Limit Order Markets

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Structural Econometrics

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What are Limit Order Markets?

Market microstructure

- Competitive equilibrium is a useful modeling tool to parsimoniously capture the fundamental reason why markets exist:
  - so that individuals can benefit from trading with one another.
- It is impossible, however, to design a noncooperative game that implements the competitive equilibrium allocation.
- Can we replace the fiction of a Walrasian auctioneer setting prices with models based on:
  - institutions, or trading rules, designed to facilitate trade
  - where behavior can be modeled as a noncooperative game.
- Focusing on one such institution, three questions frame this lecture:

1. What is a limit order market (LOM)?
2. Do LOM models have empirical content?
   - Can LOM models be tested (falsified)?
   - Note empirical content does not imply identification.
3. How efficient are LOMs in allocating resources?

What is an LOM?

The order book

- The trading mechanism for a given security in a generic limit order market can be described by:
  1. the order book
  2. the rules and procedures for submitting and withdrawing orders.
- At any given instant during business hours, there is:
  1. a list of unfilled orders to buy the security
  2. another list of unfilled orders to sell the security
- Each limit order on each list consists of:
  1. a price
  2. a quantity
  3. a submission time
- Every order on the sell list is marked with a higher price than every order on the buy list.
- The difference between the lowest unfilled sell order (the ask) and the highest unfilled buy order (the bid) is called the spread.

What is an LOM?

Orders

- An investor seeking to trade the security in this market can:
  1. add to one of the lists by placing a buy (sell) order, which is lower than the offer (higher than the bid). This is called making a limit buy (sell) order.
  2. execute a trade by accepting the ask (bid) on the other side of the market. This is called a market buy (sell) order.
- If two unfilled orders have the same price, then the order submitted earlier is executed first.
- Investors wishing to execute only a proportion of another investor's unfilled limit order with their own market order may do so.
- Investors wishing to withdraw their limit orders may so at any before a market order cancels them with a transaction.
- Summarizing limit order markets exhibit price/time precedence.

What is an LOM?

Trading window

Limit order book

Price	Quantity	Cumulati...	Player...	Player type	
6000	4	0	2	Telecommunication ...	
5800	9	4	3	Telecommunication ...	
3800	1	0	0	Telecommunication ...	
200	4	1	1	Telecommunication ...	

What is an LOM?

Data on limit order markets

- A limit order market (LOM) for financial securities offer an excellent laboratory analyzing trading mechanisms where there are many players on both sides of the market:
  1. The rules governing trading in limit order markets are transparent, and therefore easy to capture with a model (compared to labor markets and transactions in industrial organization).
  2. Different units of the securities are perfect substitutes and therefore comparable (in contrast to many real assets).
  3. The volume and value of traded securities is huge, inducing traders to perform as well as they can (unlike experimental settings).
  4. Reliable data can be obtained from several limit order exchanges because they form part of the contract to which parties agree on both sides (relative to say survey data or information small businesses provide to the government for taxation purposes).
     - one deficiency: very rarely can researchers observe the traders' identity to track their orders or observe their wealth portfolio.

- At time $t \in \{1, 2, \ldots\}$ just one trader has his only opportunity to submit an order for one (or more generally exogenously determined) unit(s) of an asset.
- Trader t is risk neutral and values the unit at:

$$v_t = u_t + y_t$$

  where:
  - u_t is independent and identically distributed with support on the real line and probability distribution function $G(u)$
  - y_t is a Martingale, meaning $E_t[y_{t+1}] = y_t$.
- We interpret y_t as the expected liquidation value of an asset that pays no dividends in the meantime.
- Trader t observes both components.

