Continuous Choices in Competitive Equilibrium

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- Competitive equilibrium is the bedrock of economics:
    - Consumers reveal their preferences through their choices (*three axioms supporting consumer choice theory*);
    - Given the price of each commodity, consumers and producers buy or sell as many units as they wish (*individual optimization*);
    - At those prices the market for each commodity clears, supply matching demand (*existence of equilibrium*);
    - All potential gains from trade are realized, the economy achieving a Pareto efficient allocation (*welfare theorems*).
- Assuming complete markets (Debreu, 1959 Chapter 7) is the dynamic analogue to a static model of competitive equilibrium:
    - Commodities are indexed by time and the state of the world;
    - States evolve over time according to a probability distribution.

Law of One Price

Auxiliary assumptions

- Most econometric analysis also make several auxiliary assumptions.

1. The expected utility hypothesis holds.
2. Subjective beliefs match probability transitions.
3. Preferences are time additively separable up to a finite vector of human capital, habit persistence and nondurables.
4. Current utility payoffs are sufficiently smooth that individual agents only require a small number of securities to achieve the equilibrium allocations of the complete markets.

Portfolio Choices in Competitive Equilibrium

The consumer optimization problem

- Suppose there are J financial securities.
- Let p_{tj} denote the price of the j^{th} security in period t consumption units, and $q_{t-1,j}$ the amount a consumer owns at the beginning of the period.
- Let r_{tj} denote the real return on assets purchased in period $t-1$.
- The investor's budget constraint is:

$$c_t + \sum_{j=1}^{J} p_{tj} q_{tj} \leq \sum_{j=1}^{J} r_{tj} p_{t-1,j} q_{t-1,j}$$

- At t the consumer maximizes a concave objective function with linear constraints, choosing $(q_{s1}, ..., q_{sJ})$ to maximize:

$$u(c_t) + E_t \left[\sum_{s=t+1}^{T} \beta^{s-t} u(c_s) \right]$$

subject to the sequence of all the future budget constraints.

- Nonsatiation guarantees:

$$c_t = \sum_{j=1}^{J} \left(r_{tj} p_{t-1,j} q_{t-1,j} - p_{tj} q_{tj} \right)$$

- The interior first order condition for each $k \in \{1, \ldots, J\}$ requires:

$$\begin{aligned} & p_{tk} u' \left(\sum_{j=1}^{J} \left(r_{tj} p_{t-1,j} q_{t-1,j} - p_{tj} q_{tj} \right) \right) \\ \geq \quad & E_t \left[p_{tk} r_{t+1,k} \beta u' \left(\sum_{j=1}^{J} \left(r_{t+1,j} p_{tj} q_{tj} - p_{t+1,j} q_{t+1,j} \right) \right) \right] \end{aligned}$$

with equality holding if $q_{tj} > 0$.

- Substituting c_t and c_{t+1} back into the marginal utilities and rearranging yields the fundamental equation of portfolio choice:

$$1 = E_t\left[r_{t+1,k}\beta\frac{u'(c_{t+1})}{u'(c_t)}\right] \equiv E_t\left[r_{t+1,k}MRS_{t+1}\right]$$

- Recall from the definition of a covariance:

$$\begin{aligned} cov\left(r_{t+1,k}, MRS_{t+1}\right) &= E_t\left[r_{t+1,k}MRS_{t+1}\right] - E_t\left[r_{t+1,k}\right]E_t\left[MRS_{t+1}\right] \\ &= 1 - E_t\left[r_{t+1,k}\right]E_t\left[MRS_{t+1}\right] \\ &= 1 - E_t\left[r_{t+1,k}\right]/r_{t+1} \end{aligned}$$

where the second line uses the fundamental equation of portfolio choice, and the third the definition of the risk free rate.

- Rearranging this equation gives the risk correction for the k^{th} asset:

$$E_t\left[r_{t+1,k}\right] - r_{t+1} = -r_{t+1}cov\left(r_{t+1,k}, MRS_{t+1}\right)$$

- For any $r \times 1$ vector x_t belonging to the information set at t and all k:

$$0 = E_t \left[r_{t+1,k} \beta \frac{u'(c_{t+1})}{u'(c_t)} - 1 \right] = E \left[r_{t+1,k} \beta \frac{u'(c_{t+1})}{u'(c_t)} - 1 \left| x_t \right. \right]$$

and hence:

$$0 = E \left\{ x_t \left[r_{t+1,k} \beta \frac{u'(c_{t+1})}{u'(c_t)} - 1 \right] \right\}$$

- Given a sample of length T we can estimate the $1 \times l$ vector (β, α) for a parametrically defined utility function $u(c_t; \alpha)$ by solving:

$$0 = A_T \sum_{t=1}^{T} x_t \left[r_{t+1,k} \beta \frac{u'(c_{t+1}; \alpha)}{u'(c_t; \alpha)} - 1 \right]$$

where A_T is an $l \times r$ weighting matrix.

