Lecture 2
Auction Formats

There are many different types of auctions. We explain some more common formats and investigate how to bid optimally. Aside from the auction format, optimal bidding depends on how correlated valuations are for the object up for auction.

There are different types of auctions

- In a first price sealed bid auction, each bidder submits his/her bid without knowing what the others are bidding, and the auctioneer sells the good to the highest bidder at the price he submitted.
- In an English auction bidders compete against each other by raising the price until everyone but one bidder drops out of the bidding.
- In a Dutch auction, the auctioneer reduces the price until a bidder indicates he/she is willing to take the object.
- In a second priced sealed bid auction, players simultaneously submit their bids, the highest bidder wins the auction, and pays the second highest bid.

Bidding strategies

- Does it matter what form the auction takes?
- Returning to basics, from SCM (45-970) a strategy is a complete description of instructions to be played throughout the game
- The strategic form of a game is the set of alternative strategies to each player and their corresponding expected payoffs from following them.
- Two games are strategically equivalent if they share the same strategic form.
- In strategically equivalent auctions, the set of bidding strategies that each potential bidders receive, and the mapping to the bidder's payoffs, are the same.

Common value auction: Oil field tract

- Consider a new oil field tract that drillers bid for after conducting seismic their individual explorations.
- The value of the oil field is the same to each bidder, but unknown. The n^{th} bidder receives a signal s_n which is distributed about the common value v, where

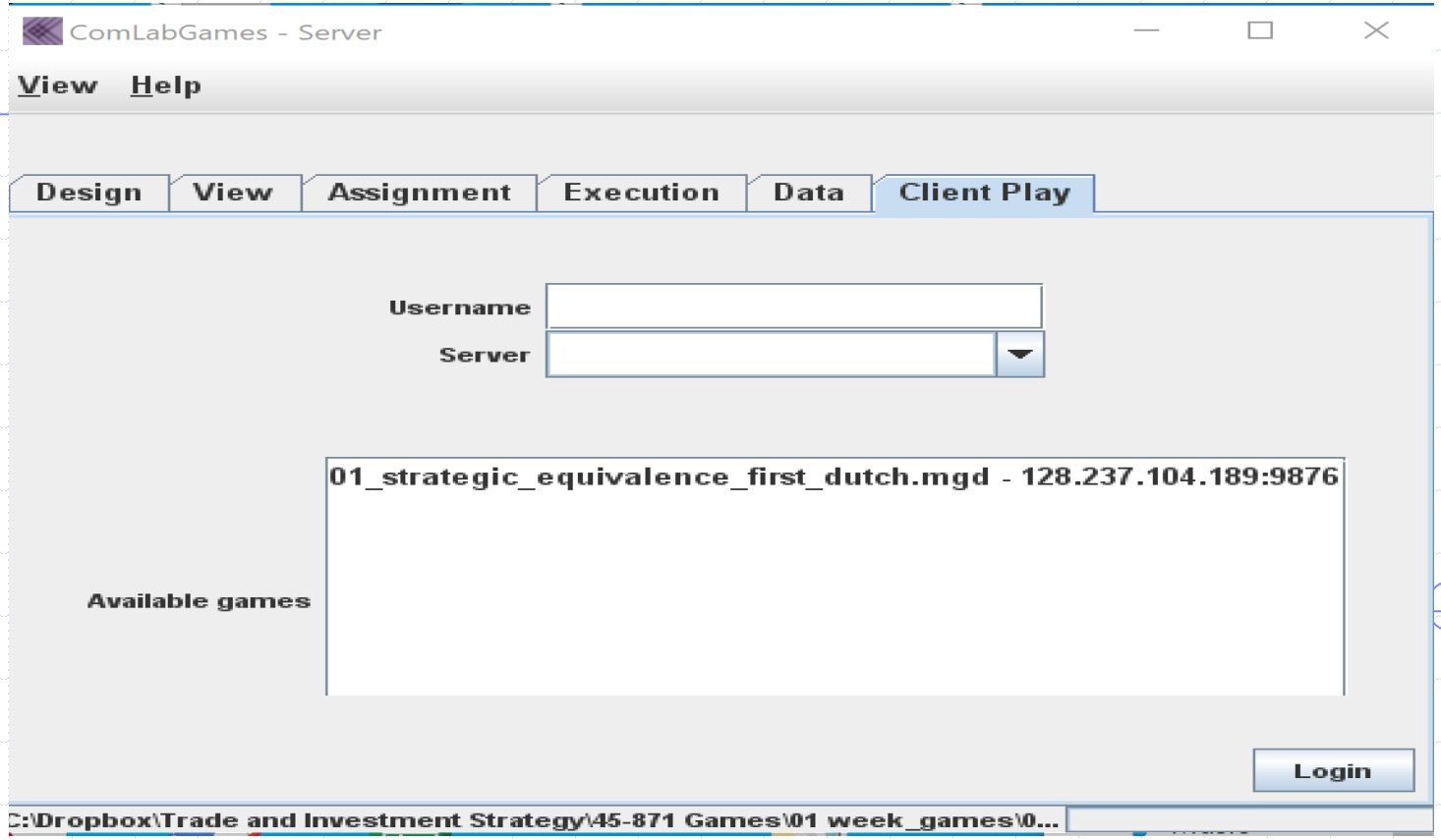
$$s_n = v + \varepsilon_n$$

and $\varepsilon_n \equiv E[v|\ s_n] - v$ is independently distributed across bidders.