An LOM Model

Prices

- Traders buy and sell on a discrete price grid $\{\ldots, p_{j-1}, p_j, p_{j+1}, \ldots\}$.
- The difference $p_{j+1} - p_j$ is called the tick size.
- Denote by $\left\{p_{0t}^{(b)}, p_{1t}^{(b)} \ldots\right\}$ the buy prices trader t can choose from:
  - $p_{0t}^{(b)}$ is the lowest limit order sell offer (the ask price)
  - Trader t submits a market buy order by selecting $p_{0t}^{(b)}$
  - $p_{kt}^{(b)}$ is k ticks below $p_{0t}^{(b)}$.
  - Trader t submits a limit buy order by selecting $p \in \left\{p_{1t}^{(b)}, p_{2t}^{(b)} \ldots\right\}$.
- Similarly $\left\{p_{0t}^{(s)}, p_{1t}^{(s)} \ldots\right\}$ are sell prices, and trader t can submit a:
  - market sell order by selecting the (highest limit order) bid price $p_{0t}^{(s)}$
  - limit sell order $p_{kt}^{(s)}$ that is k ticks above $p_{0t}^{(s)}$.
- The difference $p_{0t}^{(b)} - p_{0t}^{(s)}$ is called the spread (ask price less bid price).
- Prices on and inside the spread can be selected by a buyer or seller.

- Let $d_{kt}^{(b)} \in \{0,1\}$ and $d_{kt}^{(s)} \in \{0,1\}$ for $k \in \{0,1,2,\ldots\}$, where $d_{kt}^{(b)} = 1$ means t submits a buy order at price $p_{kt}^{(b)}$.
- Assume t submits at most one order, implying:

$$\sum_{k=0}^{\infty}\left(d_{kt}^{(b)} + d_{kt}^{(s)}\right) \leq 1$$

- Trader t chooses $d_t \equiv \left(d_{0t}^{(b)}, d_{0t}^{(s)}, d_{1t}^{(b)}, d_{1t}^{(s)}, \ldots\right)$ to maximize:

$$E_t\left\{\sum_{k=0}^{\infty}\left[\left(d_{kt}^{(b)} - d_{kt}^{(s)}\right)v_t - \left(d_{kt}^{(b)}p_{kt}^{(b)} - d_{kt}^{(s)}p_{kt}^{(s)}\right)\right]\right\}$$

- But what could happen after the order is placed, and how are expectations formed?

- Suppose $d_{kt}^{(s)} = 1$ and let $r_{k,t,t+\tau}^{(s)} \in \{0,1\}$ indicate whether the order is cancelled at $t+\tau$ (by setting $r_{k,t,t+\tau}^{(s)} = 1$) or not ($r_{k,t,t+\tau}^{(s)} = 0$).
- Assume $r_{k,t,t+\tau}^{(s)}$ and similarly defined $r_{k,t,t+\tau}^{(b)}$ are independent exogenous processes across t.
- Let $q_{k,t,t+\tau}^{(s)} \in \{0,1\}$ indicate $d_{kt}^{(s)} = 1$ is executed or filled at $t+\tau$ (by setting $q_{k,t,t+\tau}^{(s)} = 1$) or not ($q_{k,t,t+\tau}^{(s)} = 0$).
- Execution is endogenous (because another trader is involved):
  - $q_{0,t,t}^{(s)} = 1$ (because market orders execute immediately).
  - if $q_{k,t,t+\tau}^{(s)} = 1$ then $d_{0,t+\tau}^{(b)} = 1$. (Every execution crosses a limit order with a market order.)
  - if $q_{k,t,t+\tau}^{(s)} = 1$ and $d_{k't'}^{(s)} = 1$ for some $k' < k$ and $t' < t+\tau$ then $q_{k',t',\rho}^{(s)} = 1$ or $r_{k',t',\rho}^{(s)} = 1$ for some $\rho \leq t+\tau$ (reflecting price precedence).

- This is a perfect information game:
  - Each trader t observes the value of y_t and all the outstanding limit orders (comprising the limit order book)
  - Traders move sequentially each trader is fully informed about the moves of previous agents
- If the game has a finite horizon, meaning $t \in \{1, 2, \ldots, T\}$ then it is straightforward to establish that:
  - an unique equilibrium exists
- Let $\widehat{d}_{kt}^{(b)}$and $\widehat{d}_{kt}^{(s)}$ denote equilibrium $d_{kt}^{(b)}$ and $d_{kt}^{(s)}$ choices.
- similarly Let $\widehat{q}_{kt}^{(b)}$and $\widehat{q}_{kt}^{(s)}$ denote equilibrium $q_{kt}^{(b)}$ and $q_{kt}^{(s)}$ executions.

