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# ESTIMATING A FIRM'S AGE-PRODUCTIVITY PROFILE USING THE PRESENT VALUE OF WORKERS' EARNINGS\*

LAURENCE J. KOTLIKOFF AND JAGADEESH GOKHALE

In hiring new workers, risk-neutral employers equate the present expected value of a worker's compensation to the present expected value of his/her productivity. Data detailing how present expected compensation varies with the age of hire, therefore, embed information about how productivity varies with age. This paper infers age-productivity profiles using data on the present expected value of earnings of new hires of a Fortune 1000 firm. For each of the five occupation/sex groups considered, productivity falls with age, with productivity exceeding earnings when young and vice versa when old.

Understanding how productivity varies with age is important for a variety of reasons. A decline in productivity with age implies that aging societies must increasingly depend on the labor supply of the young and middle-aged. It also means that policies designed to keep the elderly in the work force, while potentially good for the elderly, may decrease overall productivity. A third implication is that, absent government intervention, employers may not be willing to hire the elderly for the same compensation as younger workers.

Labor economists are particularly interested in the relationship between productivity and age because it can help test alternative theories of the labor market. The simplest such theory is the spot market theory in which workers are paid, at least annually, their marginal product. There appear to be few, if any, economists who view the spot market theory as reasonable. Research by Kotlikoff and Wise [1989] presents fairly strong evidence against the spot market theory; it demonstrates that many, if not most, defined benefit pension plans induce very sharp discontinuities with age in vested pension accrual. Under the spot market theory there should be offsetting sharp discontinuities at these ages in wage compensation. Such offsetting wage discontinuities are not, however, evident in the data.

In contrast to the spot market theory, contract theories of

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labor markets imply only a present value relationship between compensation and productivity. Consider, for example, the contracts that would be written by risk-neutral employers. In these contracts, although earnings in any single year can exceed or be less than that year's productivity, the present expected value of the worker's output will equal the present expected value of his/her compensation.

Different contract theories have different implications concerning the relationship of productivity and wages as the worker ages. One such theory is the specific human capital model of Mincer [1974] and Becker [1975]. It suggests that if firms are free to fire older workers, the age-wage profile will be structured such that earnings exceed productivity when young and vice versa when old. On the other hand, in Becker and Stigler's [1974] and Lazear's [1979, 1981] agency models of worker shirking, the worker receives less than his/her marginal product when young, with the difference paid out in the form of wages, accrued pension benefits, or severance pay in excess of the marginal product when old. The efficiency wage models of Harris and Todaro [1970], Stofft [1982], Yellen [1984], Shapiro and Stiglitz [1984], and Bulow and Summers [1986] provided a view of the labor market similar to that of Lazear. These models stress the payment of above market clearing wages as a mechanism to induce greater worker effort when such effort is not fully observable. As shown by Akerlof and Katz [1989], these models yield predictions identical to the Lazear-Becker-Stigler agency model concerning age-earnings profiles with the difference in the models involving the use of employment fees/performance bonds to clear the market in agency models, but not in efficiency wage models.

The evidence to date on the age-productivity relationship is limited and mixed. The findings for older workers of Medoff and Abraham [1981] that pay increases although indices of productivity decline, suggest wages in excess of marginal products toward the end of the work span. Lazear and Moore [1984] report that the earnings profiles of the self-employed are flatter than those of employees, also suggesting earnings in excess of productivity among older employees. Kahn and Lang [1986], in contrast, examine responses to questions concerning desired hours of work. They suggest that older workers, with earnings in excess of their marginal products, are likely to be hours-constrained by their employers and, therefore, desire to work more. The opposite would be true if earnings of older workers are below their marginal

products. Kahn and Lang's empirical findings support the view that marginal productivity exceeds earnings for older workers.

Knowledge of the difference between age-wage and age-productivity profiles is potentially quite important to the financial valuation of firms.<sup>1</sup> Suppose, for example, that wages are less than productivity for younger workers and greater than productivity for older workers. Then for each firm the excess of its present expected payment of wages to its existing workers less the present expected productivity of these workers—its backloaded compensation—represents an implicit liability. The word implicit refers to the fact that firms do not carry such liabilities on their books. While firms do not report such liabilities, if the market is aware of these liabilities, the firm's market valuation will be less by the amount of these liabilities. Hence, the shapes of the age-compensation and age-productivity profiles are important for determining the ratio of a firm's market value to its replacement costs: its  $q$ . Summers [1981] points out the low values of firms'  $q$ 's in the United States for much of the postwar period. The low  $q$  values are surprising given findings of high price-cost margins, which imply much more market power and higher profits than is indicated by the observed values of  $q$  (see Salinger [1984]). Like Summers' tax adjustments to  $q$ , backloaded compensation may go a long way to reconciling with theory low observed values of  $q$ .

This paper assumes risk-neutral employers and estimates the age-productivity relationship for a single firm using the first-order condition that the present expected value of total compensation equals the present expected value of productivity. Workers hired at different ages have different present expected values of total compensation and, correspondingly, different present expected values of productivity. Hence, if one parameterizes the age-productivity relationship, the parameters of this relationship can be identified from information on how total present expected compensation varies with age.

The data in the study are earnings histories for over 300,000 employees of a Fortune 1000 corporation covering the period 1969–1983. While the firm's name cannot be disclosed, the firm is involved primarily in sales. These data are advantageous not only because one can control for the firm, but also because one can determine precisely the accrued pension compensation arising under the firm's defined benefit pension plan. At particular ages

1. We thank Lawrence Summers for pointing this out.

and amounts of service, pension compensation in this firm is an important component of total compensation.

The results indicate that productivity declines with age and that workers are paid more than they produce when old to offset being paid less than they produce when young. For some occupation/sex groups the difference between productivity and compensation at young and old ages is very sizable. The results support the bonding models of Becker and Stigler [1974] and Lazear [1979, 1981], as well as the efficiency wage models. The results seem less compatible with the Becker-Mincer human capital model.

There are, however, a number of reasons for viewing these results cautiously. First, they apply only to the firm in question. Similar analyses of productivity and compensation profiles for other firms could reach quite different conclusions. Second, the analysis assumes that the form of contracts remained constant over the sample period. Third, the probability of remaining employed is treated as exogenous and time invariant, rather than an endogenous choice of the employer. Fourth, the analysis assumes that the age-productivity relationship has remained constant over a sixteen-year period. Fifth, the results may be subject to selectivity bias if (1) different workers within an occupation group have contracts that differ in ways other than their initial wage and (2) if the composition of workers who join or leave the firm at particular ages is correlated with the characteristics of the contract.

The paper continues as follows. Section I presents the basic methodology. Section II presents the data. Section III examines the results. Section IV briefly considers the potential importance of the findings for firms' values of  $q$ . Finally, Section V states conclusions and suggests additional research.

## I. METHODOLOGY

To understand our multiperiod model and its use in inferring the age-productivity relationship, it may help first to consider a very simple one-good, two-period model with an interest rate of zero. Assume that some workers work both when young and old and that other workers work only when they are old, but that both types of workers are equally productive when old. Further assume that to reduce shirking by young workers, to encourage human capital formation, or for other reasons workers who are hired when young are paid less (more) than their marginal product when young and more (less) than their marginal product when old. Let  $Z_y$

and  $Z_o$  stand, respectively, for the present values of compensation of those hired when young and those hired when old. Since workers who are hired when old are paid their marginal product,  $Z_o$  is also the productivity of older workers, and since  $Z_y$  equals the sum of the marginal products of a worker when he is young and when he is old (recall that the interest rate is zero),  $Z_y - Z_o$  is the productivity of younger workers. Thus, if we know  $Z_y$  and  $Z_o$ , we can infer the age-productivity relationship. If  $Z_y - Z_o > Z_o$ , productivity falls with age; if  $Z_y - Z_o < Z_o$ , productivity rises with age. Note that if workers are paid their productivity each period, this method will also generate the correct age-productivity relationship.