- Clearly this estimator easily generalizes to any number of assets with an interior condition.

Representative Consumer Model

Estimates from aggregate consumption data (Hansen and Singleton, 1984, Table I)

Cons	Return	NLAG	$\hat{\alpha}$	$\widehat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	χ^2	DF	Prob
NDS	EWR	1	−0.9360	2.5550	.9930	.0060	5.226	1	.9774
NDS	EWR	2	0.1529	2.3468	.9906	.0056	7.378	3	.9392
NDS	EWR	4	1.2605	2.2669	.9891	.0059	9.146	7	.7577
NDS	EWR	6	0.1209	2.0455	.9928	.0054	14.556	11	.7963
NDS	VWR	1	−1.0350	1.8765	.9982	.0045	1.071	1	.6993
NDS	VWR	2	0.1426	1.7002	.9965	.0044	3.467	3	.6749
NDS	VWR	4	−0.0210	1.6525	.9969	.0043	5.718	7	.4270
NDS	VWR	6	−1.1643	1.5104	.9997	.0041	11.040	11	.5601
ND	EWR	1	−1.5906	1.0941	.9930	.0034	7.186	1	.9926
ND	EWR	2	−0.7127	0.9916	.9918	.0034	12.040	3	.9928
ND	EWR	4	−0.1261	0.8917	.9921	.0035	14.638	7	.9591
ND	EWR	6	−0.4193	0.8256	.9936	.0033	18.016	11	.9188
ND	VWR	1	−1.2028	0.7789	.9976	.0027	1.457	1	.7726
ND	VWR	2	−0.5761	0.7067	.9975	.0027	5.819	3	.8792
ND	VWR	4	−0.6565	0.6896	.9978	.0027	7.923	7	.6606
ND	VWR	6	−0.9638	0.6425	.9985	.0027	10.522	11	.5159

Representative Consumer Model

Estimates from aggregate consumption data (Hansen and Singleton, 1984, Table III)

Equally- and Value-Weighted Aggregate Returns 1959:2–1978:12								
Cons	NLAG	$\hat{\alpha}$	$\widehat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	χ^2	DF	Prob.
NDS	1	−0.5901	1.7331	.9989	.0041	18.309	6	.9945
NDS	2	1.0945	1.4907	.9961	.0040	24.412	12	.9821
NDS	4	0.3835	1.4208	.9975	.0039	40.234	24	.9798
ND	1	−0.6494	0.6838	.9982	.0025	19.976	6	.9972
ND	2	−0.0200	0.6071	.9982	.0025	27.089	12	.9925
ND	4	−0.1793	0.5928	.9986	.0025	42.005	24	.9871

Value-Weighted Aggregate Stock Returns and Risk-Free Bonds Returns 1959:2–1978:12								
Cons	NLAG	$\hat{\alpha}$	$\widehat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	χ^2	DF	Prob.
NDS	1	−.1405	.0420	.9998	.0001	31.800	8	.9999
NDS	2	−.1472	.0376	.9998	.0001	44.083	16	.9998
NDS	4	−.1405	.0320	.9996	.0001	65.250	32	.9995
ND	1	−.0962	.0461	.9995	.0001	25.623	8	.9988
ND	2	−.1150	.0377	.9995	.0001	39.874	16	.9991
ND	4	−.1611	.0364	.9994	.0001	60.846	32	.9985

Three Industry-Average Stock Returns 1959:2–1977:12								
Cons	NLAG	$\hat{\alpha}$	$\widehat{SE}(\hat{\alpha})$	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	χ^2	DF	Prob.
NDS	1	1.5517	1.8006	.9906	.0046	13.840	13	.6147
NDS	4	0.6713	1.2466	.9940	.0035	88.211	49	.9995
ND	1	0.7555	0.7899	.9924	.0029	13.580	13	.5959
ND	4	0.5312	0.5512	.9939	.0024	89.501	49	.9996

Representative Consumer Model

Interpreting estimates from aggregate data

- To interpret these results, lifetime utility is:

$$\sum_{t=1}^{\infty} \beta^t u(c_t) = (1+\alpha)^{-1} \sum_{t=1}^{\infty} \beta^t c_t^{1+\alpha}$$

  - NDS (nondurables plus services)
  - ND (nondurables)
  - EWR (NYSE equally weighted average returns)
  - VWR (NYSE value weighted average returns)
  - The 3 industries are chemicals, transportation and equipment, and other retail.

