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45-812 Econometrics II

Mini 22023

## ASSIGNMENT 2 (Hansen and Singleton 1982)

The model: Consider the following model of consumption and portfolio choices we analyzed in lectures, and recall:

$$
1=E_{t}\left[r_{t+1, k} \beta \frac{u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}\right] \equiv E_{t}\left[r_{t+1, k} M R S_{t+1}\right]
$$

where:

- $E_{t}[\cdot]$ is an expectations operator that conditions on all the information the consumer has at time $t$;
- $r_{t j}$ denotes the real return on the $j^{t h}$ asset purchased in period $t-1$;
- $\beta$ is the subjective discount factor;
- $u(\cdot)$ denotes the within-period utility function.

Suppose:

$$
\begin{equation*}
u\left(c_{t}\right)=(1+\alpha) c_{t}^{1+\alpha} \tag{Assumption1}
\end{equation*}
$$

Question 1: For the whole of the post WW2 era, plot the aggregate quarterly series of:

- nondurables per capita $c_{t}$ and the ratio of $c_{t+1} / c_{t}$
- nondurables and services per capita $c_{t}^{*}$ and the ratio of $c_{t+1}^{*} / c_{t}^{*}$
- value weighted aggregate stock returns $r_{t+1}$
- equally weighted aggregate stock returns $r_{t+1}^{*}$

Test whether any of these series have a unit root. (You should read about unit root tests first.) Is there evidence that these series are not stationary and ergodic?

Question 2: Replicate ( to the extent you can) the two top panels of Table III from Hansen and Singleton (1984), by estimating their model with their subsample (data from 1959:2 through 1978:12), and the instrument sets they used. How do your results (estimates and values of test statistics) compare with theirs? If they are not identical can you explain the discrepancies?(

Question 3: Now reestimate the model using the whole sample, where the two parameters $\alpha$ and $\beta$, are allowed to differ in the three time frames, with say $\left(\alpha_{1}, \beta_{1}\right)$ parameterizing the model before 1959:2, $\left(\alpha_{2}, \beta_{2}\right)$ in Hansen-Singleton frame, and $\left(\alpha_{3}, \beta_{3}\right)$ after 1978:12. Test the overidentifying restrictions of the model and compare the estimates for the three time frames. Interpret your results. Test whether $\left(\alpha_{1}, \beta_{1}\right)=\left(\alpha_{2}, \beta_{2}\right)=\left(\alpha_{3}, \beta_{3}\right)$ and interpret your results.

Question 4: Finally, replace Assumption 1 with:

$$
\begin{equation*}
u\left(c_{t}\right)=-e^{-\gamma c_{t}} \tag{Assumption2}
\end{equation*}
$$

Repeat the exercises entailed in Questions 2 though 4.

Question 5: On the basis of the evidence from your work, which is the more palatable parameterization. Briefly explain the reasons for your choice.