Equilibrium

Conditional choice probabilities, execution probabilities and picking off risks

- To characterize the equilibrium choices, define:
  - *conditional choice probabilities of submission*:

$$\lambda_{kt}^{(b)} \equiv \int \widehat{d}_{kt}^{(b)} dG(u)$$

  - *execution probabilities*:

$$\psi_{kt}^{(b)} \equiv E_t \left[\sum_{\tau=0}^{\infty} \widehat{q}_{kt,t+\tau}^{(b)} \prod_{\rho=1}^{\tau} \left(1 - r_{kt,t+\rho}^{(b)} \right) \right]$$

  - *picking-off risk*:

$$\xi_{kt}^{(b)} \equiv E_t \left[\sum_{k=0}^{\infty} (y_{t+\tau} - y_t) \, \widehat{q}_{kt,t+\tau}^{(s)} \right]$$

- Thus trader t chooses d_t to maximize:

$$\sum_{k=0}^{\infty} \left[d_{kt}^{(b)} \left(\psi_{kt}^{(b)} u_t + \xi_{kt}^{(b)} - p_{kt}^{(b)} \right) + d_{kt}^{(s)} \left(p_{kt}^{(s)} - \psi_{kt}^{(s)} u_t - \xi_{kt}^{(s)} \right) \right]$$

An LOM Model

A revealed preference argument (Lemma 1, HMS 2004)

- Suppose $d_{kt}^{(b)}(u) = 1$ and $d_{k't}^{(b)}(u') = 1$ Then:

$$\psi_{kt}^{(b)}\left(u + y_t - p_{kt}^{(b)}\right) + \xi_{kt}^{(b)} - c \geq \psi_{k't}^{(b)}\left(u + y_t - p_{k't}^{(b)}\right) + \xi_{k't}^{(b)} - c$$

$$\psi_{k't}^{(b)}\left(u' + y_t - p_{k't}^{(b)}\right) + \xi_{k't}^{(b)} - c \geq \psi_{kt}^{(b)}\left(u' + y_t - p_{kt}^{(b)}\right) + \xi_{kt}^{(b)} - c$$

- Add the inequalities together; then add to both sides:

$$\psi_{kt}^{(b)} p_{kt}^{(b)} + \psi_{k't}^{(b)} p_{k't}^{(b)} + \left[\psi_{kt}^{(b)} + \psi_{k't}^{(b)}\right] y_t + 2c - \xi_{kt}^{(b)} - \xi_{k't}^{(b)}$$

- Rearrange the resulting inequality to yield:

$$\left[\psi_{kt}^{(b)} - \psi_{k't}^{(b)}\right](u - u') \geq 0$$

- For example if $u \geq u'$ then $\psi_{kt}^{(b)} \geq \psi_{k't}^{(b)}$.
- Since $\psi_{kt}^{(b)}$ is decreasing in k (the number of ticks below the best sell offer), $p_{k't}^{(b)} \leq p_{kt}^{(b)}$.
- An analogous result holds for the sell side.

- Empirical content of LOM models can be derived from a monotonicity property of *threshold valuations*.
- Define $\theta_t^{(b)}(k, k')$ as the valuation of a trader indifferent between submitting $p_{kt}^{(b)}$ versus $p_{k't}^{(b)}$:

$$
\begin{aligned}
& \psi_{kt}^{(b)}\left(\theta_t^{(b)}(k,k') + y_t - p_{kt}^{(b)}\right) + \xi_{kt}^{(b)} - c \\
= \;& \psi_{k't}^{(b)}\left(\theta_t^{(b)}(k,k') + y_t - p_{k't}^{(b)}\right) + \xi_{k't}^{(b)} - c \\
\Rightarrow \theta_t^{(b)}(k,k') = \;& p_{kt}^{(b)} + \frac{\left[p_{kt}^{(b)} - p_{k't}^{(b)}\right]\psi_{k't}^{(b)} + \xi_{k't}^{(b)} - \xi_{kt}^{(b)}}{\psi_{k't}^{(b)} - \psi_{kt}^{(b)}}
\end{aligned}
$$

- Similar expressions can be defined for traders indifferent between:
  - selling at two different prices
  - buying a unit versus selling a unit (at a higher price)
  - trading at some price versus not trading at all.