- Note that:
  - 10 out of 12 specifications in Table III are rejected at the 0.05 level.
  - In the other 2 specifications in Table III $\alpha > 0$, which implies $u(c_t)$ convex increasing (so the FOC is not a maximum).

- Subscript current utility, consumption and instruments by individual $n \in \{1, \ldots, N\}$ and define the forecast error ϵ_{nt} as:

$$\epsilon_{nt} \equiv \left[r_{tk} \beta u_n' \left(c_{t+1} \right) / u_n' \left(c_t \right) \right] - 1$$

- Then if the FOC asset for k holds with equality for n:

$$E_t \left[\epsilon_{nt} \left| x_{nt} \right. \right] = 0 \Longrightarrow E \left[\epsilon_{nt} \left| x_{nt} \right. \right] = 0 \Longrightarrow E \left[x_{nt} \epsilon_{nt} \right] = 0$$

- Noting $\{\ldots, \epsilon_{nt}, \epsilon_{n,t+1}, \ldots\}$ is a Martingale difference sequence, the Euler equation approach to estimation could be applied, since for all $n \in \{1, \ldots, N\}$:

$$0 = E \left[x_{nt} \epsilon_{nt} \right] = \underset{T \to \infty}{p \lim} \left[\frac{1}{T} \sum\nolimits_{t=1}^{T} x_{nt} \epsilon_{nt} \right]$$

- The large sample properties of this estimator rely on the length of the time series, or more intuitively, the fact that successive forecast errors are uncorrelated with each other.

Relaxing the Assumption of a Representative Consumer

Using Euler equation methods on a cross section

- Suppose $u_n(c_t) = u(c_t)$ and the data only comprise a small number of periods (say 2), but a large number of individuals.
- Averaging over $n \in \{1, \ldots, N\}$ gives:

$$\frac{1}{N}\sum_{n=1}^{N} x_{nt}\left[r_{tk}\beta\frac{u'(c_{t+1})}{u'(c_t)} - 1\right] = \frac{1}{N}\sum_{n=1}^{N} x_{nt}\epsilon_{nt}$$

- Assuming each person in the sample lived in different disconnected economy from everyone, implying the forecast error ϵ_{nt} for n is independent of $\epsilon_{n't}$ for n':

$$\underset{N\to\infty}{p\lim}\left[\frac{1}{N}\sum_{n=1}^{N} x_{nt}\epsilon_{nt}\right] = 0 = \underset{T\to\infty}{p\lim}\left[\frac{1}{T}\sum_{t=1}^{T} x_{nt}\epsilon_{nt}\right]$$

- Thus $u(c_t)$ could be identified and estimated off a panel.

- Now suppose they live in the same economy. Then:

$$v_t \equiv \underset{N\to\infty}{p\lim}\left[\frac{1}{N}\sum\nolimits_{n=1}^{N} x_{nt}\epsilon_{nt}\right] \neq 0$$

- In this case v_t depends on any aggregate shock to the economy hitting everyone in the economy.
- Furthermore v_t is instrument specific, so treating v_t as a time dummy in estimation requires as many time dummies as there are instruments.
- Hence $u\left(c_t\right)$ is not identified off a panel formed from individuals who are subjected to correlated disturbances over time.

- Let $\{\mathcal{F}_t\}_{t=0}^{\infty}$ denote a sequence of σ-algebras with measure $\mathcal{P}$ that reflects how history unfolds as the economy evolves.
- Each period $t \in \{\underline{n}, \ldots, \overline{n}\}$ household n consumes $(c_{nt1}, \ldots, c_{ntK})$.
- Define a commodity by the triplet (k, t, A).
- Let $p_{tk}(A)$ denote the date zero price of receiving a unit of k at t in the event of $A \in \mathcal{F}_t$ occurring:

$$p_{tk}(A) = \int_A \lambda_{tk}(\omega) \mathcal{P}(d\omega)$$

- The Radon-Nikodym derivative $\lambda_{tk}(\omega)$ converts the probability of events into a commodity price measure.