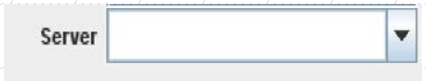
- Notice that each drilling company would have more precise estimates of the common valuation from reviewing the geological survey results of their rivals.

Login Instructions

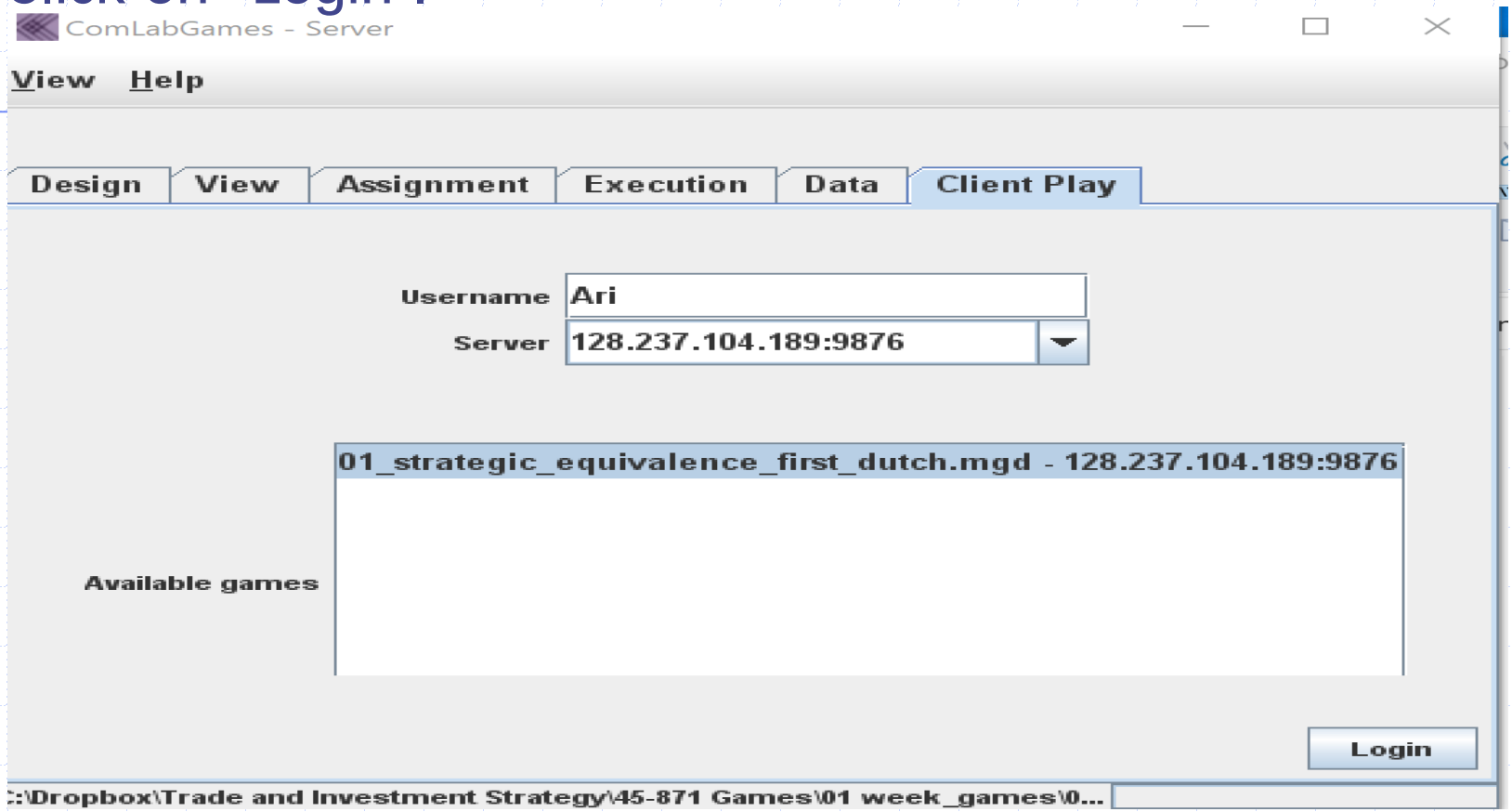
1. Click on Client Play
2. Click on Available games: “01_strategic_equivalence...”