We now consider a multiperiod model in which the interest rate is nonzero, in which workers may leave the firm, and in which productivity, in addition to depending on age, may depend on service and the date the worker is hired, as well as on the worker's individual characteristics. The firm in our model is assumed to have a constant returns production function that depends on capital and labor. Labor input is assumed to differ across workers only in terms of effective units; i.e., the labor input of one worker is a perfect substitute for that of any other, but the amount of effective labor units is different for each worker. The firm is assumed to have full knowledge of the worker's productivity at the time he or she is hired. Let  $Y_t$ ,  $L_t$ , and  $K_t$  stand for output, labor, and capital in year  $t$ , respectively. The concave production function is

$$(1) \quad Y_t = F(L_t, K_t),$$

where

$$(2) \quad L_s = \sum_{j=s-57}^s \sum_{a=18}^{75} N_{j,a} q(a+s-j, a, s) h(a+s-j, a, s).$$

Equation (2) adds up the labor input of workers hired this year and in past years. Specifically, we assume that ages 18 and 75 are the respective minimum and maximum ages of workers. Hence, the firm at time  $s$  has no workers hired before year  $s - 57$ , which is the first year included in the summation. The term  $N_{j,a}$  stands for the number of workers hired in year  $j$  at initial hiring age  $a$ . Of course, not all of the workers hired in the past stay with the firm. The term  $q(a+s-j, a, s)$  denotes the fraction of those workers who are currently age  $a+s-j$ , who joined the firm at age  $a$ , and who have remained with the firm through year  $s$ . Finally,  $h(a+s-j, a, s)$

denotes the productivity in year  $s$  of workers age  $a + s - j$  who joined the firm at age  $a$ .

The expected present value of real profits of the firm at time  $t$ ,  $\pi_t$ , is given by

$$(3) \quad \pi_t = E_t \sum_{s=t}^{\infty} [P_s Y_s - I_{s-t}] R^{s-t} \\ - \sum_{s=t}^{\infty} \sum_{a=18}^{75} N_{s,a} e_{s,a} R^{s-t} - \sum_{s=t-57}^t \sum_{a=18}^{75} N_{s,a} D_{s,a},$$

where  $E_t$  is the expectation operator at time  $t$ ,  $P_s$  is the real price of output in year  $s$ ,  $R$  is one over one plus the real interest rate,  $I_s$  is investment in year  $s$  (i.e.,  $I_s = K_{s+1} - K_s$ ),  $e_{s,a}$  is the present (discounted to year  $s$ ) expected value of compensation payments to workers hired in year  $s$  at age  $a$ , and  $D_{s,a}$  is the present expected value of remaining compensation payments to workers hired at age  $a$  in year  $s < t$ . Equation (3) states that the present expected value of profits equals the present expected value of output, less the present expected value of compensation paid to current and future hires, and less the present expected value of remaining compensation paid to past hires. At time  $t$  the future values of  $P_s$  are uncertain; as a consequence, the future values of  $Y_s$  are also uncertain.

In maximizing the present expected value of profits, firms are constrained to structure compensation payments to provide workers with competitive levels of expected utility. In addition, they may face anti-shirking constraints, requiring that they structure the time path of compensation to reduce or eliminate worker malfesance. Regardless of these side constraints, the first-order condition for hiring workers age  $a$  in year  $t$  is that the present expected value of marginal output equals the present expected value of compensation; i.e.,

$$(4) \quad E_t \sum_{s=t}^{t+75-a} P_s F_{l_s} q(a + s - t, a, t) h(a + s - t, a, s) R^{s-t} = e_{t,a},$$

where  $F_{l_s}$  is the marginal product of labor in year  $s$ . The summation in (4) runs from year  $t$  to the year the worker, who is now age  $a$ , reaches age 75, which is  $75 - a$  years from year  $t$ . The product  $P_s F_{l_s}$  gives the marginal revenue product of one unit of effective labor in year  $s$ . Multiplying this product by  $h(a + s - t, a, s)$  gives the marginal revenue product in year  $s$  of the worker hired at age  $a$  and

who is, in year  $s$ ,  $a + s - t$  years of age. The term  $q(, , )$  adjusts for the probability that the worker hired at age  $a$  in year  $t$  is still with the firm in year  $s$  (when he is age  $a + s - t$ ).

The present expected value of compensation of a worker hired in year  $t$  at age  $a$ ,  $e_{t,a}$ , can be expressed in terms of the time path of future annual compensation. Let  $w(i, a, s)$  stand for the total annual compensation paid to workers who are age  $i$  in year  $s$  and who joined the firm at age  $a$ . Then

$$(5) \quad e_{t,a} = \sum_{s=t}^{t+75-a} w(a + s - t, a, t) q(a + s - t, a, t) R^{s-t}.$$

According to (5), the present expected value of total compensation of the worker who is hired in year  $t$  when he is age  $a$  ( $e_{t,a}$ ) equals the present value sum of the products of annual compensation, given by the  $w(, , )$ 's, times the probabilities, given by the  $q(, , )$ 's, that the worker will remain with the firm until the year in question to collect the compensation.

While the length of employment is uncertain, the assumption of risk-neutral employers and risk-averse workers, whose productive characteristics are fully known by the firm, implies that the actual annual compensation payments (the  $w(, , )$ s in (5)) are specified with certainty at the time the worker joins the firm.

Assuming that the structure of the compensation contract is constant through time, the ratio of compensation at age  $i + 1$  to compensation at age  $i$  is independent of time; i.e.,

$$(6) \quad w(i + 1, a, t) / w(i, a, t - 1) = \mu(i + 1, a).$$

If the age-productivity relationship and the probabilities of departure are also assumed to be time invariant, the third arguments in the functions  $h(, , )$  and  $q(, , )$  can be dropped.

Letting  $\theta_s$  stand for the marginal revenue product in year  $s$  of an effective unit of labor ( $P_s F_{ls}$ ), equations (4), (5), and (6) imply that

$$(7) \quad w(a, a, t) \sum_{s=t}^{t+75-a} \mu(a + s - t, a) q(a + s - t, a) R^{s-t} \\ = \sum_{s=t}^{t+75-a} E_t \theta_s q(a + s - t, a) h(a + s - t, a) R^{s-t}.$$

In (7) the left-hand side expresses the present expected value of compensation payments paid to a worker hired at age  $a$  in year  $t$  in

terms of the worker's first year compensation,  $w(a, a, t)$ , and his expected on-the-job wage growth, which is given by the  $\mu(, , t)$ 's multiplied by the probability of remaining with the firm, the  $q(, , )$ 's, and then discounted.

The assumption of myopic expectations permits writing  $E_t \theta_s = \theta_t$ , and (7) can be expressed as

$$(8) \quad C(a, t) = \theta_t \sum_{s=t}^{t+75-a} q(a + s - t, a) h(a + s - t, a) R^{s-t} \equiv \theta_t H(a),$$

where  $C(a, t)$  stands for the left-hand side of (7) (the present expected compensation of a worker hired at age  $a$  in year  $t$ ). Equation (8) indicates that, based on the stated assumptions, the present expected value of the productivity of a worker hired at age  $a$  can be written as the product of a term involving the firm's expected, as of year  $t$ , overall productivity per unit of effective labor input ( $\theta_t$ ) and a term indicating the present expected number of units of effective labor input of a worker hired at age  $a$ ,  $H(a)$ .