Complete Markets

A lifetime budget constraint

- Assume markets are complete and there is a competitive market for every commodity defined on k, t, and $\{\mathcal{F}_t\}_{t=0}^{\infty}$.
- The assumption of complete markets implies the consumer budget set is a single lifetime budget constraint, rather than a sequence of period-specific budget constraints.
- The lifetime budget constraint for n is:

$$E_0 \left[\sum_{t=\underline{n}}^{\overline{n}} \sum_{k=1}^{K} \lambda_{tk} c_{ntk} \right] \leq B_n \tag{1}$$

- Compared with the representative consumer model, we:
  - drop the representative consumer assumption;
  - assume a competitive equilibrium exhausts the gains from trade.

- Suppose households obey the expected utility hypothesis, preferences taking the time additive form:

$$E_0 \left[\sum_{t=\underline{n}}^{\overline{n}} \beta^{t-\underline{n}} u_{t-\underline{n}} \left(c_{nt1}, \ldots, c_{ntK} \right) \right] \tag{2}$$

- Let:
  1. η_n denote the Lagrange multiplier associated with (1)
  2. p_{tk} denote the spot price of k at t (conditional the state)
  3. the first good be a numeraire and define $\lambda_t \equiv \lambda_{t1}$.
  4. $t_n \equiv t - \underline{n}$ denote the age of the household
  5. $c_{nt} \equiv \left(c_{nt1}, \ldots, c_{ntK} \right)$ denote the consumption vector of n at t.
- Household n maximizes (2) subject to (1).
- Then the first order condition for an interior solution for k is:

$$\beta^{t_n} u_{t_n,k} \left(c_{nt} \right) \equiv \beta^{t_n} \frac{\partial u_{t_n} \left(c_{nt} \right)}{\partial c_{ntk}} = \eta_n \lambda_{tk} \equiv \eta_n \lambda_t p_{tk} \tag{3}$$

- Suppose $K = 1$ and z_{nt} is a vector differentiating consumers:

$$u_{nt}\left(c_{nt}\right) \equiv \left(\alpha+1\right)^{-1}\delta\left(z_{nt}\right)c_{nt}^{\alpha+1} \tag{4}$$

- Then (3) simplifies to:

$$\beta^{t_n}\delta\left(z_{nt}\right)c_{nt}^{\alpha} = \eta_n\lambda_t$$

- Hence the asset pricing equation can be expressed as:

$$1 = \beta E_t\left[r_{t+1,k}\frac{\delta\left(z_{n,t+1}\right)}{\delta\left(z_{nt}\right)}\left(\frac{c_{n,t+1}}{c_{nt}}\right)^{\alpha}\right] = E_t\left[r_{t+1,k}\frac{\lambda_{t+1}}{\lambda_t}\right] \tag{5}$$

- Solving for consumption and averaging over a population of N yields:

$$c_t \equiv \sum\nolimits_{n=1}^{N} c_{nt} = \lambda_t^{\frac{1}{\alpha}}\sum\nolimits_{n=1}^{N}\left[\eta_n/\beta^{t_n}\delta\left(z_{nt}\right)\right]^{\frac{1}{\alpha}}$$

or the shadow value of consumption:

$$\lambda_t = c_t^{\alpha}\left\{\sum\nolimits_{n=1}^{N}\left[\eta_n/\beta^{t_n}\delta\left(z_{nt}\right)\right]^{\frac{1}{\alpha}}\right\}^{-\alpha}$$

Complete Markets

Econometric implications of heterogeneity in CRRA preferences with complete markets

- Substituting the expression for λ_t into (5) gives: the Euler equation for a representative consumer:

$$1 = \beta E_t \left[r_{t+1,k} \left\{ \frac{\sum_{n=1}^{N} \left[\eta_n / \beta^{t_n} \delta\left(z_{nt}\right) \right]^{\frac{1}{\alpha}}}{\sum_{n=1}^{N} \left[\eta_n / \beta^{t_n+1} \delta\left(z_{n,t+1}\right) \right]^{\frac{1}{\alpha}}} \right\}^{\alpha} \left(\frac{c_{t+1}}{c_t} \right)^{\alpha} \right]$$

1. If consumers are only differentiated by some fixed characteristics and their initial wealth, and $z_{nt} \equiv z_n$ then:

$$\beta \left(c_{t+1} / c_t \right)^{\alpha} = \lambda_{t+1} / \lambda_t$$

2. However to achieve a representative consumer style portfolio equation if z_{nt} changes over time, returns must be reweighted to reflect shifting priorities for consumption in t by different segments of the population.

