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45-812 Econometrics II

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ASSIGNMENT 2

Overview The purpose of this assignment is to introduce you to actually doing structural econometrics. The assignment should be undertaken in groups of about three. I do not object to discussion taking place across groups in the early stages of the work either. I do insist that each group should refine its own answers and submit a unique report. For example you might collectively derive the perfect set of answers and then negotiate with each other about where to inject uniquely defined errors.

Due data and assessment Please submit your answers in the form of a report by **Sunday November 17, 11:59 p.m.** with your code attached as an appendix. Hand written work will not be graded. The code must be clearly written with comments where appropriate so that a reader can easily follow. Poor grammar, unclear expression, and lack of precision, will be graded as if I have very limited expertise in this area. All questions carry equal weights.

Model Consider the following model of invention. If you decide to be an inventor (perhaps only for a while) here is what happens. Choices for period t are indicated by $d_t \in \{0, 1\}$ where $d_t = 0$ means "doing something else" in period t and $d_t = 1$ means "inventing". Your lifetime choices is a sequence $\{d_t\}_{t=1}^{T}$ where T is the last period of your life.

In each period you invent there is a fixed probability ξ, your ability parameter, that you "draw" a successful new product/firm in which case you get a "prize" value of "start-up" equal to a normalized value of one. Otherwise you get zero in that period if you made that choice. These draws distributed independently over time. Let $x_t \in \{0, 1\}$ indicate whether your invention is successful ($x_t = 1$) or not ($x_t = 0$). The value of "doing something else" in period t is a constant w. You have a subjective discount factor of β and your lifetime utility is:

$$\sum_{t=1}^{T} \beta^{t-1} \left[d_t I \{x_t = 1\} + (1 - d_t) w \right]$$

If you knew ξ then this would be an easy problem to solve: choose $d_t = 1$ for all t if $\xi > w$; otherwise choose $d_t = 0$ for all t. The difficulty arises because when you start working at the beginning of your career at period don't your own ξ. You do, however know that your ability parameter ξ is drawn from a Beta probability distribution, with parameters (γ, δ) say. Thus the model is fully characterized by parameters, namely w, the outside wage, β the discount factor, and the parameters (γ, δ) defining the distribution from which your ξ is drawn.

Data The data set consists of the choice of the individual. In other words, the data analyst does not observe whether the invention of each individual was successful or not. (If such data was available the beta distribution could be estimated from output in the first period, and an estimator of w could be obtained from the fraction of individuals choosing to invent.

Tutorial In the tutorial file, we discuss three cases: static ($T = 1$), two-period ($T = 2$), and infinite-horizon ($T = \infty$). You can use the content from the tutorial to answer the questions below.

Question 1. Prove that it is never optimal to "do something else" in any period t and invent in period $t + 1$. Thus, the optimal career strategy is to spend no periods inventing, or spend one or more periods inventing at the beginning of your career.

Question 2. Let's consider $T = 2$ case. Can you explain why not all the parameters in your model are identified from the solution to the quit rates without imposing restrictions (such as knowing the values of some of the parameters)?

Question 3. The tutorial file illustrates how to estimate γ when the other parameters are given to the econometrician in $T = 2$ case. Repeat the exercise where we instead estimate δ with the same parameter values. That is, set $(\gamma, \delta, w, \beta, T) = (3.0, 2.0, 0.55, 0.96, 2)$ in simulation and i) provide a closed-form MLE estimator and Fisher information for δ, ii) generate a table of estimation result for δ similar to the one presented in the tutorial file, and iii) verify that your estimates match with `Optim.jl` (or any optimization package in your programming language of choice). You are welcome to try out other number of agents or use other parameter values.

Question 4. The tutorial file provides a flexible enough program that can be adapted to other finite horizon models and a way to compute hazard rates $h_2, \dots, h_5$. Let's use these and repeat the exercise of estimating γ in a five-period model ($T = 5$). Discuss what you find as i) N increases and/or ii) more information on h_t is used. The following steps might be helpful to guide you to answer this question:

1. Provide an algorithm (perhaps some 'pseudo-code') that performs backward induction for the $T = 5$ model.

2. Set some values for $(\gamma, \delta, w, \beta, T)$ so that all agents will choose to invent in the first period (and of course, you should choose $T = 5$).

3. Write a code that performs backward induction that solves this model. It might be computationally cheaper to store the decision rules for each period given initial parameters γ and δ (and remember, go backwards!).

4. Using the code, simulate the data up to $T = 5$.Compute the hazard rates $h_2, \dots, h_5$. Use these to get a MLE for γ (you are welcome to choose δ instead)

Question 5. Compute the hazard rate at the sixth period, h_6. What is the hazard rate given your structural parameters? Do you think this hazard rate would be useful to improve the MLE estimate? Present a table of estimation result of the structural parameter of your choice (either γ or δ) in $T = \infty$ case. Discuss what you find as i) N increases and/or ii) more information on h_t is used.