Continuing Login Instructions

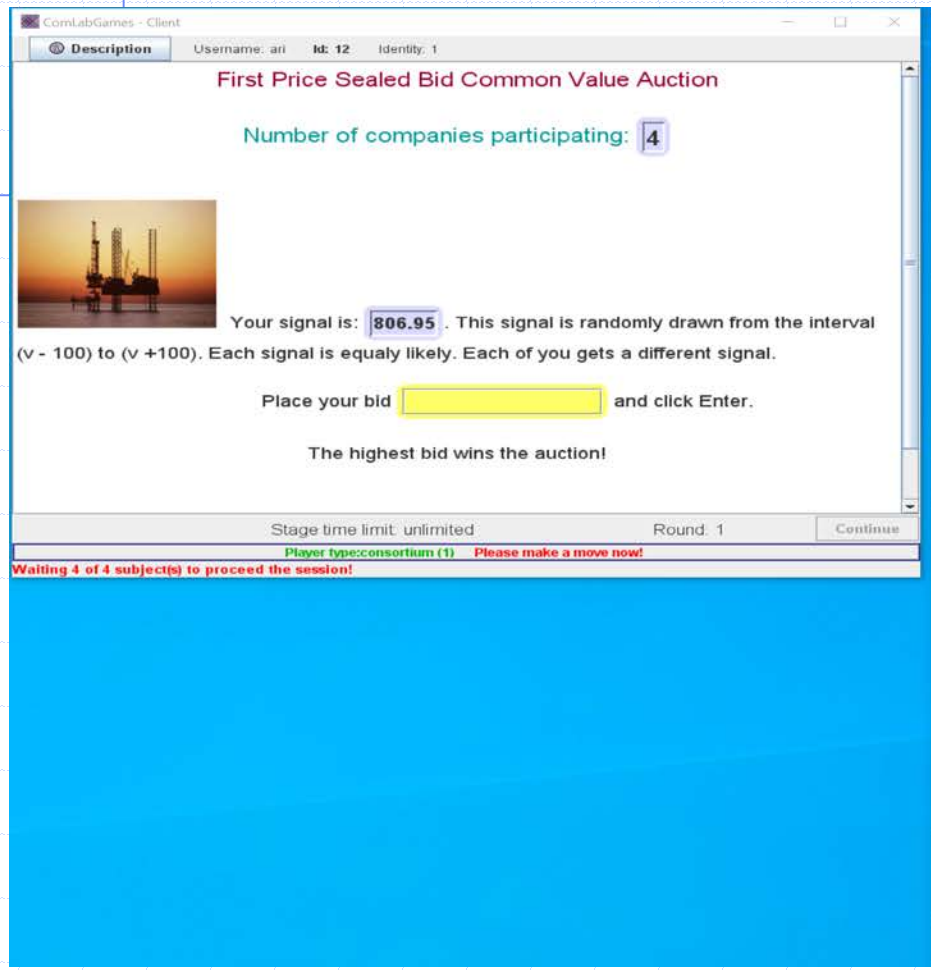
1. Server address: 128.237.104.189 will appear automatically after selecting “01_strategic...”. If not type 128.237.104.189 in 

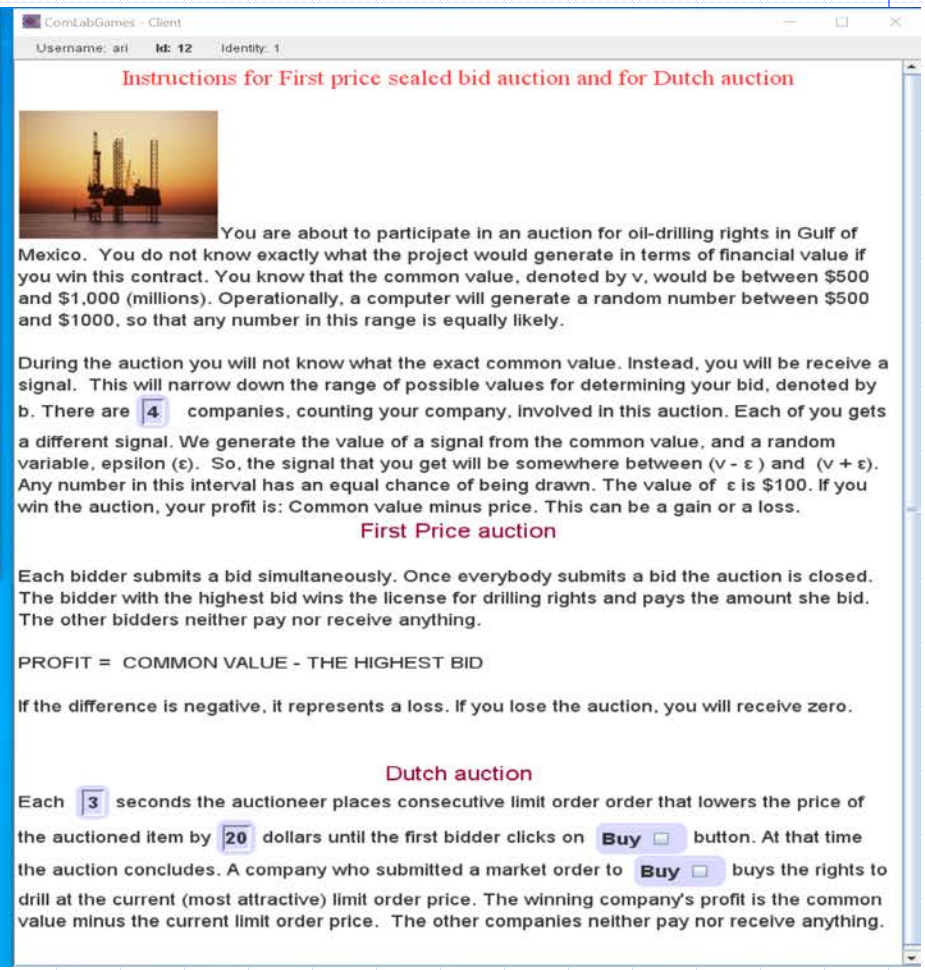
.
2. Write a username.
3. Click on “Login”.



