do not know whether they are making a limit or a market order).
  - ② permission to place both
    - limit and market orders.
    - buy and sell orders

## 2 Similarities with Other Trading Mechanisms

Financial trading mechanisms similar to limit order markets are pervasive

- A limit order market is a real world institution for characterizing many of the exchanges in the financial sector.
- Electronic limit order markets are amongst the fastest growing markets within the financial and retail sectors.
- Whether market makers set the spread (NASDAQ), specialists oversee transactions between investors (NYSE), or the market admits anyone in good standing to submit buy and sell orders (EBAY), these exchanges have a common structure.
- On the traditional NYSE exchange:
  - ① both investors and dealers make market and limit orders;
  - ② the only restriction on the dealer is that he must process the orders (including his own) in the order in which they are received.
- On the NASDEC and HEX exchanges investors contact brokers who place limit and market orders.



## 2 Similarities with Other Trading Mechanisms

Auctions, retail markets and bargaining models

- Several types of auctions are nothing but simple limit order markets:
  - ① First price sealed bid (or procurement) auction. Bidders simultaneously make limit orders, the auctioneer a market order.
  - ② Discriminatory sealed bid auction: Bidders do not see the book, place multiple limit orders and auctioneer places market orders.
  - ③ Descending (Dutch) auction. Auctioneer makes sequential limit sell orders, until one bidder places a market buy.
  - ④ English, Japanese or button auctions. Bidders sequentially make limit buy orders, the auctioneer a market sell order.
- In a retail market stores place limit orders and customers place market orders.
- In a sequential bargaining mechanism traders alternate with by placing limit orders until one accepts a limit order of the other by a market order.
- Thus limit order markets provide a useful paradigm for analyzing market microstructure.

# 3 Can LOM Models be Tested?

## A simple model of limit order markets

- Can the underlying model be rejected with data? Suppose:

- 1 The valuation of investor  $i$  for a unit of a security is:

$$v_{it} = u_i + y_t$$

where  $u_i$  is independent across investors, and  $y_t$  is common to all and simultaneously observed.

- 2 On a discrete price grid  $\{\dots, p_{j-1}, p_j, p_{j+1}, \dots\}$  investors trade off more favorable prices against lower transaction probabilities that limit orders give when choosing the type buy or sell order to place.
- 3 In addition limit orders are exposed to picking off risk: they are more likely to be transacted when the common shock moves against them.
- 4 Higher (lower)  $u_i$  lead to higher (lower) limit orders to buy (sell)
- 5 If  $u_i$  is high (low) enough, the trader submits a market buy (sell) order.

### 3 Can LOM Models be Tested?

Inequalities of identified threshold valuations in equilibrium

- Hollifield, Miller and Sandas (2004) test a monotonicity condition these models should satisfy. They show:
  - threshold valuations,  $\theta_{kt} < \theta_{k+1,t}$  exist such that investors with valuation  $\theta_{kt}$  at  $t$  are indifferent between submitting an order at  $p_k$  versus an order at  $p_{k+1}$  where:

$$\begin{aligned} & \int [\theta_{kt} - p_k + y_\tau] f(y_\tau | y_t) g[\tau | p_k, y_t, b_t] dy_\tau d\tau \\ = & \int [\theta_{kt} - p_{k+1} + y_\tau] f(y_\tau | y_t) g[\tau | p_{k+1}, y_t, b_t] dy_\tau d\tau \end{aligned}$$

- $f(y_\tau | y_t)$  is the density of  $y_\tau$  conditional on  $y_t$
- $g[\tau | p_k, y_t, b_t]$  is the (incomplete) density of transacting at  $\tau$  conditional on  $(p_k, y_t, b_t)$
- If  $(y_t, b_t)$  is observed then  $f(y_\tau | y_t)$  and  $g[\tau | p_k, y_t, b_t]$  are identified so the inequalities  $\theta_{kt} < \theta_{k+1,t}$  can be tested.
- HMS use data on Ericsson taken from the Stockholm Automated Exchange system in 1991- 92.