- Therefore one reason for rejecting the representative consumer model is time varying heterogeneity.

Complete Markets

Marginal rates of substitution in equilibrium

- Temporarily dropping for convenience the subscript n, the individual identifier, and setting $p_{t1} \equiv 1$, there are:
  1. $(K-1)(\overline{n}-\underline{n})$ equations corresponding to the spot markets:

$$MRS_{tk}(c_t) \equiv \frac{u_{tk}(c_t)}{u_{t1}(c_t)} = p_{tk}$$

  2. $(\overline{n}-\underline{n})-1$ equations pertaining to the numeraire that intertemporally balance consumption:

$$MRS_t(c_t, c_{t+1}) \equiv \frac{\beta u_{t+1,1}(c_{t+1})}{u_{t1}(c_t)} = \frac{\lambda_{t+1}}{\lambda_t}$$

- Given η_n, we can show that these $K(\overline{n}-\underline{n})-1$ marginal rates of substitution equations fully characterize an interior equilibrium consumption of n.

- All remaining marginal rates of substitution functions are fully described by $MRS_{tk}(c_t)$ and $MRS_t(c_t, c_{t+1})$ because:

$$MRS_{t,s,k,l}(c_t, c_s) \equiv \frac{\beta^{s-t} u_{sl}(c_{sl})}{u_{tk}(c_{nt})} = \frac{MRS_{sl}(c_s)}{MRS_{tk}(c_t)} \prod_{r=t}^{s} MRS_r(c_r, c_{r+1})$$

- Likewise all the remaining contingent prices are described by the vector sequence of spot prices $\{(p_{t2}, \ldots, p_{tK})\}_{t=\underline{n}}^{\overline{n}}$ along with the contingent price sequence for the numeraire $\{\lambda_t\}_{t=\underline{n}}^{\overline{n}}$, because:

$$\frac{p_{sl}}{p_{tk}} \prod_{r=t}^{s-1} \frac{\lambda_{r+1}}{\lambda_r} = \frac{\lambda_s p_{sl}}{\lambda_t p_{tk}}$$

- It follows that additional equalities of the form:

$$MRS_{t,s,k,l}(c_t, c_s) = \frac{\lambda_s p_{sl}}{\lambda_t p_{tk}}$$

provide neither additional restrictions nor additional parameters.

Complete Markets

Example (Altug and Miller, 1990)

- For example suppose:

$$u_t\left(c_{nt}\right) \equiv \sum_{k=1}^{K} \frac{\exp\left(x_{nt}B_k + \epsilon_{ntk}\right)}{\alpha_k + 1} c_{ntk}^{\alpha_k + 1}$$

- Focusing on the first two goods we have:

$$\begin{aligned} p_{t2} &= MRS_{t2}\left(c_{nt}\right) \\ &= \exp\left[x_{nt}\left(B_2 - B_1\right) + \epsilon_{nt2} - \epsilon_{nt1}\right] \frac{c_{nt2}^{\alpha_2}}{c_{nt1}^{\alpha_1}} \end{aligned}$$

Taking logarithms:

$$\begin{aligned} & \epsilon_{nt2} - \epsilon_{nt1} \\ = \; & x_{nt}\left(B_1 - B_2\right) + \alpha_1 \ln\left(c_{nt1}\right) - \alpha_2 \ln\left(c_{nt2}\right) + \ln p_{t2} \end{aligned}$$

- For any instrument vector z_{nt} satisfying:

$$E\left[\epsilon_{nt}\left|z_{nt}\right.\right]=0$$

we have:

$$E\left\{z_{nt}\left[x_{nt}\left(B_1-B_2\right)+\alpha_1\ln\left(c_{nt1}\right)-\alpha_2\ln\left(c_{nt2}\right)+\ln p_{t2}\right]\right\}=0$$

- A GMM estimator now comes from setting

$$0=A\sum_{n=1}^{N}z_{nt}\left[x_{nt}\left(B_1-B_2\right)+\alpha_1\ln\left(c_{nt1}\right)-\alpha_2\ln\left(c_{nt2}\right)+\ln p_{t2}\right]$$

- The usual large sample properties apply.