An LOM Model

Monotonicity of threshold valuations (Lemmas 2 and 3, HMS 2004)

- Index the set of buy orders optimal for some trader by a subscript "j" instead of "k".
- Lemma 1 implies:

$$\theta_t^{(b)}(j, j+1) > \theta_t^{(b)}(j+1, j+2)$$

- An analogous argument applies to the sell side:

$$\theta_t^{(s)}(j, j+1) < \theta_t^{(b)}(j+1, j+2)$$

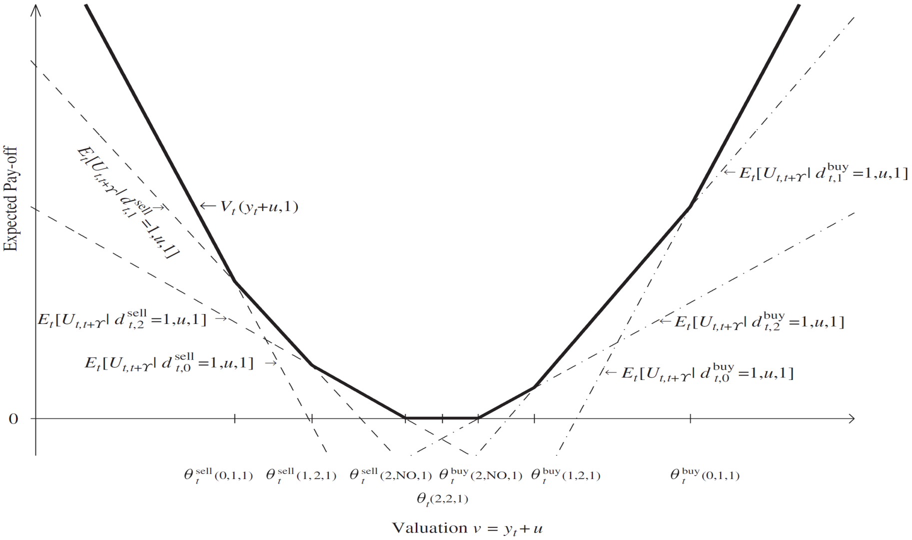
- Using similar reasoning we can show:

$$\theta_t^{(b)}(0, 1) < \theta_t^{(2)}(0, 1)$$

- These inequalities yield a characterization of the optimal submission strategy.

An LOM Model

Figure 2, HMS 2004



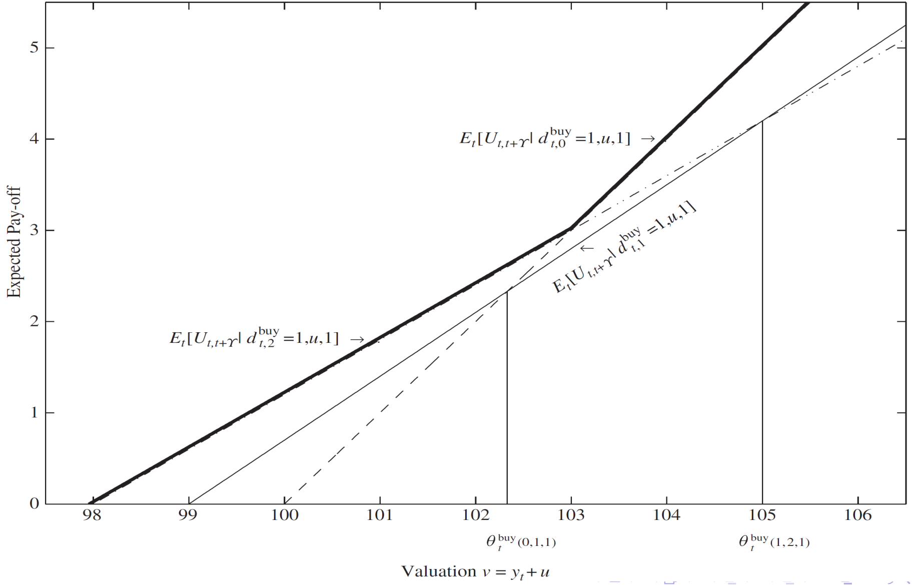
- Consider a market in which:
  - The tick size is a unit: $p_{kt}^{(b)} - p_{0t}^{(b)} = k$.
  - The market buy price is one hundred: $p_{0t}^{(b)} = 100$.
  - There is no common component: $y_t \equiv 0$.
  - Traders submit orders at $p_{0t}^{(b)}, p_{1t}^{(b)}$ and $p_{2t}^{(b)}$.
  - By definition $\psi_{0t}^{(b)} = 1$.
  - Furthermore $\psi_{1t}^{(b)} = 0.7$ and $\psi_{2t}^{(b)} = 0.6$.
- Using the formula for calculating threshold valuations:

$$\begin{aligned} \theta_t^{(b)}(0,1) &= 100 + 0.7 \big/ (1 - 0.7) = 102.33 \\ \theta_t^{(b)}(1,2) &= 99 + 0.6 \big/ (0.7 - 0.6) = 105.00 \end{aligned}$$