# 3 Can LOM Models be Tested?

## Monotonicity tests

Threshold valuation difference	Constant	Order quantity	Ask depth	Instruments		Lagged volume	Index volatility	Time of day	Joint $M_{PC}$ statistic
				Bid depth					
Buy threshold valuations									
$\theta^{\text{buy}}(0, 1, X_T)$	2.15	14.82	4.66	5.11	5.38	1.42	25.76	0.00	
$-\theta^{\text{buy}}(1, 2, X_T)$	(0.15)	(1.03)	(0.37)	(0.40)	(0.40)	(0.12)	(1.84)		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	
$\theta^{\text{buy}}(1, 2, X_T)$	1.21	8.34	2.73	3.02	2.93	0.78	14.58	0.00	
$-\theta^{\text{buy}}(2, 3, X_T)$	(0.14)	(0.93)	(0.33)	(0.35)	(0.38)	(0.12)	(1.68)		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	
Sell threshold valuations									
$\theta^{\text{sell}}(1, 2, X_T)$	2.02	13.89	4.58	5.06	5.02	1.32	24.28	0.00	
$-\theta^{\text{sell}}(0, 1, X_T)$	(0.16)	(1.20)	(0.38)	(0.44)	(0.44)	(0.14)	(1.83)		
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	
$\theta^{\text{sell}}(2, 3, X_T)$	0.24	1.61	0.47	0.54	0.57	0.16	2.82	0.00	
$-\theta^{\text{sell}}(1, 2, X_T)$	(0.49)	(3.61)	(1.14)	(1.29)	(1.44)	(0.41)	(5.48)		
	0.69	0.67	0.66	0.66	0.65	0.65	0.70	0.99	
Buy and sell threshold valuations									
$\theta^{\text{buy}}(2, 3, X_T)$	-1.43	-9.84	-3.26	-3.62	-3.47	-0.92	-17.27	13.16	
$-\theta^{\text{sell}}(2, 3, X_T)$	(0.42)	(3.01)	(1.00)	(1.11)	(1.25)	(0.38)	(4.77)		
	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	
Joint $M_{D\theta}$ statistic									
Buy thresholds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.76	0.74	0.75	0.75	0.76	0.75	1.00	
Sell thresholds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.75	0.75	0.75	0.75	0.75	0.75	1.00	
Buy and sell thresholds	70.35	76.33	80.97	76.43	50.46	32.90	79.49	99.31	
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

### 3 Can LOM Models be Tested?

What parts of the model are rejected?

- Rejections only occur for investors who are almost indifferent between placing a high limit order sell versus a low limit order buy.
- According to our parameter estimates:
  - ① investors placing high sell limit orders should be placing low buy limit orders instead
  - ② investors placing low buy limit orders should be placing high limit sell orders instead
- If the model included expected value maximizing speculators, with these threshold parameter estimates, they would place low buy limit orders and high sell limit orders.
- Moreover if many stocks have this feature it becomes a near arbitrage opportunity (through diversification).
- Conversely short sale restriction might mitigate opportunities to speculate in this fashion.

# 4 Do LOM Outcomes mimic Competitive Equilibrium?

## Liquid limit order markets

- Can competitive equilibrium be adapted to limit order markets?
- Suppose there are many fully informed potential buyers and sellers and no one believes he or she can influence transaction prices. Then:
  - 1 all trades transact at the same price.
  - 2 the transaction price switches between being the bid and the ask.
  - 3 the spread might be infinite. Since no trading occurs at other prices placing buy (sell) limit orders behind an existing one, or placing an order that reduces the spread from infinity, ensures it will not be transacted.
  - 4 the usual welfare theorems "probably almost apply". (Imagine modeling a perfect Bayesian equilibrium in a Markov game where the probability of arrivals on the one side of the market increase with the queue on the other.)
- For future reference, we say this market is perfectly liquid.
- Because every (buy and sell) limit order is transacted at one price, no trading occurs at any other price, there is no reason for a traders to believe s/he can individually affect her/his own transaction price.