To gain some intuition about the relationship between the present expected value of compensation (the  $C(, , )$ 's) and the age and age of hire productivity relationship (the  $h(, , )$ 's), consider the simple case in which there is a constant probability  $p$  of staying each year with the firm, i.e.,  $q(i, a) = p^{i-a}$ , that  $h(, , )$  depends only on age, i.e.,  $h(i, a) = v(i)$ , and that  $\theta_t$  equals unity (i.e., it is time-invariant). In this case the present expected value of compensation paid to a worker hired at age  $a$  can be expressed as a time-invariant function  $C^*(a)$ , where  $C(a, t) = C^*(a)$ . A little manipulation of (8) now leads to

$$(9) \quad v(a) = C^*(a) - pRC^*(a + 1).$$

Equation (9) expresses the worker's productivity at age  $a$  in terms of the difference in the present value of compensation paid to workers hired at age  $a$  and workers hired at age  $a + 1$ . This equation is the analogue to the difference  $Z_y - Z_o$  in the very simple model discussed above.

The first difference of equation (9) gives the growth in productivity with age; i.e.,

$$(10) \quad v(a + 1) - v(a) = [C^*(a + 1) - C^*(a)] \\ - pR[C^*(a + 2) - C^*(a + 1)].$$

From (9), if the product of the survival rate and the interest rate,  $pR$ , equaled unity, productivity at age  $a$ ,  $v(a)$ , would just equal the

difference in the present expected value of compensation of workers hired at age  $a$  and at age  $a + 1$ . In this case the present expected value of compensation of younger hires would always exceed that of older hires (assuming positive values of  $v(a)$  at all ages). On the other hand, if the annual probability of departing the firm is high,  $pR$  will be much less than unity, and a value of  $C^*(a + 1)$  in excess of  $C^*(a)$  is consistent with positive values of  $v(a)$ .

The formula for changes in productivity with age is given in (10). In some cases one can read the age-productivity relationship from the slope of the age-present expected compensation profile,  $C^*(a)$ , and knowledge that  $pR < 1$ . For example, productivity is constant with age in the range of ages over which the  $C^*(a)$  profile is flat. One can also tell that productivity rises with age over ranges in which  $C^*(a)$  is rising, but at a decreasing rate; the intuition here is that a positive, but flattening slope of  $C^*(a)$  means that the immediate positive slope of  $C^*(a)$  (the difference in  $C^*(a + 1)$  and  $C^*(a)$ ) is due to productivity at age  $a + 1$ ,  $v(a + 1)$ , exceeding productivity at age  $a$ ,  $v(a)$ , rather than to later marginal products exceeding  $v(a)$ . If  $C^*(a)$  is increasing, but at an increasing rate, one cannot say whether productivity at age  $a + 1$  exceeds or falls short of productivity at age  $a$ . Similarly, one can tell that productivity declines with age over ranges of ages in which  $C^*(a)$  declines with age at a decreasing rate. However, if  $C^*(a)$  declines with age at an increasing rate, one cannot tell whether productivity is decreasing or increasing with age.

Returning to the general case, equation (8) can be transformed into an econometric relation by appending a multiplicative error term,  $e^{\epsilon_{a,t,j}}$ , where the subscript  $j$  references the individual worker. The error term can be viewed as a worker-specific productivity factor. Its inclusion in the model means that workers hired at the same age in the same occupation/sex category may have different initial salaries. Hence, the model permits worker heterogeneity as well as selectivity based on the  $e^{\epsilon_{a,t,j}}$   $s$ . While workers hired at particular ages, or in certain years, may be more or less productive than workers hired at other ages or in other years without biasing the results, the model does require the same wage-growth contract and the same departure rates for all workers within an occupation/sex group. Taking logarithms of the resulting expression yields

$$(11) \quad c_{a,t,j} = \log \theta_t + \log H(a) + \epsilon_{a,t,j}.$$

In (11),  $c_{a,t,j}$  is the logarithm of  $C(a,t)$  for worker  $j$  who is age  $a$  in year  $t$ . While  $h(\cdot)$  can, in principle, be parameterized as a function

of service as well as age, in practice, the resulting cumulative age and cumulative service variables are too collinear to estimate separate age and service coefficients. Hence, we parameterize the productivity function  $h(\cdot, \cdot)$  as simply a cubic function of age, and acknowledge that the age-productivity results reported below confound service-productivity effects.<sup>2</sup> Letting  $h(k, a) = \alpha_1 k + \alpha_2 k^2 + \alpha_3 k^3$ ,  $H(a)$  can be written as

$$\begin{aligned}
 (12) \quad H(a) &= \alpha_1 \sum_{s=t}^{t+75-a} q(a + s - t, a)(a + s - t)R^{s-t} \\
 &+ \alpha_2 \sum_{s=t}^{t+75-a} q(a + s - t, a)(a + s - t)^2 R^{s-t} \\
 &+ \alpha_3 \sum_{s=t}^{t+75-a} q(a + s - t)(a + s - t)^3 R^{s-t}.
 \end{aligned}$$

One cannot separately identify all four of the parameters in (11) and (12):  $\theta_t$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ . To see this, substitute from (12) into (11) and divide both sides of the resulting expression by  $\alpha_1$ , and

2. To see why the estimation might confound age and service effects if service as well as age affects productivity, consider the case that productivity at a point in time is a linear function of age and service; i.e., let  $h(k, a) = \beta k + \lambda(k - a)$  (recall that  $k$  stands for age and  $k - a$  for service). Consider first the case that the probability of leaving employment with the firm prior to a given age,  $D$ , is zero, but it is unity after age  $D$ . In this case the function  $H(a)$  is given by

$$H(a) = \sum_{k=a}^D [\beta k + \lambda(k - a)] = \beta a + \frac{\lambda(D - a + 1)(D - a)}{2} = \phi + \left( \beta - \frac{\lambda}{2} - \lambda D \right) a + \frac{\lambda a^2}{2},$$

and the estimation of (8) would yield two coefficients, one for  $a$  (age of hire) and one for  $a^2$  (age of hire squared). The coefficient on  $a$  would combine both  $\beta$  and  $\lambda$  (age and service effects), while the coefficient on  $a^2$  would indicate the effect of service.

Next consider the case of a constant probability  $p$  of remaining with the firm regardless of one's age and  $R$  equals unity, the term  $H(a)$  in (8) would be given by

$$H(a) = \sum_{k=a}^{\infty} p^{k-a} [\beta k + \lambda(k - a)] \equiv \phi + \frac{\beta p a}{1 - p}.$$

In this case the present expected contribution of service to productivity is identical for all hires (and is captured by the constant  $\phi$ ) and the estimation of equation (8) would recover only the coefficient  $\beta$ .