Auction window and Instruction window

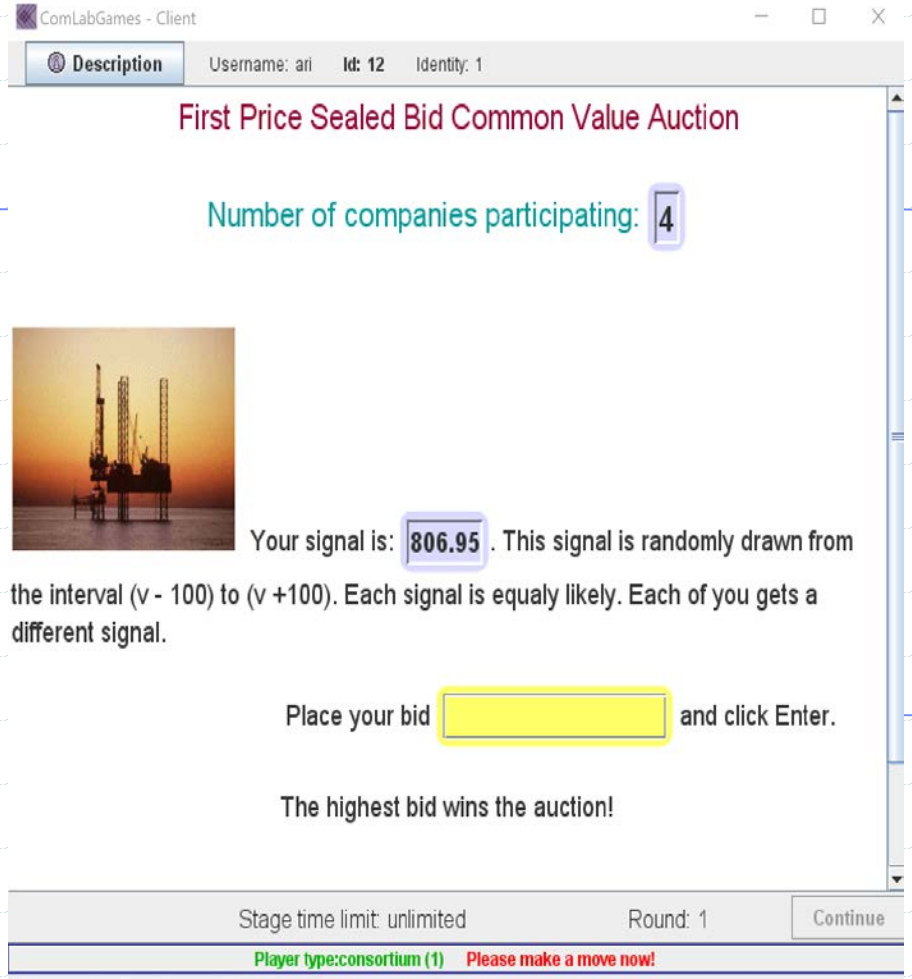
1. Instruction and auction window appears on your screen.
2. To close the instruction window click on “x”. To retrieve it click on “Description”.