## 4 Do LOM Outcomes mimic Competitive Equilibrium?

Diffusing information through limit order markets exhibiting price taking behavior

- Now suppose that an announcement fully and simultaneously informs everybody about a financial event, and that after some time elapse this new information is fully absorbed into buy and sell orders. Say the security will trade at a higher price than before.
- In a fully informed price taking market:
  - If the new information raises the value of the asset to enough investors:
    - 1 every limit sell order lower than the new ask will be withdrawn immediately
    - 2 transaction prices after the announcement will immediately reflect the new information, jumping once and stabilizing instantaneously.
  - Similarly if new information lowers the value of the asset to enough investors:
    - 1 every limit buy order higher than the new bid will be withdrawn immediately.
    - 2 transaction prices after the announcement will immediately reflect the new information, dropping once and instantaneously stabilizing.

## 4 Do LOM Outcomes mimic Competitive Equilibrium?

Trading off immediacy of transaction time against price

- Now suppose that orders on both sides of the market arrive infrequently.
- In this situation Walras law could be adapted to say that the rate at which buy orders that are transacted must equal with the rate at which sell orders that transact.
- If individuals don't care when their orders transact, then the law of one price could be applied here too.
- But if enough of them care how quickly their order is filled, then they might be willing to pay a premium to transact earlier.
- This breaks down the law of one price (with deviations occurring from conjectured equilibrium strategies).
- In fact because investors now have preferences over "immediacy", it is useful to interpret a limit order market for a single security as a mechanism for trading multiple products differentiated by their placement/transaction/withdrawal time window.



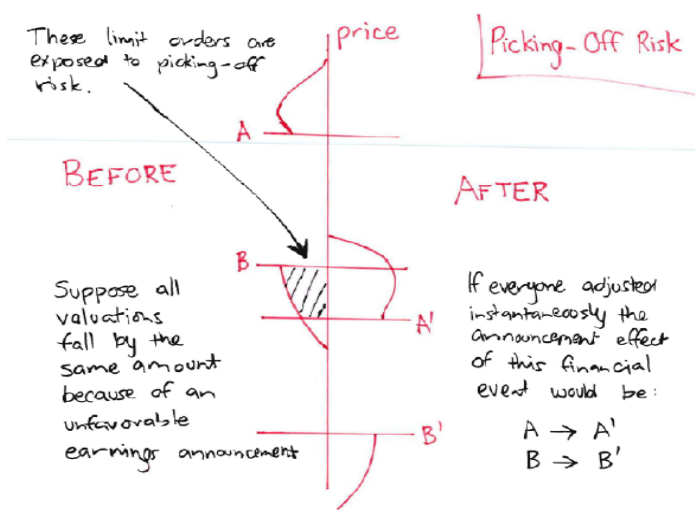
## 4 Do LOM Outcomes mimic Competitive Equilibrium?

Trading off monitoring financial events against price

- Alternatively investors do not value transaction immediacy, but have differential access to financial events.
- When new information arrives that raises (lowers) the value of a security to everybody, new market buy (sell) orders quickly snap up the most attractive limit order sell (buy) orders if they are not withdrawn or reset quickly enough.
- Investors with slower reaction times are disadvantaged, because they are more likely to transact when the underlying value of the security moves against them.
- When informationally disadvantaged investors place limit orders, they incur picking off risk, selling (buying) shortly after the common value rises (falls), before they can adjust their orders to the new market conditions.
- It creates an incentive for them to break the law of one price by offering a more attractive price to reduce their exposure to picking off risk through a faster expected transaction time:

# 4 Do LOM Outcomes mimic Competitive Equilibrium?

Picking off risk and insider trading illustrated



# 5 Inefficiency from Breaking the Law of One Price

## Four types of inefficiency in limit order markets

- Illiquid limit order markets for financial securities might not realize all the potential gains from trade for four reasons:
  - ① Limit orders are not executed when they should be (that is to maximize the total gains from trade).
  - ② Traders do not submit orders when they should (deterred by the order submission cost, the low probability of execution and the picking-off risk).
  - ③ Traders submit wrong sided" orders, such as a buy order rather than a sell order (to profit from stale limit sells after value of stock has risen).
  - ④ Traders submit orders when they should not (because of order submission costs) .