- Similarly:

$$\frac{\lambda_{t+1}}{\lambda_t} = \beta \exp\left[\left(x_{n,t+1} - x_{nt}\right) B_1 + \epsilon_{n,t+1,1} - \epsilon_{nt1}\right] \left(\frac{c_{nt+1,1}}{c_{n,t+1,1}}\right)^{\alpha_1}$$

or in logarithmic form:

$$\Delta \ln \lambda_t - \ln \beta = \Delta x_{nt} B_1 + \Delta \epsilon_{nt1} + \alpha_1 \Delta \ln c_{nt1}$$

where:

$$\begin{aligned} \Delta x_{nt} &\equiv \left(x_{n,t+1} - x_{nt}\right) & \Delta \epsilon_{nt1} &\equiv \left(\epsilon_{n,t+1,1} - \epsilon_{nt1}\right) \\ \Delta \ln \lambda_t &\equiv \ln \lambda_{t+1} - \ln \lambda_t & \Delta \ln c_{nt1} &\equiv \ln c_{nt+1,1} - \ln c_{nt1} \end{aligned}$$

- If $E\left[\epsilon_{nt} \left| z_{nt}\right.\right] = 0$ then:

$$E\left\{z_{nt}\left[\Delta \ln \lambda_t - \ln \beta - \alpha_1 \Delta \ln c_{nt1} - \Delta x_{nt} B_1\right]\right\} = 0$$

- A GMM estimator with the usual large sample properties can be formed from the sample analogue.

Complete Markets

The rate of time discounting cannot be identified off short panels

- Suppose the model is generated by contingent prices λ_t^e and β_0.
- Assume neither λ_t^e nor β_0 are directly observed.
- Treat β and $\Delta \ln \lambda_t$ as parameters in the estimated model. Setting:

$$\begin{aligned} \beta &= \beta^* \\ \Delta \ln \lambda_t^* &= \Delta \ln \lambda_t^e - \ln \beta_0 + \ln \beta^* \end{aligned}$$

- It immediately follows that a specification with:

$$(\beta, \Delta \ln \lambda_t) = (\beta^*, \Delta \ln \lambda_t^*)$$

is observationally equivalent to:

$$(\beta, \Delta \ln \lambda_t) = (\beta_0, \Delta \ln \lambda_t^e)$$

- Thus the rate of time discounting is not identified off short panels.

Complete Markets

The degree of aggregate uncertainty cannot be identified off short panels

- In a world of perfect foresight where the one period interest rate is i_t, then:

$$\lambda_t = 1 \big/ (1 + i_t)$$

and:

$$MRS_{t_n}\left(c_{nt}, c_{n,t+1}\right) \equiv \frac{\beta u_{t_n+1,1}\left(c_{n,t+1}\right)}{u_{t_n,1}\left(c_{nt}\right)} = \frac{1}{\left(1 + i_t\right)}$$

- We cannot tell from panel data whether this is one of several paths the world could be taking, or whether it is the unique path.
- Confusing "complete markets" with a "full insurance" model, as some authors have done, is misleading.
- Complete markets might fail because of borrowing constraints, rather than a lack of insurance opportunities.

An International Comparison (Miller and Sieg, 1997)

Descriptive statistics for the U.S. and Germany

Variables	1983	1984	1985	1986	1987	1988	1989	1990
Household size	3.45	3.48	3.50	3.53	3.53	3.53	3.53	3.49
	(1.06)	(1.04)	(1.06)	(1.07)	(1.04)	(1.06)	(1.09)	(1.08)
Number of children under 16	1.10	1.09	1.04	1.03	1.01	.96	.93	.89
	(.97)	(.99)	(1.02)	(1.05)	(1.05)	(1.08)	(1.08)	(1.07)
Number of rooms	4.14	4.19	4.19	4.14	4.16	4.19	4.16	4.18
	(1.38)	(1.36)	(1.40)	(1.38)	(1.40)	(1.43)	(1.39)	(1.41)
Rent[a]	—	705.92	745.59	787.36	813.38	850.08	924.48	1,002.57
	—	(346.91)	(352.71)	(388.56)	(409.85)	(413.93)	(456.43)	(486.23)
Hours worked$^{b}_{m}$	44.28	44.38	44.26	43.62	43.73	43.83	43.19	43.29
	(8.31)	(6.62)	(7.36)	(7.40)	(7.45)	(6.61)	(5.91)	(6.53)
Gross labor income$^{a}_{m}$	3,578.63	3,693.23	3,936.23	4,122.32	4,247.84	4,434.63	4,623.92	4,884.37
	(1,170.48)	(1,254.28)	(1,526.49)	(1,632.20)	(1,622.73)	(1,621.31)	(1,712.98)	(1,927.18)
Hours worked$^{b}_{f}$	30.40	27.45	28.21	28.44	27.91	27.73	26.99	25.90
	(14.29)	(12.81)	(13.41)	(11.61)	(13.21)	(13.11)	(13.29)	(12.89)