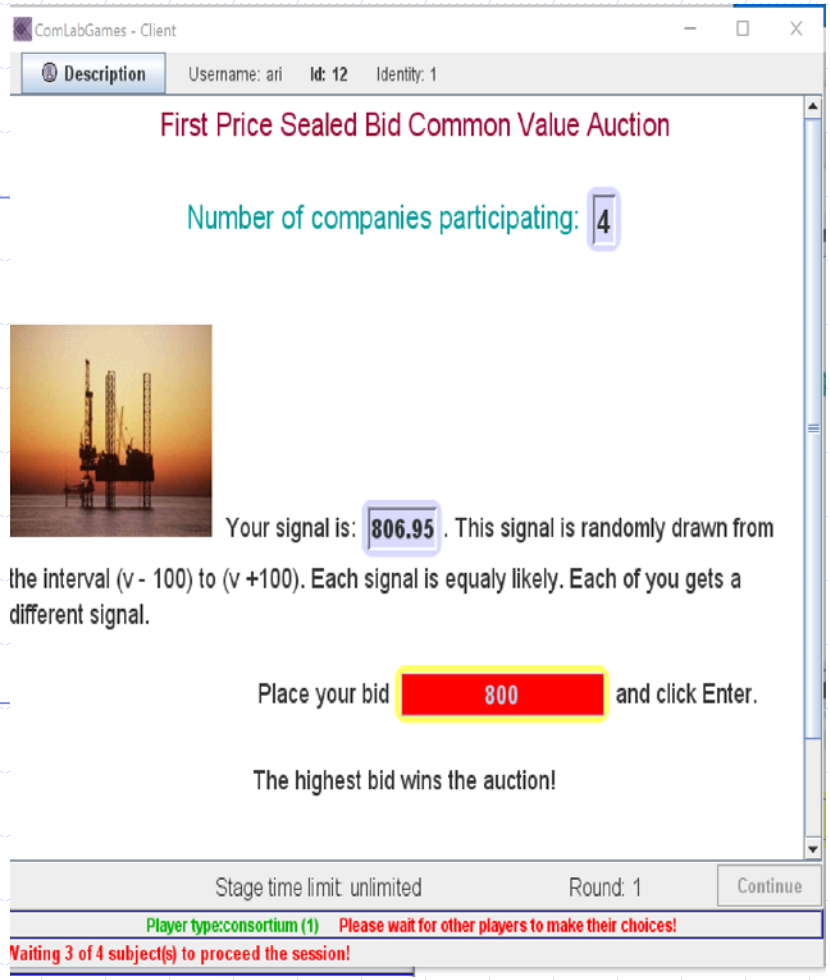




Write a bid in First price sealed bid auction

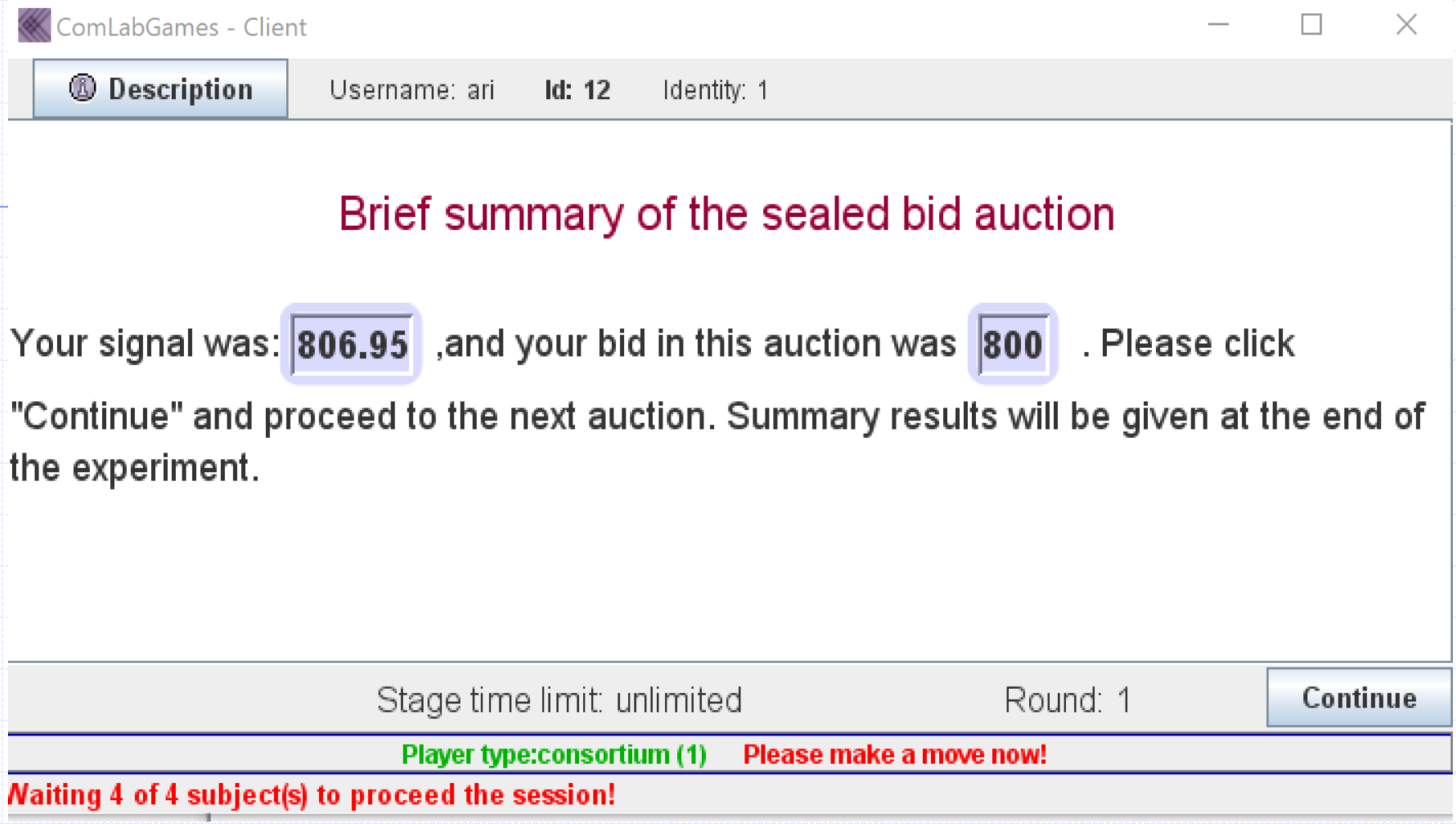
1. Write a number and click Enter
2. Your bid should be colored in red.
3. Wait for all players in your session to submit bids.