## 5 Inefficiency from Breaking the Law of One Price

What kind of exchange might be a good candidate for estimating these costs?

- The Vancouver Stock Exchange (VSE) was incorporated 1906, merged into the Canadian Venture Exchange (CDNX) in 1999, which was subsequently absorbed into the Toronto Stock Exchange (TSE).
- When it became fully automated by 1990, the VSE listed 2,300 stocks, more than two-thirds in the gold, silver, oil and gas businesses.
- Annual trading increased from roughly C\$4 billion in 1991 to \$6.7 billion in 1993.
- Yet the VSE had an unsavory reputation reminiscent of the wild west:
  - In 1989, Forbes magazine christened it "scam capital of the world".
  - A 1994 report by James Matkin (of the Vancouver Stock Exchange & Securities Regulation Commission) referred to "shams, swindles and market manipulations" within the VSE.
  - The summary judgement of Investopedia.com is that "the VSE is an example of one of the world's less successful stock exchanges."

# 5 Inefficiency from Breaking the Law of One Price

Structural estimates from Hollifield, Miller, Sandas and Slive (2006)

	BHO	ERR	WEM
	Gains		
	Maximum gains as a % of the common value		
	9.07	8.61	6.75
	Current gains as a % of the common value		
Lower bound	7.88	8.09	6.08
Upper bound	8.45	8.31	6.40
Average	8.16	8.20	6.24
	Maximum gains minus current gains		
Lower bound	0.62	0.30	0.35
Upper bound	1.20	0.52	0.67
Average	0.91	0.41	0.51
	Current gains as a % of maximum gains		
Lower bound	86.79	93.97	90.07
Upper bound	93.13	96.57	94.81
Average	89.96	95.27	92.44
	Decomposition of Losses		
	No execution as a % of total losses		
Sell side	32.32	31.20	33.05
Buy side	40.10	39.01	41.85
Subtotal	72.42	70.21	74.90
	No submission as a % of total losses		
Sell side	2.24	0.62	0.41
Buy side	1.98	0.15	0.71
Subtotal	4.22	0.77	1.12
	Wrong direction as a % of total losses		
Sell side	0.86	0.02	0.39
Buy side	0.20	0.05	0.63
Subtotal	1.06	0.07	1.02
	Extramarginal submissions as a % of total losses		
Sell side	9.81	11.87	10.30
Buy side	12.49	17.07	12.66
Subtotal	22.30	28.94	22.96
Total	100.00	100.00	100.00
	Monopoly Gains		
	Monopoly gains as a % of the common value		
	5.02	5.57	4.18
	Monopoly gains as a % of maximum gains		
	55.31	64.71	61.87
	Current gains as a % of monopoly gains		
	162.65	147.23	149.41

# 5 Inefficiency from Breaking the Law of One Price

## Comparing the two assessments

- The HMSS high frequency time series data based estimates paint a glowing picture of capitalism at work in the VSE, in stark contrast to the historical narrative.
- Indeed the HMSS estimates probably *understate* the efficiency of limit order markets because in computing the maximal gains from trade, we did not account for coordination problems between investors arriving at the market at different times.
- Thus the (uncorrected) potential gains from trade are *overstated*.
- One puzzling feature of the historical narrative is that it does not explain why trading volume on the VSE grew substantially:
  - ① If the VSE was so unsuccessful, surely disgruntled exploited investors would shift their activities to other exchanges.
  - ② The amalgamation into the TSE occurred at roughly the same time several European exchanges were merging, quite possibly driven by the electronic exchange technology. It is hard to argue this was evidence of an unsuccessful exchange.