More generally, when we allow for more complicated departure processes as well as productivity functions that are nonlinear in age and service the  $H(a)$  function will be a highly nonlinear function of age and service parameters. Unfortunately, collinearity precludes estimating separate age and service parameters, and it proved necessary to make the identifying assumption of zero service effects. The literature is mixed with respect to the effects of service on wages. Depending on one's model of labor contracts, the findings of Altonji and Shakotko [1987] (but not Lang [1988] or Topel [1988]) that wages do not rise with service may imply that productivity also does not rise with service.

observe that the resulting constant term will equal  $\log \theta_t + \log \alpha_1$ . Since this poses no problem for estimating the age-productivity relationship, the parameter  $\alpha_1$  is normalized to unity. With this normalization and using (12), equation (11) can now be expressed as

$$(11') \quad c_{a,t,j} = \log \theta_t + \log [X_1(a) + \alpha_2 X_2(a) + \alpha_3 X_3(a)] + \epsilon_{a,t,j},$$

where  $X_1(a)$ ,  $X_2(a)$ , and  $X_3(a)$  are the respective sums on the right-hand side of (12). Equation (11') can be estimated nonlinearly. Since time enters only through the intercept term  $\log \theta_t$ , data for workers hired in different years can be pooled by simply entering year dummies. Given the estimated value of the  $\alpha_2$  and  $\alpha_3$  and the normalization  $\alpha_1 = 1$ , we can determine the shape of the  $h(k,a) = \alpha_1 k + \alpha_2 k^2 + \alpha_3 k^3$  function.

## II. THE DATA AND EMPIRICAL IMPLEMENTATION

The large firm's data used in this study are earnings histories covering the period 1969 through 1983 of workers employed in the firm at some time during the period 1980 through 1983. The workers are classified into three rather broad occupation/sex groups: male office workers, female office workers, salesmen, saleswomen, and male managers. There are too few female managers to warrant their analysis. Unfortunately, there are no additional demographic variables that could be included in the analysis. The Appendix presents the distribution of the observations by age of hire and occupation/sex groups.

The firm has a defined benefit plan with a fairly complex set of age- and service-related benefits. The benefit formula is a percent of earnings formula in which the basic retirement annuity equals a percentage rate times the number of years of service for workers with fewer than 26 years of service. For those with more service, the formula equals 25 times the former percentage rate, plus the additional service beyond 25 times a lower percentage rate. The basic benefit is offset by the amount of Social Security benefits the firm predicts the worker will receive. The predicted Social Security benefit is derived from another age- and service-related formula unique to the firm.

The pension plan's normal retirement age is 65, and its early retirement age is 55. For workers who retire after the early retirement age, but before the normal retirement age, there is a special early retirement benefit reduction table that is based on the

worker's age and service. Workers who terminate employment before age 55 are not eligible for the quite generous early retirement reduction rates and face instead actuarially reduced benefits. Another very important penalty for terminating before the early retirement age is that workers retiring after the early retirement age have their Social Security offset deferred until they reach age 65. These provisions of the firm's pension plan can produce quite substantial vested pension accrual at age 55, but rather modest accrual prior to age 55. After age 55 the accrual is much smaller and, indeed, can become negative.

The survival probabilities, the  $q(\cdot, \cdot)$ 's, used in constructing  $c_{a,t,j}$  and the variables in equation (10') and (13) were calculated separately for each of the five age-occupation/sex groups in the following manner. First, the fraction of workers at a given age and initial age of hire who remain in the firm from one year to the next was calculated. Next, these annual survival hazards were smoothed using a second-order polynomial in age, age squared, years of service, years of service squared, and age times years of service. Finally, the cumulative survival probabilities, the  $q(\cdot, \cdot)$ 's, were computed based on the smoothed annual survival probabilities.

The data used in the regressions of annual survival hazards are taken from the years 1980 through 1984. For these years we have complete employment duration data on all workers in our five categories who were employed with the firm at any time during these years. Unfortunately, while we have the complete employment/earnings histories going back to 1969 for those workers hired prior to 1980 who were still employed with the firm from 1980 through 1984, we do not have any information on those workers hired prior to 1980 who did not remain with the firm through 1980. Hence, in forming the empirical hazards, we can only use data from 1980 through 1984. The  $R^2$ 's in these regressions are 0.23 for male office workers, 0.29 for female office workers, 0.12 for salesmen, 0.01 for saleswomen, and 0.21 for male managers. The respective number of observations in these regressions are 1344, 1387, 1274, 630, and 963. The smaller number of observations for saleswomen reflects the fact that we do not have data in certain age and age of hire cells on the fraction of saleswomen remaining with the firm between one year and the next. The missing data typically involve saleswomen hired at older ages and, for a given age of hire, saleswomen who are older. The explanation for the missing data is that most saleswomen in the firm are hired at young ages and have high probabilities of leaving the firm within a few years.

Table I presents the smoothed survival function  $q(\cdot, \cdot)$  for the different occupation/sex groups at selected ages and ages of hire. Table I indicates very substantial differences in job survival rates across the five groups; 34.3 percent of male managers who hire on at age 30 are predicted to remain with the firm 25 years later. For male and female office workers the comparable percentages are

TABLE I  
PREDICTED PROBABILITIES OF REMAINING WITH THE FIRM FROM AGE OF HIRE  
TO SPECIFIED AGE BY OCCUPATION/SEX GROUP<sup>a</sup>

Age of hire	Age				
	25	35	45	55	65
<b>Male office</b>					
20	0.461	0.303	0.335	0.218	0.024
30		0.699	0.485	0.215	0.017
40			0.791	0.334	0.029
50				0.681	0.092
60					0.435
<b>Female office</b>					
20	0.472	0.300	0.289	0.144	0.010
30		0.688	0.420	0.142	0.007
40			0.792	0.298	0.018
50				0.735	0.101
60					0.543
<b>Salesmen</b>					
20	0.286	0.084	0.049	0.020	0.002
30		0.420	0.149	0.054	0.007
40			0.496	0.145	0.021
50				0.480	0.077
60					0.379
<b>Saleswomen</b>					
20	0.301	0.053	0.015	0.004	0.001
30		0.373	0.083	0.023	0.005
40			0.431	0.105	0.026
50				0.467	0.111
60					0.474
<b>Male managers</b>					
20	0.622	0.505	0.488	0.215	0.013
30		0.885	0.768	0.343	0.024
40			0.900	0.431	0.038
50				0.657	0.079
60					0.321

a. These predicted probabilities are based on separate regressions (one for each occupation/sex group) of the annual hazard of those in an age/age of hire cell of remaining with the firm against a second-order polynomial in age, age squared, years of service, years of service squared, and age times years of service.

21.5 and 14.2, respectively. For salesmen and saleswomen the respective percentages are 5.4 and 2.3. The table also demonstrates that workers hired at older ages, at least through age 50, have larger probabilities of remaining in the firm for a given period of time than workers hired at younger ages.

The  $\mu(\cdot, \cdot)$ 's in the above discussion have stood for the growth in total compensation, including pension compensation; but in order to determine the course of pension compensation, first one needs to know the course of nonpension compensation. Hence, the function  $\mu^*(\cdot, \cdot)$ , which gives the growth in nonpension compensation, was first estimated by regressing observed growth rates in earnings, excluding pension compensation, against a second-order polynomial in age, age squared, service, service squared, age times service, age squared times service, service squared times age, and age squared times service squared. In these regressions we used data on workers' earnings histories going back to 1969. We eliminated the first and last year (for those workers who departed) of earnings because we were not sure those earnings represented a full year's nonpension compensation. Hence a worker needs to remain with the firm for at least four years to have his wage growth data included in the regression; e.g., a worker who remains with the firm for only three years will have only one year—his second year—of usable earnings data, which is insufficient to calculate a value for wage growth.

We have a large number of observations in these regressions since each worker who remains with the firm for several years supplies more than one observation on the growth in nonpension compensation. The number of observations in these regressions are 71,903 for male office workers, 132,543 for female office workers, 201,467 for salesmen, 6,482 for saleswomen, and 33,285 for male managers. The smaller number of observations for saleswomen reflects the fact that a much smaller fraction of saleswomen than other types of workers remains with the firm for the four years needed to enter our regression sample. Given the large number of observations and the small number (eight) of regressors, it may not be surprising that the  $R^2$ 's are small. The  $R^2$ 's in these regressions are 0.04 for male office workers, 0.04 for female office workers, 0.01 for salesmen, 0.01 for saleswomen, and 0.03 for male managers.

