

45-971

The Last Test for some of you  
I hope to see the rest of you Mini 2 next year for  
45-871

Due Monday May 4 or email Prof Miller to request an extension

(team work is fine)

### Question 1 on technology, bargaining, profits and wages

Suppose there are 10 recruiting firms and 10 graduating students. Student recruit quality is given by their grade, a positive integer, denoted by  $q$ . The grade to each student recruit is unique and ranges from 1 to 10. Firms differ by the amount of capital invested in them. The stock of capital invested in a firm is denoted by  $k$ . Each firm has a different amount of capital, and  $k$  also ranges from 1 to 10. We look at two types of production technology:

1. When a firm with capital stock  $k$  is matched with a student of quality, the joint output is  $kq$ . This is the one we look at in class and is called a multiplicative production technology.
2. When a firm with capital stock  $k$  is matched with a student of quality, the joint output is  $k + q$ . This is called an additive production technology.

Total output is obtained by matching each employer with a recruit and summing over the output from the matches. It is possible that total output might depend on how the matches are formed.

**Question 1.1:** To maximize total output how should employers match up with recruits when the production technology is multiplicative? What is the highest total output that can be attained with this technology, and what is the lowest?

**Question 1.2:** To maximize total output how should employers match up with recruits when the production technology is additive? Does it matter? What is total output is attained with this technology?

Firms are permitted to make up to one offer at a time, but can withdraw at any time an offer that has not been accepted. The labor market is stratified by student quality. Thus every offer is directed to a student of a given quality, say  $q$ , and consists of a wage, say  $w(q)$ . Students cannot make offers themselves but can only accept offers that are made to them. When a student of quality  $q$  accepts an

offer of  $w(q)$  from a firm with capital of  $k$ , both the firm and the student exit the market. In that case the student receives a wage of  $w(q)$  and firm reaps profits of output less wage bill, that is  $kq - w(q)$  in the multiplicative case and  $k + q - w(q)$  in the additive case.

In class we discussed how the Nash equilibrium would be defined when there is a multiplicative production function. In that case a Nash equilibrium is a wage offer by each firm  $k \in \{1, \dots, N\}$  to just one student  $q \in \{1, \dots, N\}$ , denoted  $w(q, k)$ , which is accepted, with two special properties:

1. For all  $(k', q')$ :

$$kq - w(q, k) \geq kq' - w(q', k)$$

2. If  $w'(q, k) < w(q, k)$  then there exists another firm  $k^*$  such that:

$$k^*q - w'(q, k) > k^*q^* - w(q^*, k^*)$$

The first inequality says that the profits to firm  $k$  from the equilibrium assignment  $q$  and wage  $w(q, k)$  are no less than the profits to the same firm from making a competitive offer to any other recruit. The second property is that if firm  $k$  offered less than  $w(q, k)$  then another firm  $k^*$  would make more profits from making a more attractive offer worker to  $q$  than offering its equilibrium wage  $w(q^*, k^*)$  to the recruit it is assigned to in equilibrium,  $q^*$ .

**Question 1.3:** Write down how the inequalities change when there is an additive technology, interpret them, and show they simplify in the additive case.

In class we showed that if the economy was efficiently matching recruits to employers in the Nash equilibrium for the multiplicative technology firms would set the equilibrium wage so that:

$$w_q = \frac{1}{2}(q-1)q$$

**Question 1.4:** What are the profits of the firms in this case?

**Question 1.5:** In the additive technology case show that setting  $q = w(q, k)$ , that is setting the wage of the worker equal to their skill, is the Nash equilibrium wage because it satisfies the special properties you derived in Question 1.3.

**Question 1.6:** What are firm profits when the technology is additive?

**Question 1.7:** Now change the bargaining mechanism so that the roles of the recruits and their potential employers are reversed; that is the recruits demand a wage and the firms simply respond. How do your answers about matches wages and profits change?

**Question 1.8:** Use your analysis to explain how the bargaining mechanism can sometimes but not always interact with the production technology in the determination of wages and employment and profit outcomes.

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## Question 2 on outsourcing

Read the article on Nike.

**Question 3.1:** In bullet point form what are the main issues preceding the Nike's decision to withdraw?

Review Slides 8 and 9 on industrial relations.

**Question 3.2:** Using an analogous diagram, relabel the horizontal axis so that it measures Bangladesh workers working for American firms versus say Japanese firms, and explain what happens when Nike withdraws from Bangladesh using a similar analysis to what we did in class for the unionized sector. Note that in this case when Nike withdraws from the Bangladesh labor market the demand by American firms shifts to the left. Are Bangladesh workers better off?

**Question 3.3:** Weigh both the costs and benefits of Nike's chosen course of action for Nike.

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## Question 3 on benefits

A firm is considering what package of wages and flexible hours to attract employees who have the same productivity. There are two types of workers: One type, young and wild, benefits from flexible hours, the other type, old and stodgy, not so much. The optimal compensation package for the young and wild is the cheapest combination package of flexibility and wages for the firm that the young and wild are willing to accept. Suppose the old and stodgy would definitely accept the optimal compensation package designed for the young and wild.

**Question 3.1:** Show in a diagram, similar to but not the same as Slide 31 or Slide 33, why it is cheaper to hire the old and stodgy.

Suppose the firm cannot be sure it can meet its human resource needs without offering two contracts,

but is allowed to offer one contract to the old and stodgy and another contract to the young and wild. For example the firm might have one plant in Florida and another in Oregon. We suppose it can meet only half of its workforce requirements by hiring old and stodgies.

**Question 3.2:** Illustrate on a second diagram similar to Slide 36 the ideal two packages it would offer if it could perfectly discriminate.

Of course laws against age discrimination limits the degree to which the firm can exclude one group from accepting contracts it offers another group. Accordingly we now suppose that both groups can take any package the firm offers.

**Question 3.3:** Using a set of diagrams analogous to the figure in Slide 39 , describe the possibilities that the firm faces and how it should legally proceed to minimize the cost of hiring its workforce. Label everything fully.

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## Question 4 on financial incentives

Read the article on the radiologist from Georgia. Perry hospital (not to mention the community it serves), is in need of your services. You should adapt the experiment we ran on major and minor discoveries to this case by setting the value of minor discoveries to zero (meaning the mammogram found nothing), and interpret  $x$  as the amount of additional work and documentation required when the diagnosis indicates evidence of cancer. Perry hospital knows that if it creates proper incentives for its staff, then it will attract demand from further afield from who are worried they have cancer but afraid their concerns will be dismissed at hospitals that are not incentivized to catch early symptoms of cancer (which is much easier and cheaper to treat than at later stages). Thus the probability of "discovery" is anticipated to rise to about ten percent. For argument's sake let us assume that all the other parameters of the model are the same.

**Question 4.1:** What is the optimal contract that should be offered to radiologists like Rachael Rapraeger?