# Disseminating Information in a LOM

## Three core research issues

- A tenet of Hayek (1944) is that decentralized trading facilitates the diffusion of new information about value, and consequently resources are allocated more efficiently by markets than centralized mechanism.
- ① How are portfolio choices made in limit order markets?
  - How is portfolio choice theory connected to LOM models?
  - How and when does dynamic rebalancing occur in a LOM?
- ② How do limit order markets form and dissolve?
  - When do limit order markets function better than the alternative?
  - What does the alternative trading mechanism look like?
  - Is the transition between one trading form and another efficient?
- ③ Do limit order markets disseminate new information efficiently?
  - Are well informed investors exploiting poorly informed investors by subjecting them to picking off risk?
  - Do high frequency traders deter long range investors from acquiring costly information that would spur technological progress?

## 6 How is Portfolio Analysis Connected to LOM Models?

Adapting the Euler equation approach to empirical models of a LOM

- Buying and selling securities are central to both market microstructure and portfolio choice theories.
- Yet the "how" (of market microstructure) and "why" (of portfolio theory) are not connected (Parlour and Seppi, 2008).
- In HMS and HMSS traders enter at exogenous random times, place a unit order, and limit orders expire at an exogenous rate.
- Dynamic issues, such as timing order placement, withdrawals and replacement, key choices in portfolio analysis, are ignored.
- Typically researchers do not observe the originator of an order, so order replacement is unobserved.
- Finally  $y_t$  is identified off observed factors:
  - HMS assume  $y_t \equiv \beta X_t$  is  $I(1)$ , where  $X_t$  is the value weighted index of the 30 most traded stocks at  $t$  on the exchange.
  - HMSS proxy  $y_t$  with a centered moving average of mid-quotes over a 20-minute window.
- Thus  $y_t$  is endogenous to the LOM model.



# 7 How do Limit Order Markets Form and Dissolve?

## The HEX exchange

- The Helsinki Stock Exchange formed in 1912, became electronic in 1990, renamed HEX with Finnish acquisitions in 1997, merged with Stockholm exchange in 1993, now called Nasdaq Stockholm AB.
- The limit order data are from the HEX from September 18, 1998 to October 23, 2001. Each entry in this data set is a single order entered into the trading system.
- Each entry contains a unique order ID, including: entry date and time-stamp, limit price, quantity, the brokerage room identity, and a set of codes to track the life of the order.
- An order can expire, be partly or completely filled, withdrawn, or have its price amended.
- From these data we can reconstruct the limit order book for each second of every trading day for all the stocks.

# 7 How do Limit Order Markets Form and Dissolve?

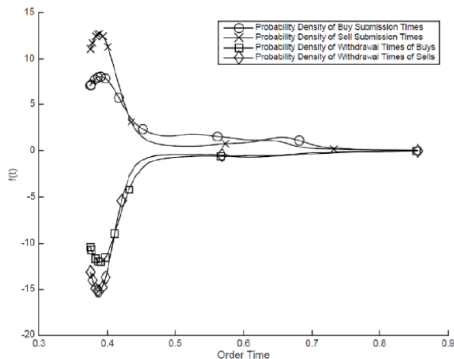
Summary statistics from the exchange

	Mean	Std	Min	Max
Number of Daily Shares Traded	126502.82	1281292.44	0.00	105639789.0
Number of Daily Buy Orders	64.30	282.30	0.00	13088.00
Number of Daily Sell Orders	67.16	259.92	0.00	11379.00
Fraction of Buy Orders Executed	0.33	0.34	0.00	1.00
Fraction of Sell Orders Executed	0.29	0.31	0.00	1.00
Number of Shares Traded in Opening Session	246.24	2423.02	0.00	229200.00
Number of Buy Orders in Opening Session	14.43	27.79	0.00	685.00
Number of Sell Orders in Opening Session	9.55	20.30	0.00	609.00
Total Number of Equities	329			
Total Number of Brokers	41			

Observations averaged across  
all trading days and equities

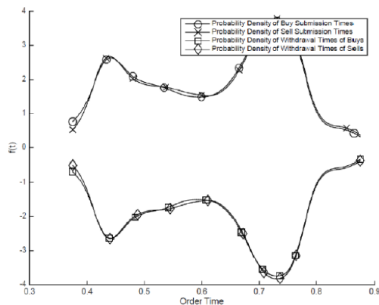
# 7 How do Limit Order Markets Form and Dissolve?