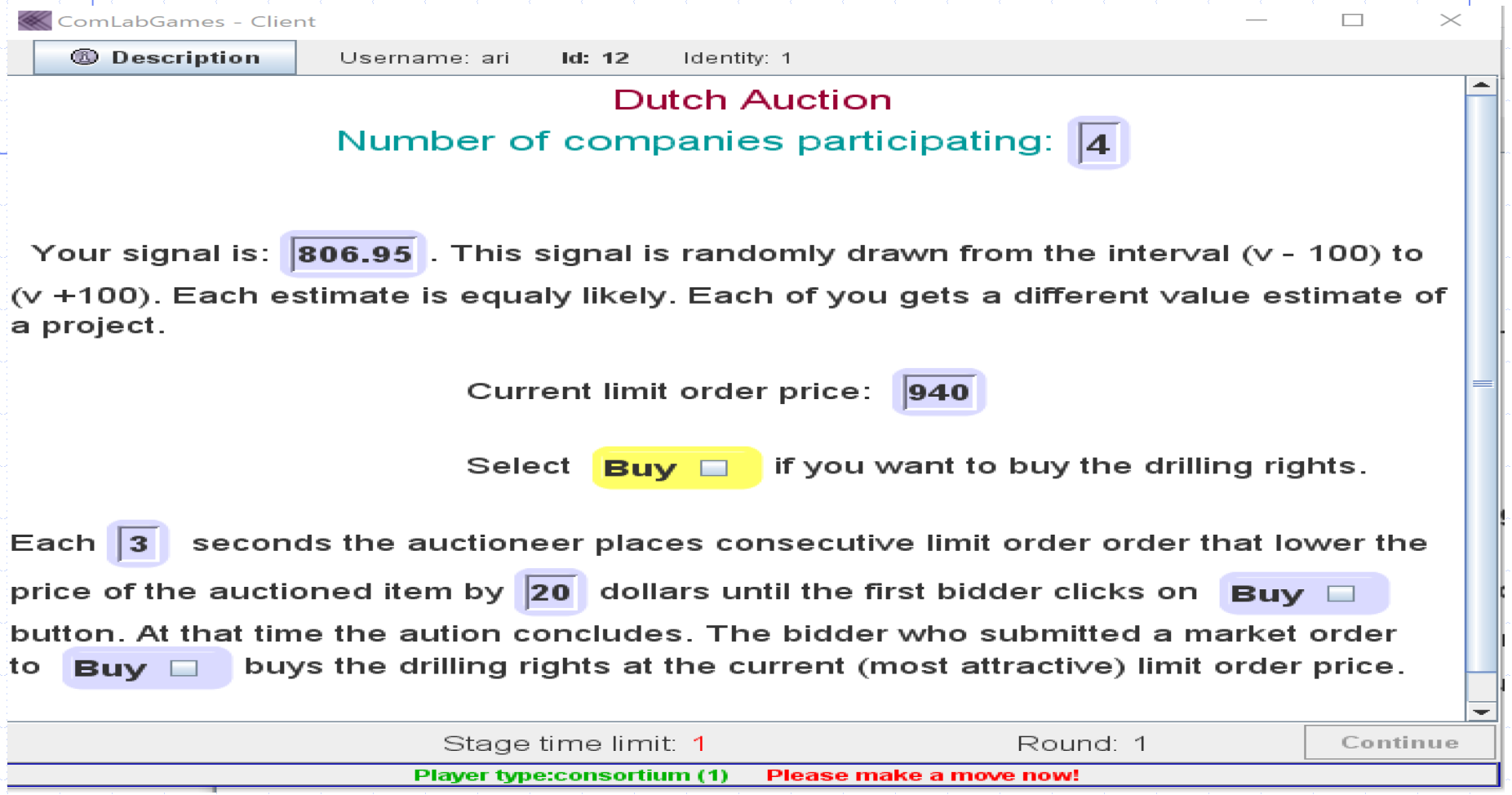
Summary page for sealed bid auction

1. Click “Continue” to move to Dutch auction.
2. Wait for all players in your session to click “Continue”.

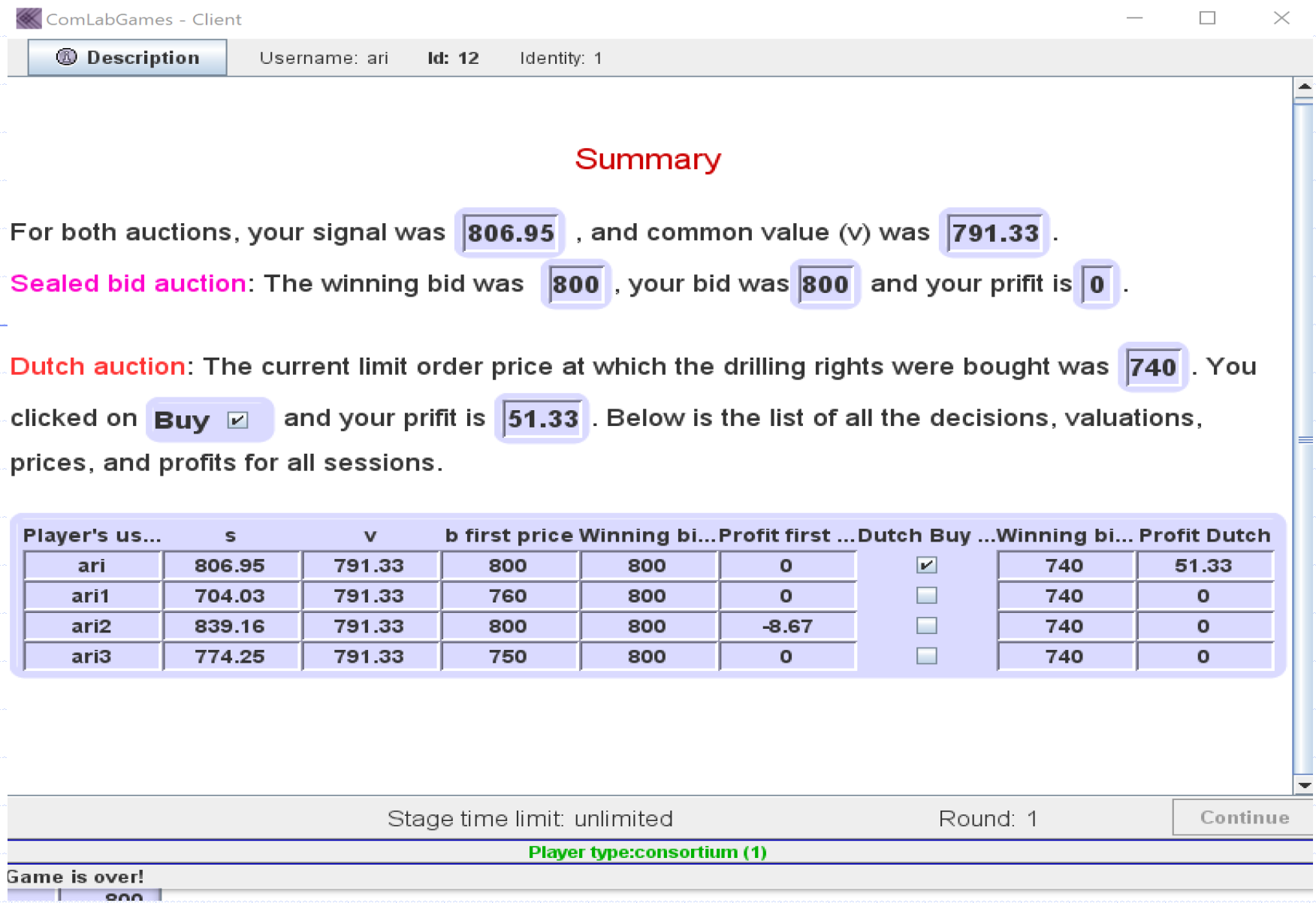


Dutch auction page

1. Every 3 seconds limit order price is lowered by $20.
2. The first subject in a session who click on Buy buys the drilling rights.



Summary page



The expected value of the item upon winning the auction

- If the n^{th} bidder wins the auction, he realizes his signal exceeded the signals of everybody else, that is

$$s_n \equiv \max\{s_1,\ldots,s_N\}$$

so he should condition the expected value of the item on this new information.

- His expected value is now the expected value of v_n conditional upon observing the maximum signal:

$$E[v_n | \; s_n \equiv \max\{s_1,\ldots,s_N\}]$$

- This is the value that the bidder should use in the auction, because he should recognize that unless his signal is the maximum he will receive a payoff of zero.

The Winner's Curse

- Conditional on the signal, but before the bidding starts, the expectation of the common value is:

- We define the winner's curse as:

$$E\left[v\middle|s_n\right]-E\left[v\middle|s_n=\max\left\{s_1,\ldots,s_N\right\}\right]$$
$$=s_n-E\left[v\middle|s_n=\max\left\{s_1,\ldots,s_N\right\}\right]>0$$

- Although bidders should make due allowance for the fact that their valuation will typically overstate the true value of the object if they win the auction, novice bidders typically do not take it into account when placing a bid.

Descending auctions are strategically equivalent to first-price auctions

- During the course of a descending auction no information is received by bidders.
- Each bidder sets his reservation price before the auction, and submits a market order to buy if and when the limit auctioneer's limit order to sell falls to that point.
- Dutch auctions and first price sealed bid auctions share strategic form, and hence yield the same realized payoffs if the initial valuation draws are the same.