Table 2. Descriptive Statistics for the PSID Subsample

	1980	1981	1982	1983	1984	1985	1986	1987	1988
Household size	3.98	3.92	3.87	3.82	3.84	3.83	3.78	3.83	3.82
	(1.68)	(1.60)	(1.52)	(1.40)	(1.42)	(1.42)	(1.26)	(1.25)	(1.23)
Number of children under 16	1.57	1.55	1.54	1.53	1.58	1.63	1.64	1.65	1.65
	(1.34)	(1.33)	(1.28)	(1.24)	(1.25)	(1.24)	(1.20)	(1.19)	(1.17)
Number of rooms	5.40	5.42	5.56	5.52	5.59	5.61	5.59	5.69	5.91
	(1.53)	(1.54)	(1.49)	(1.45)	(1.53)	(1.51)	(1.53)	(1.64)	(1.71)
Rent[a]	221.03	241.85	266.84	273.22	301.70	322.37	334.87	—	—
	(110.76)	(126.11)	(139.86)	(127.97)	(147.72)	(161.70)	(166.73)	—	—
Hours worked$^{b}_{m}$	42.76	41.48	40.57	41.97	42.85	42.86	43.33	43.96	43.72
	(12.10)	(12.15)	(12.00)	(12.13)	(11.70)	(11.37)	(11.86)	(11.58)	(11.88)
Gross labor income$^{a}_{m}$	1,425.05	1,550.07	1,605.56	1,733.35	1,886.44	1,957.76	2,052.60	2,234.64	2,334.64
	(838.74)	(946.49)	(1,007.32)	(1,072.58)	(1,337.93)	(1,198.64)	(1,247.06)	(1,456.30)	(1,391.11)
Hours worked$^{b}_{f}$	26.52	25.66	26.07	26.17	27.73	27.58	28.34	28.87	29.07
	(14.67)	(13.78)	(14.51)	(14.46)	(14.76)	(14.23)	(12.99)	(13.37)	(13.14)

NOTE: Standard errors are given in parentheses.
[a] Measured on a monthly basis in U.S. dollars.
[b] Measured on a weekly basis in U.S. dollars.
$_m$ Variable refers to male.
$_f$ Variable refers to female.

An International Comparison

A model of male labor supply and housing demand

- The following notation applies to household n at time t:
  - l_{0nt} female leisure
  - l_{1nt} male leisure
  - h_{nt} housing services
  - x_{nt} observed demographics
  - $(\epsilon_{0nt}, \ldots, \epsilon_{3nt})$ unobserved disturbance *iid* over n
- Current utility takes the form:

$$\begin{aligned} u\left(l_{0nt}, l_{1nt}, h_{nt}, , x_{nt}\right) &\equiv \alpha_0^{-1} \exp\left(x_{nt} B_0 + \epsilon_{0nt}\right) h_{nt}^{\alpha_0} l_{0nt}^{\alpha_2} \\ &\quad + \alpha_1^{-1} \exp\left(x_{nt} B_1 + \epsilon_{1nt}\right) l_{1nt}^{\alpha_1} l_{0nt}^{\alpha_3} + \ldots \end{aligned}$$