- Since $\theta_t^{(b)}(1,2) > \theta_t^{(b)}(0,1)$ the monotonicity condition is violated.
- In Figure 4 (next slide) the solid line for indirect utility lies strictly above the utility benefit from submitting a limit buy order at 99.

Testing an LOM model

Figure 4, Hollifield, Miller and Sandas (2004)



- Given a typical inequality implied by the model, say:

$$\theta_t^{(b)}(k, k+1) - \theta_t^{(b)}(k+1, k+2) > 0$$

for any $z_t \in \mathcal{F}_t$, where $\mathcal{F}_t$ denotes the information set of trader t:

$$E\left[\theta_t^{(b)}(k, k+1) - \theta_t^{(b)}(k+1, k+2)\,|z_t\right] > 0$$

- Therefore for any $z_t^{++} \in \mathcal{F}_t$, a strictly positive (vector of) element(s):

$$E\left\{\left[\theta_t^{(b)}(k, k+1) - \theta_t^{(b)}(k+1, k+2)\right] z_t^{++}\right\} > 0$$

- The test statistic is based on:

$$T^{-1}\sum_{t=1}^{T}\left\{\left[\widetilde{\theta}_t^{(b)}(k, k+1) - \widetilde{\theta}_t^{(b)}(k+1, k+2) - LB\right] z_t^{++}\right\}$$

where:

  - $\widetilde{\theta}_t^{(b)}(k, k+1)$ is a consistent estimator for $\theta_t^{(b)}(k, k+1)$
  - and $0 < LB \leq \theta_t^{(b)}(k, k+1) - \theta_t^{(b)}(k+1, k+2)$ for all (t, k, z_t).

Testing an LOM model

Overview of test procedure

- To implement the test we must first identify the
  1. find subsequences of conditional choice submission probabilities $\left\{\lambda_{jt}^{(b)}\right\}_j$ and $\left\{\lambda_{jt}^{(s)}\right\}_j$ that are strictly positive
  2. estimate the execution probabilities $\psi_{kt}^{(b)}$ and $\psi_{kt}^{(s)}$ for the elements in the subsequence
  3. estimate the y_t process
  4. estimate the picking-off risk $\xi_{kt}^{(b)}$ and $\xi_{kt}^{(s)}$
  5. form the threshold values $\theta_t^{(b)}(k, k')$ and $\theta_t^{(s)}(k, k')$.
  6. test the inequalities that apply to $\theta_t^{(b)}(k, k')$ and $\theta_t^{(s)}(k, k')$.
- Strictly speaking the null hypothesis is a joint hypothesis combining all six steps.
- Therefore the *size* of the test (the probability of being in the tail of a test statistic) is affected by all the sources of sampling variation.

- The sample comprises data on Ericsson taken from the Stockholm Automated Exchange system in 1991- 92.
- We focus on $\left\{p_{0t}^{(b)}, p_{1t}^{(b)}, p_{2t}^{(b)}, p_{3t}^{(b)}\right\}$ and $\left\{p_{0t}^{(s)}, p_{1t}^{(s)}, p_{2t}^{(s)}, p_{3t}^{(b)}\right\}$.
- We conducted the tests of strictly positive submission probabilities, strictly positive differences in execution probabilities, and monotonicity in threshold valuations separately.
- Consequently the critical values for the tests aren't adjusted properly for sampling error in prior stages.
- We cannot reject the (separately tested) hypotheses that for probabilities of:

$$\text{submission } \lambda_{jt}^{(b)} > 0 \text{ and } \lambda_{kt}^{(b)} > 0 \text{ for } j \in \{0, 1, 2, 3\}$$
$$\text{execution } \psi_{jt}^{(b)} > \psi_{j+1,t}^{(b)} \text{ and } \psi_{jt}^{(s)} > \psi_{j+1,t}^{(s)} \text{ for } j \in \{0, 1, 2\}$$

- We defer to later lectures more detail about nonparametric estimation and testing inequalities.