There is obviously a lot of variation in nonpension compensation as well as the survival hazards that is not dependent on age or age of hire. This does not appear to present a problem for our

analysis because we are interested in determining the expected (ex ante) present value of compensation, not the realized (ex post) present value of compensation. While random factors may raise or lower a worker's survival probabilities or wage growth above or below that which would be forecast ex ante, it is only the ex ante forecast that we need to accurately assess. We should also note, in this context, that despite the low  $R^2$ 's in the survival and wage growth regressions, the predicted survival rates and wage growth rates differ considerably across workers who are in different occupation/sex groups, but were hired at the same age, and across workers in the same occupation/sex group, but were hired at different ages. It is these differences that provide the identification needed for this analysis.

The initial wage together with the smoothed function for growth in nonpension compensation ( $\mu^*(, )$  function) provides a path of nonpension compensation that can be used to calculate the path of pension accrual. The path of nonpension plus pension compensation is then used to form the present expected value of total compensation, the  $c_{a,t,j}S$ .

Table II presents the smoothed nonpension compensation growth rate function  $\mu^*(, )$  for the different occupation/sex groups at selected ages and ages of hire. Table II indicates that the age of hire also is an important factor in real wage growth. According to the regression, workers hired at later ages often experience greater real wage growth than workers hired at younger ages. In addition, wage growth for female office workers and saleswomen at particular combinations of age and age of hire often exceeds that of their male occupational counterparts.

A reduced-form regression can help illustrate the shape of the age-profile of the present expected value of compensation. This regression relates the logarithm of the present expected value of compensation (calculated using the initial wage, the  $q(, )$  survival function, and the  $\mu(, )$  compensation growth function) to a set of year dummies and a polynomial in age. The exponent of the coefficients of this polynomial in age multiplied by their respective variables indicates the shape of the age-present expected value of compensation profile. Figure I presents this profile for each of the five occupation/sex groups normalized by the age 40 level of this profile. Notice that each of the normalized profiles of present expected compensation rises at early ages at a decreasing rate, suggesting, as indicated above, that productivity rises with age at these ages. In addition, each of the profiles, except that of

TABLE II  
 PREDICTED ANNUAL WAGE COMPENSATION GROWTH RATES FOR SPECIFIC AGES  
 AND AGES OF HIRE BY OCCUPATION/SEX GROUP<sup>a</sup>

Age of hire	Age				
	25	35	45	55	65
<b>Male office</b>					
20	0.071	0.028	0.028	0.017	0.005
30		0.047	0.021	-0.002	-0.032
40			0.030	0.003	-0.043
50				0.019	-0.027
60					0.014
<b>Female office</b>					
20	0.047	0.027	0.030	0.007	-0.006
30		0.048	0.008	-0.010	-0.015
40			0.043	-0.001	-0.013
50				0.034	-0.002
60					0.019
<b>Salesmen</b>					
20	0.016	-0.024	-0.013	-0.015	-0.173
30		0.010	-0.008	-0.030	-0.164
40			0.004	-0.025	-0.128
50				0.000	-0.076
60					-0.004
<b>Saleswomen</b>					
20	0.042	0.072	0.076	0.124	0.128
30		0.012	0.023	0.023	0.005
40			-0.004	-0.020	-0.057
50				-0.008	-0.058
60					-0.038
<b>Male managers</b>					
20	0.090	0.054	0.062	-0.009	-0.230
30		0.079	0.026	-0.017	-0.148
40			0.068	0.005	-0.075
50				0.057	-0.010
60					0.047

a. These predicted annual wage compensation growth rates are based on separate regressions (one for each occupation/sex group) of workers' annual earnings growth against a second-order polynomial in age, age squared, service, service squared, age times service, age squared times service, service squared times age, and age squared times service squared.

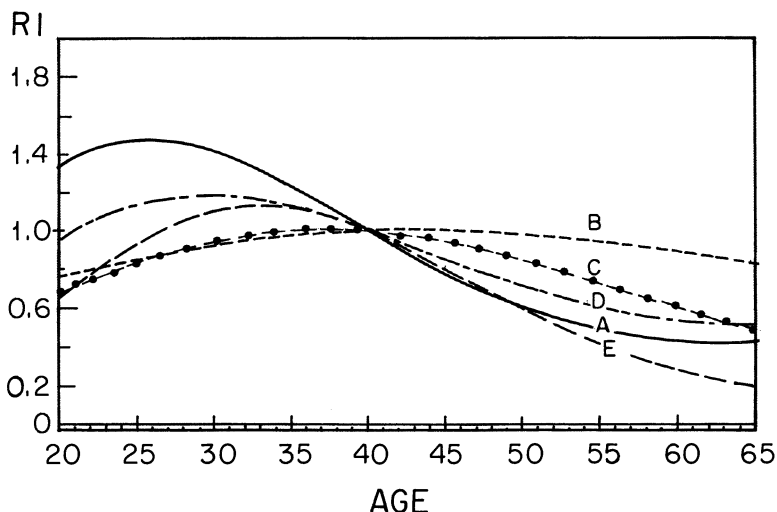


FIGURE I

Relative Profile of Present Expected Compensation

- A = Male managers
- B = Saleswomen
- C = Salesmen
- D = Female office workers
- E = Male office workers.

saleswomen, declines at a decreasing rate in old age, suggesting that productivity declines with age at these ages for at least the other occupation/sex groups.

### III. ESTIMATES OF THE AGE-PRODUCTIVITY PROFILE

Table III presents the regression results from estimating (11'), assuming a 6 percent interest rate. Recall that this regression relates the logarithm of the present expected value of compensation to year dummies and the logarithm of the sum of three nonlinear functions of age multiplied by three coefficients, one of which is normalized to unity. In this regression only observations on workers hired during the years 1970 through 1983 are included, since pension accrual for workers hired prior to 1970 could not be determined. All of the age-squared and age-cubed coefficients reported in the table are highly significant. Many of the year dummies are also significant, suggesting that the modeling of expectations of future  $\theta$ 's may be important. The regression