- The wage rate is the value of the marginal product for a standard labor unit times the efficiency rating of n:

$$w_{nt} \equiv w_t \exp\left(x_{nt} B_2 + \epsilon_{2nt}\right)$$

- Similarly:

$$r_{nt} \equiv r_t \exp\left(x_{nt} B_3 + \epsilon_{3nt}\right)$$

An International Comparison

Estimates of the marginal rate of substitution functions

Parameters of utility function	Variable	I SOEP	I PSID	II SOEP	II PSID	III SOEP	III PSID	IV SOEP	IV PSID
$\alpha_0 - 1$		−2.02	−2.08	−4.58	−2.32	−4.19	−1.91	−3.17	−.91
		(.22)	(1.13)	(.24)	(.59)	(.26)	(.38)	(1.46)	(1.00)
$\alpha_1 - 1$		−1.87	−2.15	−2.46	−2.53	−3.88	−1.95	−3.76	−1.83
		(.93)	(2.81)	(1.07)	(1.68)	(.73)	(1.22)	(1.51)	(1.97)
$\alpha_2 - \alpha_3$		−.92	−1.23	−.83	−1.74	—	—	—	—
		(.89)	(2.81)	(1.04)	(1.67)				
α_2		—	—	—	—	−.31	−.66	−.29	−.61
						(.84)	(2.14)	(3.00)	(4.20)
α_3		—	—	—	—	2.09	.47	2.29	.56
						(.39)	(2.11)	(2.19)	(2.58)
ΔB	Household size	−.21	−.16	−.46	.50	—	—	—	—
		(.20)	(.26)	(.21)	(.36)				
	Number of children	.07	.10	.29	−.15	—	—	—	—
		(.22)	(.30)	(.12)	(.29)				
B_0	Household size	—	—	—	—	.33	.39	.40	.15
						(.26)	(.48)	(.58)	(1.01)
	Number of children	—	—	—	—	−.09	−.11	−.23	−.05
						(.20)	(.53)	(.81)	(.96)
B_1	Household size	—	—	—	—	.02	.13	.13	.11
						(.21)	(.31)	(.51)	(.44)
	Number of children	—	—	—	—	.22	−.15	−.13	−.12
						(.17)	(.46)	(.64)	(.42)
B_3	Size of housing unit	.28	.19	.47	.27	.48	.37	.41	.27
		(.03)	(.08)	(.02)	(.06)	(.02)	(.06)	(.11)	(.16)
	City indicator	.76	.52	−.11	.63	.94	.55	1.37	.54
		(.05)	(.24)	(.04)	(.15)	(.05)	(.02)	(.39)	(.23)
	J value	197.74		333.48		331.46		242.42	
	Degrees of freedom	207		319		321		209	

An International Comparison

Interpreting Table 3

- The column key is:
    1. MRS between housing and male leisure plus housing rental function
    2. Adds wage equation
    3. Adds intertemporal MRS for male leisure over consecutive periods and subtracts rent equation
    4. Both MRS conditions plus wage and rent equations
- The number of observations is about 400 so $\sqrt{N}$ is about 20.
- J is asymptotically χ_d where $d = \#$ overidentifying restrictions.
- None of the specifications is rejected, all the coefficients are significant and are signed according to economic intuition.
- Contingent claims prices (inversely) track aggregate consumption quite well.

An International Comparison

Aggregate consumption (solid line) and estimated contingent prices (dotted) for Germany and the U.S.

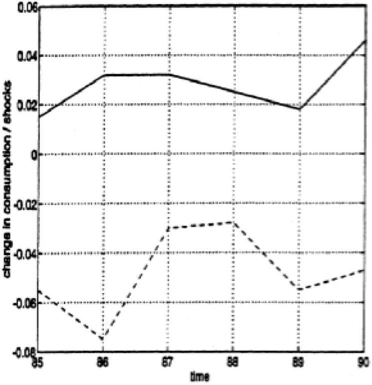

Figure 1.Aggregate Consumption and Shocks in Germany

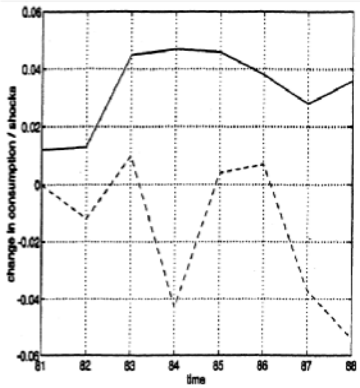

Figure 2. Aggregate Consumption and Shocks in the US

An International Comparison

Testing equality of prices, preferences and efficiency ratings

- We reject the null hypotheses that:
  - contingent claims prices between Germany and US are equal
  - contingent claims prices between different regions in the US are equal at the 0.05 but not at the 0.1 level
  - preferences between the two countries are the same
- With respect to purchasing power parity we:
  - do not reject the null that the value of marginal product of labor is equalized across both countries
  - reject the null that the premium to education is the same.