Test Results

Monotonicity test results (Table 8, HMS,2004)

Threshold valuation difference	Constant	Order quantity	Ask depth	Bid depth	Lagged volume	Index volatility	Time of day	Joint M_{PC} statistic
				Instruments				
				Buy threshold valuations				
$\theta^{buy}(0, 1, X_t)$	2·15	14·82	4·66	5·11	5·38	1·42	25·76	0·00
$-\theta^{buy}(1, 2, X_t)$	(0·15)	(1·03)	(0·37)	(0·40)	(0·40)	(0·12)	(1·84)	
	1·00	1·00	1·00	1·00	1·00	1·00	1·00	0·99
$\theta^{buy}(1, 2, X_t)$	1·21	8·34	2·73	3·02	2·93	0·78	14·58	0·00
$-\theta^{buy}(2, 3, X_t)$	(0·14)	(0·93)	(0·33)	(0·35)	(0·38)	(0·12)	(1·68)	
	1·00	1·00	1·00	1·00	1·00	1·00	1·00	0·99
				Sell threshold valuations				
$\theta^{sell}(1, 2, X_t)$	2·02	13·89	4·58	5·06	5·02	1·32	24·28	0·00
$-\theta^{sell}(0, 1, X_t)$	(0·16)	(1·20)	(0·38)	(0·44)	(0·44)	(0·14)	(1·83)	
	1·00	1·00	1·00	1·00	1·00	1·00	1·00	0·99
$\theta^{sell}(2, 3, X_t)$	0·24	1·61	0·47	0·54	0·57	0·16	2·82	0·00
$-\theta^{sell}(1, 2, X_t)$	(0·49)	(3·61)	(1·14)	(1·29)	(1·44)	(0·41)	(5·48)	
	0·69	0·67	0·66	0·66	0·65	0·65	0·70	0·99
				Buy and sell threshold valuations				
$\theta^{buy}(2, 3, X_t)$	−1·43	−9·84	−3·26	−3·62	−3·47	−0·92	−17·27	13·16
$-\theta^{sell}(2, 3, X_t)$	(0·42)	(3·01)	(1·00)	(1·11)	(1·25)	(0·38)	(4·77)	
	0·00	0·00	0·00	0·00	0·00	0·01	0·00	0·01
				Joint $M_{D\theta}$ statistic				
Buy thresholds	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
	0·75	0·76	0·74	0·75	0·75	0·76	0·75	1·00
Sell thresholds	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
	0·75	0·75	0·75	0·75	0·75	0·75	0·75	1·00
Buy and sell thresholds	70·35	76·33	80·97	76·43	50·46	32·90	79·49	99·31
	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00

Empirical Content of LOM models

Where does the model succeed?

- For several positive valued components z_t^{++} the estimated differences:

$$E\left\{\left[\theta_t^{(b)}(j,j+1)-\theta_t^{(b)}(j+1,j+2)\right]z_t^{++}\right\}$$

  are positive and significant, as the model predicts.
- On the sell side:

$$E\left\{\left[\theta_t^{(s)}(1,2)-\theta_t^{(s)}(0,1)\right]z_t^{++}\right\}$$

  is positive and significant, but:

$$E\left\{\left[\theta_t^{(s)}(2,3)-\theta_t^{(s)}(1,2)\right]z_t^{++}\right\}$$

  is positive but not significant.
- Summarizing, the null hypothesis of monotonicity is not rejected when buy and sell thresholds are considered separately.

Test Results

Where does the model fail?