TABLE III  
AGE-PRODUCTIVITY REGRESSIONS<sup>a</sup>

Variable	Males			Females	
	Office workers	Salesmen	Managers	Office workers	Saleswomen
$\alpha_2$	50.530 (1.105)	38.343 (0.618)	97.888 (3.336)	27.721 (0.426)	45.183 (9.696)
$\alpha_3$	-0.744 (0.166E-1)	-0.573 (0.942E-2)	-1.578 (0.561E-1)	-0.416 (0.645E-2)	-0.678 (0.145)
D71	-0.951E-2 (0.304E-1)	0.240E-1 (0.219E-1)	0.346E-2 (0.452E-1)	-0.515E-1 (0.210E-1)	-0.674 (0.244)
D72	0.438E-1 (0.291E-1)	0.189E-1 (0.222E-1)	-0.248E-1 (0.454E-1)	0.357E-1 (0.208E-1)	-0.520 (0.238)
D73	0.323E-1 (0.285E-1)	-0.142E-1 (0.220E-1)	-0.253E-1 (0.451E-1)	0.558E-1 (0.190E-1)	-0.646 (0.229)
D74	-0.268E-1 (0.288E-1)	-0.485E-1 (0.204E-1)	-0.125E-1 (0.416E-1)	0.718E-2 (0.188E-1)	-0.565 (0.221)
D75	0.272E-1 (0.281E-1)	-0.462E-1 (0.196E-1)	-0.727E-1 (0.411E-1)	0.915E-1 (0.185E-1)	-0.484 (0.219)
D76	-0.379E-1 (0.261E-1)	-0.685E-1 (0.191E-1)	-0.584E-1 (0.411E-1)	0.331E-1 (0.173E-1)	-0.423 (0.217)
D77	-0.325E-1 (0.256E-1)	-0.874E-1 (0.184E-1)	-0.545E-1 (0.406E-1)	0.843E-1 (0.171E-1)	-0.435 (0.215)
D78	-0.734E-1 (0.257E-1)	-0.172 (0.179E-1)	-0.589E-1 (0.396E-1)	0.356E-1 (0.168E-1)	-0.530 (0.215)
D79	-0.139 (0.258)	-0.268 (0.181E-1)	-0.223 (0.409E-1)	-0.108E-1 (0.168E-1)	-0.604 (0.215)
D80	-0.153 (0.265)	-0.258 (0.181E-1)	-0.163 (0.389E-1)	-0.099E-1 (0.167E-1)	-0.578 (0.215)
D81	-0.160 (0.252E-1)	-0.303 (0.179E-1)	-0.265 (0.389E-1)	0.189E-1 (0.165E-1)	-0.618 (0.215)
D82	-0.440E-1 (0.266E-1)	-0.366 (0.176E-1)	-0.219 (0.391E-1)	0.683E-1 (0.174E-1)	-0.637 (0.215)
D83	-0.176 (0.271E-1)	-0.417 (0.174E-1)	-0.149 (0.446E-1)	-0.180E-1 (0.173E-1)	-0.698 (0.215)
Number of obser.	7,083	19,696	2,116	20,753	3,217
$R^2$	0.276	0.075	0.204	0.134	0.086

a. Regressions of logarithm of the present value of compensation (assuming a 6 percent interest rate) against year dummies and the logarithm of the sum of three nonlinear functions of age. D71-D83 are the year dummies. The coefficients  $\alpha_2$  and  $\alpha_3$  multiply two of the three nonlinear functions of age (see equation (11')).

coefficients are little affected by the choice of interest rate; the regressions were repeated assuming interest rates of both 3 percent and 9 percent, and the coefficients are very similar to those reported in Table III.

Figures II through VI are based on the 6 percent interest rate

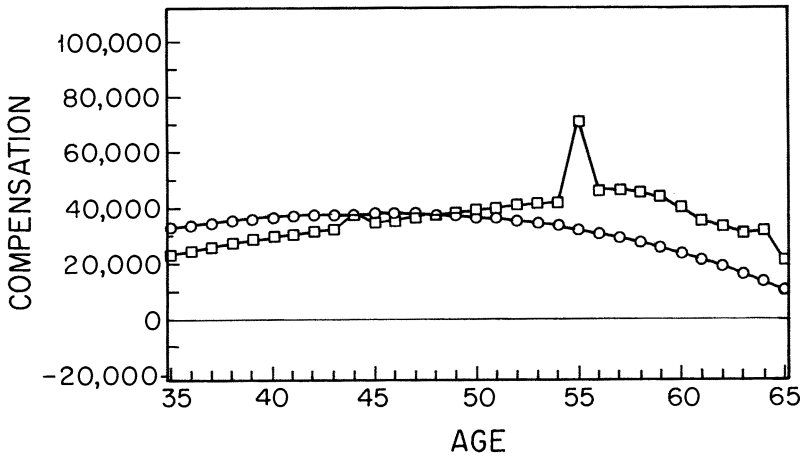


FIGURE II  
 Total Compensation and Productivity Profiles (1980 Dollars) Present Value = 500,000;  $R = 6$  percent, Male Office Workers  
 □ = Total compensation  
 ○ = Productivity.

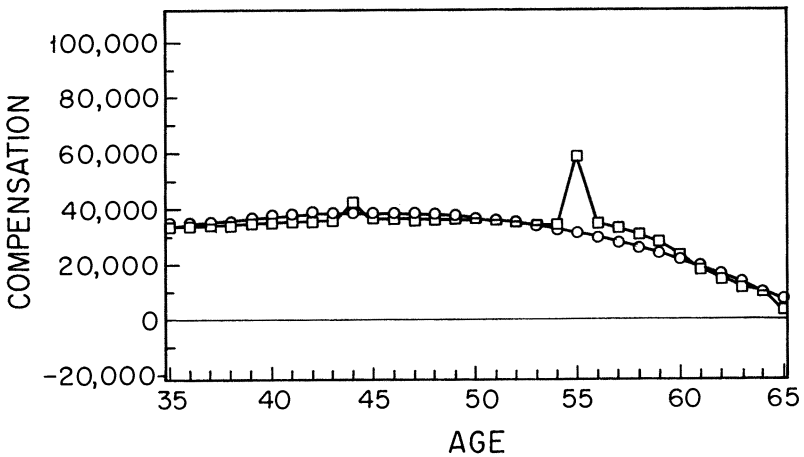


FIGURE III  
 Total Compensation and Productivity Profiles (1980 Dollars) Present Value = 500,000;  $R = 6$  percent, Male Salesworkers  
 □ = Total compensation  
 ○ = Productivity.

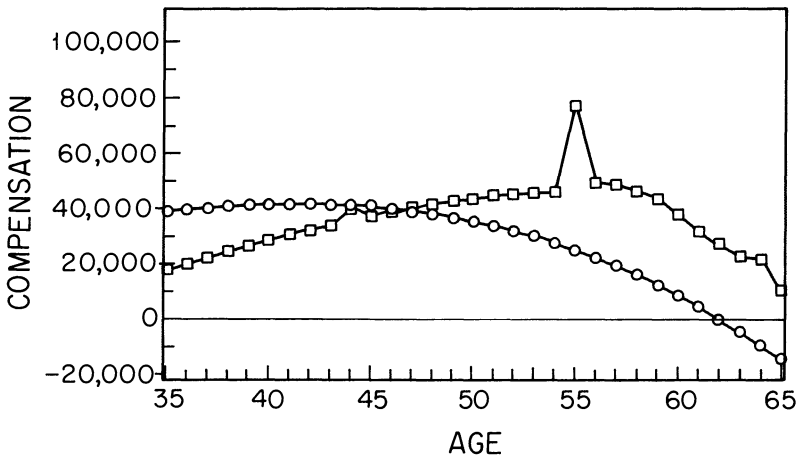


FIGURE IV

Total Compensation and Productivity Profiles (1980 Dollars) Present Value = 500,000;  $R = 6$  percent, Male Manager  
 □ = Total Compensation  
 ○ = Productivity.

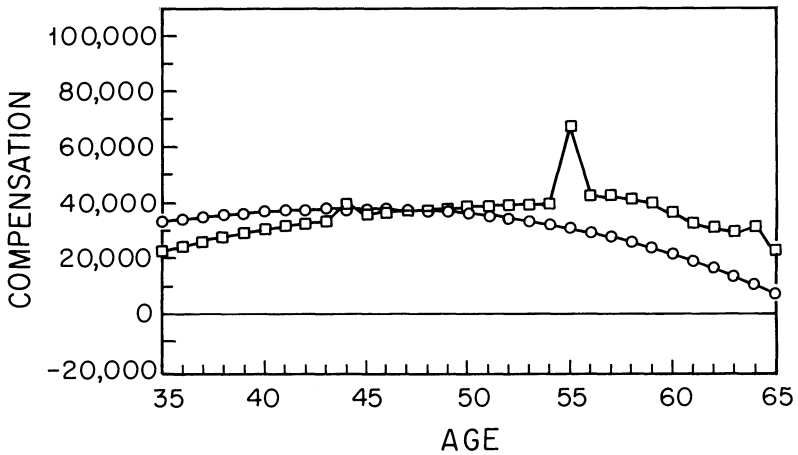


FIGURE V

Total Compensation and Productivity Profiles (1980 Dollars) Present Value = 500,000;  $R = 6$  percent, Female Office Workers  
 □ = Total Compensation  
 ○ = Productivity.