Trading intensity of lightly traded stocks



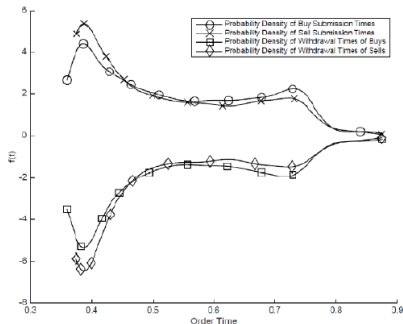
# 7 How do Limit Order Markets Form and Dissolve?

Trading intensity of medium volume stocks



# 7 How do Limit Order Markets Form and Dissolve?

Trading intensity of heavily traded stocks



# 7 How do Limit Order Markets Form and Dissolve?

What can we learn?

- There are several reasons why more trading takes place at the beginning of the day than within trading session:
  - ① For example new information received overnight greatly increasing the value of one stock might arrive leading to a stockholders to rebalance at the beginning of the next day.
  - ② Anticipating this focal point of increased trading volume, where there is likely to be more trade, other investors:
    - discount picking off risk when trading in that time window.
    - reduce liquidity in other time windows.
- The less liquid the market, the more important these two factors. In extreme cases a limit order market might degenerate to a double auction at the beginning of the day, where orders are simultaneously processed, with little activity after an initial spurt.

# 8 The Acquisition and Diffusion of New Information

## Contrasting market clearing mechanisms with LOM outcomes

- Modeling an LOM can be used to analyze the acquisition and dissemination of information because it:
  - typically supports a perfect equilibrium with differentially informed players;
  - is institutionally realistic in securities markets;
  - is a powerful paradigm for many other trading arrangements.
- For example in both HMS and HMSS investors arriving after a change in  $y_t$  are informed relative to those who arrived just before. Sandas (2001) estimates a static structural model of differential information (although the model doesn't follow LOM trading protocols).
- In contrast rational expectations competitive equilibrium models are not ideal vehicles for analyzing differential information:
  - Radner (1979) provides general conditions where with where there is no incentive to gather information;
  - Grossman and Stiglitz (1980) develop a fairly contrived model to exhibit the value of investing in information.

# 8 The Acquisition and Diffusion of New Information

## Valuing a firm traded on a limit order market

- In a competitive equilibrium the value of a firm is the product of the number of shares and the price of a share. According to the efficient markets hypothesis price follows a random walk so empirically price might be measured as the most recent transaction price (the best offer or the highest bid). Denote this value by  $V_c$ .
- What is the value of firm trading on a limit order market?
  - 1 To buy the firm we might integrate "up" the limit order sell prices, to obtain say  $V_b$ .
  - 2 To sell the firm we might integrate "down" the limit order buy prices, to obtain say  $V_s$ .
- Clearly  $V_s \leq V_c \leq V_b$ , and unless the market is perfectly liquid the inequalities are strict.



# 8 The Acquisition and Diffusion of New Information

## Valuing a firm traded on a limit order market

- This also raises the question of whether controlling interests in a firm are sold in a limit order market:
  - The short answer is "no". Large block trades can preserve the ownership structure of a firm.
  - They facilitate trade in ownership while preserving the equilibrium balance between portfolio diversification and better corporate governance due to more concentrated ownership.
- But how does a group of outside investors gain from investing in information in order to benefit from taking over the firm and reorganizing it?
  - In a perfectly liquid market where information is instantly diffused throughout the population, there is no incentive for outside investors to privately inform themselves.
  - More generally there is a trade-off between:
    - 1 how quickly information is disseminated through the market;
    - 2 the amount of investment undertaken by potential acquirers.