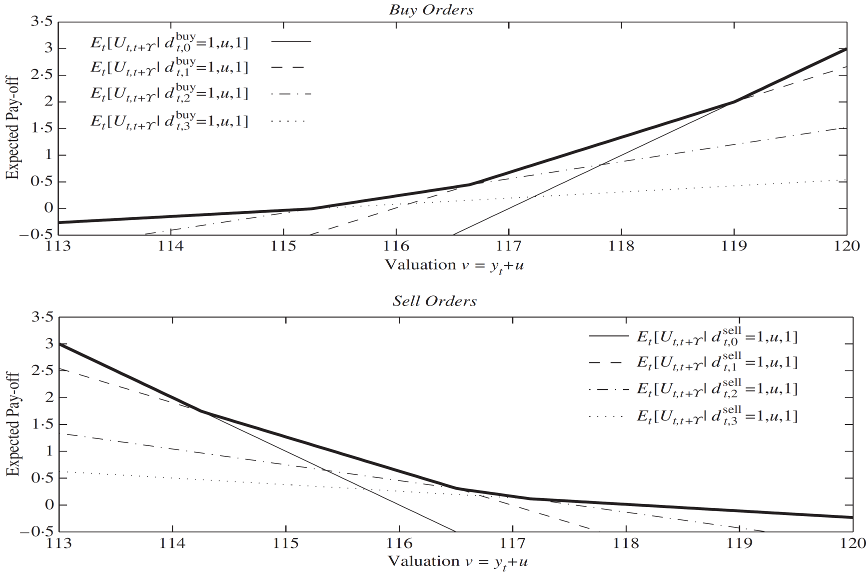
- Contrary to the predictions of the model, the sample:
  - gives negative point estimates of:

$$E\left\{\left[\theta_t^{(b)}(2,3)-\theta_t^{(s)}(2,3)\right]z_t^{++}\right\}$$

  - rejects the null hypothesis that buyer threshold valuations are higher than seller threshold valuations.
- Thus rejections only occur for investors who are almost indifferent between placing a high limit sell order versus a low limit buy order.
- According to our parameter estimates, as the next slide illustrates:
  1. investors placing high sell limit orders should be placing low buy limit orders instead
  2. investors placing low buy limit orders should be placing high limit sell orders instead

Test Results

Illustrating the model rejection (Figure 5, HMS,2004)



Estimating an LOM model

Adapting the model to continuous time

- We now assume traders arrive sequentially at rate:
  - $\Pr\{\text{Trader arrives in interval } [t, t + \Delta t] \,|x_t\} = \lambda(t; x_t)\, dt$
  - where x_t is an exogenous vector of state variables
- Following the same notation as before:
  - $d_{kt}^{(b)} \in \{0, 1\}$ and $d_{kt}^{(s)} \in \{0, 1\}$ for $k \in \{0, 1, 2, \ldots\}$
  - with the same constraint $\sum_{k=0}^{\infty} \left(d_{kt}^{(b)} + d_{kt}^{(s)} \right) \leq 1$.
- As above we assume traders are risk neutral with valuations
  - differ in their private valuation $v_t = y_t + u_t$
  - where u_t is distributed independently with $\Pr(u_t \leq u \,|x_t) \equiv G(u \,|x_t)$
- If and when he has the opportunity the trader:
  - can submit an order to trade one unit
  - pays c_0 to placing an order
  - pays a further c_e if the order executes.

Estimating an LOM model

An estimation strategy

- Note that $G\left(u\,|x\right)$ must be estimated to obtain estimates of the gains from trade $L\left(z_t\right)$.
- One strategy is to:
  1. Follow the same procedure as above to
     - determine orders with positive submission probabilities $\lambda_{jtj}^{(b)}$ and $\lambda_{jt}^{(s)}$
  2. Then estimate
     - their execution probabilities $\psi_{kt}^{(b)}$ and $\psi_{kt}^{(s)}$ (nonparametrically)
     - the y_t process
     - the picking-off risks $\xi_{kt}^{(b)}$ and $\xi_{kt}^{(s)}$
  3. Apply a competing hazards framework to jointly estimate:
     - the arrival rate of traders $\lambda\left(t; x_t\right)$
     - and $G\left(u\,|x\right)$, the distribution of their valuations.
- The details explaining how to estimate frameworks with competing risks are deferred to a future lecture.