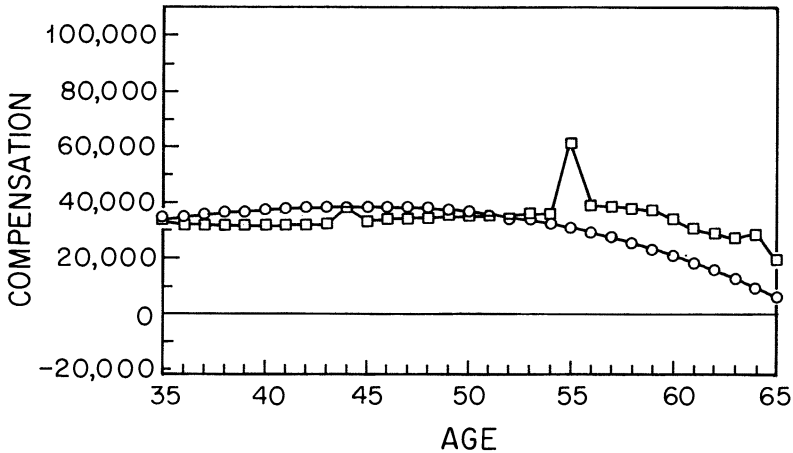


FIGURE VI

Total Compensation and Productivity Profiles (1980 Dollars) Present Value = 500,000;  $R = 6$  percent, Female Salesworkers

- = Total Compensation  
○ = Productivity.

regressions of Table III. They present the age-productivity profiles (dashed lines) predicted by the regressions for the five occupation/sex groups for workers hired initially at age 35. They also present the age-total compensation profile implied by the smoothed compensation growth function  $\mu(\cdot, \cdot)$ 's and the pattern of pension accrual. The age 35 initial level of productivity ( $\theta_t$  in equation (8)) and compensation ( $w(a, a, t)$  in equation (7)) are chosen to insure that both the present expected value of compensation and the present expected value of marginal product equal \$500,000.

While productivity initially rises with age in each diagram, it eventually starts declining with age. For male office workers productivity peaks at age 45 and declines thereafter. For this group age 65 productivity is less than one third of peak productivity. The female office workers' productivity profile is quite similar to that of the male office workers. The productivity profiles of both the salesmen and saleswomen peak a few years later than those of office workers, but their rate of decline with age is quite similar. Productivity for male managers peaks at age 43; by age 60 productivity is less than one third of peak productivity, and productivity actually becomes negative after age 62.

In four of the diagrams productivity exceeds total compensation while the worker is young and then falls below total compensa-

tion; in the remaining case, that of salesmen, the relationship of compensation and productivity is quite similar to the other four groups, except after age 61 when productivity again exceeds compensation. Except for the kinks in the age-compensation profiles associated with pension accrual, the age-compensation profiles and age-productivity profiles for salesmen and saleswomen are very close to one another at each age. This is what one would predict since salesworkers in this firm are paid, in large part, on a commission basis.

In contrast, to the results for salesworkers, one might expect the weakest connection between annual earnings and annual productivity among male managers. Figure IV indicates that this is indeed the case. At age 35 productivity for male managers exceeds total compensation by more than a factor of two, while compensation is over twice as high as productivity by age 57. The discrepancies between total compensation and productivity at these ages are somewhat smaller for office workers, but still quite important. For example, age 35 total compensation for female office workers is \$22,616, while age 35 productivity is \$33,604. In contrast, age 57 total compensation is \$42,526, although productivity is only \$28,117.

The results depicted in Figures II through VI are not sensitive to the inclusion of pension accrual in total compensation; if one ignores pension accrual in the estimation, the age-earnings and age-productivity profiles have the same relative shapes as those presented. Of course, the age-earnings profile does not exhibit the kinks of the age-total compensation profile since these kinks arise from pension accrual. If one ignores pension accrual, one can then use the data on workers hired prior to 1970. While the initial wage of those hired prior to 1969 is not reported, this wage can be inferred based on the wage observed in 1969 and the compensation growth function  $\mu(\cdot)$ ; i.e., one can impute backwards the wage at the initial age of hire. The results based on this larger data set are very similar to those presented in Figures II through VI. The general shapes of the age-total compensation profiles and age-productivity profiles are also insensitive to the choice of interest rate.

Another concern about the results is the extent to which the profiles described here as age-productivity profiles confound service-productivity effects. Unfortunately, the collinearity between cumulated service and age variables precludes modeling the  $h(\cdot, \cdot)$  function as a continuous function of both age and age of hire. An

alternative way to explore this issue is to model  $h(\cdot)$  as depending only on age, but to estimate the model separately for workers hired at different ages. If one estimates the model separately for those hired prior to age 35 and those after age 35, the resulting general shapes of the productivity profiles are quite similar to those based on the entire sample. The post-age 35 profiles are indeed very similar, while the pre-age 35 profiles exhibit a steeper decline in productivity with age, with negative predicted productivity after roughly age 55. This prediction of negative productivity late in the work span may simply represent a poor fit in the tail of the estimated polynomial.

IV. CAN DIFFERENCES IN AGE-PRODUCTIVITY AND AGE-COMPENSATION PROFILES EXPLAIN LOW VALUE OF FIRMS'  $q$ 'S?

In paying workers less than their productivity when young, firms incur implicit obligations to pay their workers more than their productivity when workers are old. While this implicit financial obligation does not show up on a firm's books (given standard accounting practices), it will be reflected in the firm's market value, making the ratio of the market value of a firm to the replacement cost of its capital less than unity.

To see why deferred labor obligations reduce  $q$ , consider equation (3'), the expression for the firm's market value (present value of expected profits) in year  $t$ ,  $\pi_t$ , and equation (4), the firm's rule for hiring new workers:

$$(3') \quad \pi_t = E_t \sum_{s=t}^{\infty} [P_s Y_s - I_s] R^{s-t} - \sum_{s=t}^{\infty} \sum_{a=18}^{75} N_{s,a} e_{s,a} R^{s-t} - \sum_{s=t-57}^t \sum_{a=18}^{75} N_{s,a} D_{s,a}.$$

Recall that  $E_t$  is the expectation operator at time  $t$ ,  $P_s$  is the real price of output  $Y_s$  in year  $s$ ,  $R$  is one over one plus the real interest rate,  $I_s$  is investment in year  $s$  (i.e.,  $I_s = K_{s+1} - K_s$ ),  $e_{s,a}$  is the present (discounted to year  $s$ ) expected value of compensation payments to workers hired in year  $s$  at age  $a$ ,  $N_{s,a}$  is the number of workers hired at age  $a$  in year  $s$ , and  $D_{s,a}$  is the present expected value of remaining compensation payments to workers hired at age  $a$  in year  $s < t$ . In (3') output in year  $s$ ,  $Y_s$ , may be written as the marginal product of labor in year  $s$ ,  $F_{ls}$ , times the supply of labor,  $L_s$ , plus the marginal product of capital in year  $s$ ,  $F_{ks}$  times the

supply of capital,  $K_s$ . Dividing (3') by  $K_t$  and applying the first-order condition (4) leads to expression (14) for  $q_t = \pi_t/K_t$ :

$$(14) \quad q_t = E_t \sum_{s=t}^{\infty} \frac{[P_s F_{ks} K_s - I_s] R^{s-t}}{K_t} \\ + E_t \sum_{s=t}^{t+56} P_s F_{ls} \sum_{j=s-57}^s \sum_{a=18}^{75} \frac{N_{j,a} q(a+s-j, a, s) h(a+s-j, a, s)}{K_t} \\ - \sum_{s=t-57}^t \sum_{a=18}^{75} \frac{N_{s,a} D_{s,a}}{K_t}.$$

Equation (14) indicates that  $q_t$ , the ratio of the firm's market value to its replacement cost, equals (a) the present value of expected total returns from current and future capital less the present value costs of current and future investment—all divided by  $K_t$ , plus (b) the present value of expected productivity of labor hired prior to year  $t$ , less (c) the present value of compensation still owed to labor hired prior to year  $t$ . If the labor market were a spot market, then the present expected value of workers' future productivities would equal the present expected value of workers' compensation, since each year's compensation would equal each year's productivity. In this case the last two terms in (14) would cancel, and  $q$  would simply equal the expected present discounted value of returns to capital less the cost of investment. With the condition that the marginal revenue product of capital in year  $s$  equals the interest rate, it is easy to show that the firm's market value at time  $t$ ,  $\pi_t$ , simply equals  $K_t$ , the replacement value of its capital; i.e., in the case of a spot labor market (and ignoring capital adjustment costs and inframarginal capital income taxes) the firm's  $q$ —the ratio of its market value to its replacement cost—equals unity.

While the firm's  $q$  is unity assuming a spot labor market, it is less than unity if the firm pays its workers less than their productivity when the workers are young and more than their productivity when the workers are old. To see this, note that the difference between the last two terms in (14) equals the present value difference between the productivity and compensation of all existing workers at time  $t$  divided by  $K_t$ . Since each of these workers was hired subject to the first-order condition that productivity equals compensation in present value over the work span, and since each of these workers was underpaid at some point in the past, the difference for each worker between the present value of his future productivity and his compensation will be negative (this

ignores unexpected changes in the firm's price of output and production technology and assumes that productivity and compensation profiles cross only once). Hence,  $q$  in this case will be less than unity.

In determining the amount of backloaded compensation (the present value difference between expected future compensation and productivity), we considered each of the workers in our data in 1980 with at least one year of service. For each of these workers we first determined their past (back to their age of hire) and future wage earnings using their 1980 reported earnings and our calculated wage compensation growth profile. To this absolute wage compensation profile we added the appropriate yearly pension accrual. We then calculated the present value of the worker's total expected compensation as of his date of hire. Next we adjusted the level of the worker's age-productivity profile such that the present expected value of the absolute level of productivity as of the worker's age of hire equals the present expected value of the worker's total compensation as of his age of hire. Benchmarking the productivity profile against the compensation profile in this manner provides us with the worker's level of productivity in 1980 and all future years. We use the 1980 and subsequent productivity and compensation levels to compute the present value difference between expected future compensation and productivity.

To get a rough feeling for the potential impact on  $q$  of backloaded compensation, denote the difference between the last two terms in (14) multiplied by  $K_t$  as  $B_t$ , the present value of backloaded compensation, and denote  $Z_t$  as total year  $t$  compensation payments to the firm's workers. We can now write

$$(15) \quad q_t = 1 - (B_t/Z_t)(Z_t/rK_t)r.$$

In evaluating (15), we assume that  $Z_t/rK_t$ , the ratio of current earnings to capital income equals four, the national average. We also assume a value of the interest rate  $r$  equal to 0.1. Then  $q_t$  equals unity minus 0.4 times the ratio of the year  $t$  present value of backloaded compensation to total compensation payments in year  $t$ . If this ratio equals 1 (0.5), it means that backloaded compensation can explain a value of  $q$  that differs from unity by 0.4 (0.2). For all the workers included in our data in 1980, which do not include all the firm's employees, the ratio of  $B_t$  to  $Z_t$  equals 1.16. It equals 2.29 for male office workers, 1.38 for female office workers, 4.88 for male managers, -0.30 for salesmen, and 0.76 for saleswomen. While additional data which are not available would be needed to

assess fully the impact of backloaded compensation on the firm's value of  $q$ , the values of  $B_t/Z_t$  for the five occupation/sex groups are sufficiently large to suggest an important role for backloaded compensation in the firm's value of  $q$ .

## V. CONCLUSION

The findings that productivity decreases with age must be viewed cautiously. Contrary to what has been assumed, it may be that some workers within an occupation/sex category receive contracts that are different from the contracts of others. Suppose that within an occupation/sex category there are A and B type workers and that A type workers receive contracts with steeper compensation profiles than B type workers. Also assume that A type workers have smaller probabilities of remaining with the firm than B type workers. If the compensation of workers remaining with the firm changes, the estimated compensation growth function and the estimated job survival function would differ from those for either A or B separately or from those that would arise if the separate job survival and compensation growth functions for A and B were averaged using constant weights. As a consequence, the age-productivity profile derived using the method presented here could differ substantially from either the profile for group A workers or the profile for group B workers. Similar biases may arise if the composition of type A and type B workers among new hires changes as the age of hire increases. These potential biases need to be explored more formally as does the possible bias arising from assuming static expectations of overall worker productivity.

These concerns notwithstanding, the results are fairly striking. Productivity falls with age; compensation at first lies below and then exceeds productivity; and the discrepancy between compensation and productivity can be very substantial. Interestingly, there is a much closer correspondence of productivity to compensation for salesworkers, who are compensated more on a spot market basis, than for other types of workers. Also, the relationship of productivity to compensation is weakest for male managers, who, one would expect, are most likely to be hired on a contract rather than a spot market basis. In addition to confirming contract theory, the results lend support to the bonding wage models of Becker and Stigler [1974] and Lazear [1979, 1981].

Finally, the results may help explain low ratios of firms' market values to the replacement costs (their  $q$ 's) of their capital.

When future compensation exceeds future productivity for a firm's workers, as is the case for the firm considered here, this difference represents a liability, which, presumably, will be reflected in a lower market value of the firm and a lower value of  $q$ . While the results reported here must be viewed cautiously, if for no other reason than they apply to only a single firm, they raise the possibility that backloaded compensation is an important determinant of firms'  $q$ 's.

APPENDIX:  
DISTRIBUTION OF WORKERS BY AGE OF HIRE AND OCCUPATION/SEX GROUP<sup>a</sup>  
(PERCENT OF WORKERS HIRED IN GIVEN AGE RANGE)

	< 20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55+
Male									
managers	0	31.0	33.9	19.1	9.2	4.6	1.6	0.5	0.1
Salesmen	0	18.4	28.1	22.5	14.6	9.1	4.6	1.9	0.6
Saleswomen	0	12.1	19.7	20.3	21.1	15.2	8.0	3.1	0.5
Male office									
workers	0	45.3	29.0	11.8	5.3	3.2	2.3	1.9	1.0
Female office									
workers	0	44.9	17.3	11.4	9.7	7.5	5.0	2.9	1.2

a. Rows may not add to 100 percent due to rounding. This table is based on 3,860 male managers, 25,858 salesmen, 2,054 saleswomen, 9,220 male office workers, and 22,361 female office workers.

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