Estimating an LOM model

Estimating the arrival of treaders and the distribution of private valuations

- Briefly, we partition each time interval $[t, t+dt)$ by each possible event, and estimate the probability of its occurrence:
- For example a crude partition of events is that in $[t, t+dt)$:

1. A market buy order arrives:

$$\Pr\left\{\widehat{d}_{0t}^{(b)} = 1 \text{ in } [t, t+dt) \,|z_t \right\} = \left\{1 - G\left[\theta_t^{(b)}(0,1)\,|x\right]\right\}\lambda(t; x_t)\,dt$$

2. There is a market sell order:

$$\Pr\left\{\widehat{d}_{0t}^{(s)} = 1 \text{ in } [t, t+dt) \,|z_t \right\} = G\left[\theta_t^{(s)}(0,1)\,|x\right]\lambda(t; x_t)\,dt$$

3. Either a limit order arrives or there is no order:

$$\begin{aligned} & \Pr\left\{\widehat{d}_{0t}^{(b)} + \widehat{d}_{0t}^{(s)} = 0 \text{ in } [t, t+dt) \,|z_t \right\} \\ = \; & 1 - \lambda(t; x_t)\,dt \\ & + \left\{G\left[\theta_t^{(b)}(0,1)\,|x\right] - G\left[\theta_t^{(b)}(0,1)\,|x\right]\right\}\lambda(t; x_t)\,dt \end{aligned}$$

How Efficient is an LOM?

Equilibrium gains from trade from behind the Rawlsian (1971) veil of ignorance

- The gains from trade do not depend on the transaction price or the picking off risk, which are transfers between buyer and seller.
- When the buyer places a limit order at t and the seller places a market order at $t + \tau$ cancelling the buy order, the gains from trade are:

$$u_{t+\tau} - u_t - 2\left(c_0 + c_e\right)$$

- More generally, the expected gains from a new trader arriving at t are:

$$V = E\left\{\begin{array}{c} \sum_{k=0}^{\infty} \widehat{d}_{kt}^{(b)}\left(u_t, z_t\right)\left[\psi_{kt}^{(b)}\left(z_t\right)\left(u_t - c_e\right) - c_0\right] \\ -\sum_{k=0}^{\infty} \widehat{d}_{kt}^{(s)}\left(u_t, z_t\right)\left[\psi_{kt}^{(s)}\left(z_t\right)\left(c_e + u_t\right) + c_0\right] \end{array}\right\}$$

How Efficient is an LOM?

Maximal gains from exchange from behind the Rawlsian (1971) veil of ignorance

- We compare the expected gains from trade in an LOM with the *potential gains from exchange*, obtained by choosing between:
  - immediately executing a new order
  - or placing the order in inventory
  - where orders in the inventory are subjected to cancellation risk
  - to maximize the expected gains from exchange.
- We can categorize the reasons why limit order markets do not realize all the potential gains from exchange.
  1. Limit orders are not executed when they should be.
  2. Traders do not submit orders when they should.
  3. Trader submits a "wrong sided" order that executes.
  4. Traders submit orders when they should not.

How Efficient is an LOM?

Structural estimates from Hollifield, Miller, Sandas and Slive (2006)

	BHO	ERR	WEM
Gains			
Maximum gains as a % of the common value			
	9.07	8.61	6.75
Current gains as a % of the common value			
Lower bound	7.88	8.09	6.08
Upper bound	8.45	8.31	6.40
Average	8.16	8.20	6.24
Maximum gains minus current gains			
Lower bound	0.62	0.30	0.35
Upper bound	1.20	0.52	0.67
Average	0.91	0.41	0.51
Current gains as a % of maximum gains			
Lower bound	86.79	93.97	90.07
Upper bound	93.13	96.57	94.81
Average	89.96	95.27	92.44
Decomposition of Losses			
No execution as a % of total losses			
Sell side	32.32	31.20	33.05
Buy side	40.10	39.01	41.85
Subtotal	72.42	70.21	74.90
No submission as a % of total losses			
Sell side	2.24	0.62	0.41
Buy side	1.98	0.15	0.71
Subtotal	4.22	0.77	1.12
Wrong direction as a % of total losses			
Sell side	0.86	0.02	0.39
Buy side	0.20	0.05	0.63
Subtotal	1.06	0.07	1.02
Extramarginal submissions as a % of total losses			
Sell side	9.81	11.87	10.30
Buy side	12.49	17.07	12.66
Subtotal	22.30	28.94	22.96
Total	100.00	100.00	100.00
Monopoly Gains			
Monopoly gains as a % of the common value			
	5.02	5.57	4.18
Monopoly gains as a % of maximum gains			
	55.31	64.71	61.87
Current gains as a % of monopoly gains			
	162.65	147.23	149.41