Rule 1: Pick the same reservation price in Dutch auction that you would submit in a first price auction

Second-price versus ascending auctions

- When there are only 2 bidders, an ascending auction mechanism is strategically equivalent to the second price sealed bid auction (because no information is received during the auction).
- More generally, both auctions are (almost) strategically equivalent if all bidders have independently distributed valuations (because the information conveyed by the other bidders has no effect on a bidder's valuation).
- In common value auctions the two mechanisms are not strategically equivalent if there are more than 2 players.

Rule 2: If there are only two bidders, or if valuations are independently distributed, choose the same reservation price in English and second price auctions.

Bidding in a second-price auction

- If you know your own valuation, there is a general result about how to bid in a second price sealed bid auction, or where to stop bidding in an ascending auction.
- Bidding should not depend on what you know about the valuations of the other players, nor on what they know about their own valuations.
- It is a dominant strategy to bid your own valuation.
- A corollary of this result is that if every bidder knows his own valuation, then the object will be sold for the second highest valuation.

Rule 3 : In a second price sealed bid auction, bid your valuation if you know it.

Proving the third rule

- Suppose you bid above your valuation, win the auction, and the second highest bid also exceeds your valuation. In this case you make a loss. If you had bid your valuation then you would not have won the auction in this case. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding above it.
- Suppose you bid below your valuation, and the winning bidder places a bid between your bid and your valuation. If you had bid your valuation, you would have won the auction and profited. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding below it.
- The proof is completed by combining the two parts.

SHOULD YOU BID
Value or B_H?

Bidding in an English or a Second Price Sealed Bid Auction

Bid B_H, Value or B_L and lose auction if highest bid is B_1 and make zero.

Bid Value, lose auction to B_{first}, and make zero, or bid B_H, win auction and lose $(B_{first} - Value)$.

Bid Value, win auction, and make $(Value - B_{second})$, or bid B_L and lose auction.

Bid B_H, Value or B_L and win auction to make $(Value - B_0)$ if next highest bid is B_0.

SHOULD YOU BID
Value or B_L?

Asymmetric valuations

- In a private valuation auctions the bidders have different uses for the auctioned object, and this fact is common knowledge to every bidder.
- Each bidder knows the that everyone else is drawing their valuations from the same probability distribution, and uses that information when making her bid.
- What happens if the private valuations of bidders are not drawn from the same probability distribution function?

Bidding with differential information

- For example one bidder might know more about the value of the object being auctioned than the others.
- What happens if they are asymmetrically informed about a common value?
- An extreme form of dependent signals occurs when one bidder know the signal and the others do not. How should an informed player bid? What about an uninformed player?

Second price sealed bid auctions

- In a second price sealed bid auction, Rule 3 implies the informed player optimally bids his true value.
- The uninformed player bids any pure or mixed distribution. If he wins the auction he pays the common value, if he loses he pays nothing, and therefore makes neither gains or losses on any bid.
- This implies the revenue from the auction is indeterminate.

Perspective of the less informed bidder in a first price auction

- Suppose the uninformed bidder always makes the same positive bid, denoted b_{fixed}. This is an example of a pure strategy.
- Is this pure strategy part of a Nash equilibrium?
- The best response of the informed bidder is to bid a little more than b_{fixed} when the value of the object v is worth more than b_{fixed}, and less than b_{fixed} otherwise.
- Therefore the uninformed bidder makes an expected loss by playing a pure strategy in this auction. A better strategy would be to bid nothing.

Equilibrium bidding in a FPSB auction

- The argument in the previous slide shows that the uninformed bidder plays a mixed strategy in this game.
- One can show that in equilibrium when the auctioned item is worth v the informed bidder bids:

$$\beta(v) = E[V|V \leq v]$$

- Furthermore the uninformed bidder chooses a bid at random from the interval [0, E[V]] according to the probability distribution H defined by:

$$H(b) = \text{Prob}[\beta(v) \leq b]$$

Return to the uninformed bidder

- If the uninformed player bids more than E[V], then his expected return is negative, since he would win the auction every time $v < E[V]$ but less frequently when $v > E[V]$.

- We now show that if his bid $b \leq E[V]$, his expected return is zero, and therefore any bid $b < E[V]$ is a best response to the informed player's bid.

- If the uninformed bids less than E[V] and loses the auction, his return is zero. If he bids less than E[V], and wins the auction, his return is:

$$E[V|\ \beta(V) < b] - b = E[V|\ V < \beta^{-1}(b)\ ] - b$$
$$= \beta(\beta^{-1}(b)\ ) - b$$
$$= 0$$

Return to the informed bidder

- Since the uninformed player bids less than E[v] with unit probability, so does the informed player.

- Noting that $\beta(w)$ varies from $\underline{v}$ to E[v], we prove it is better to bid $\beta(v)$ rather than $\beta(w)$. Given a valuation of v, the expected net benefit from bidding $\beta(w)$ is:

$$H(\beta(w))[v - \beta(w)] = \Pr\{V \leq w\}[v - \beta(w)] = P(w)[v - \beta(w)]$$

- Differentiating with respect to w, using derivations found on the next slide, yields P'(w)[v - w] which is positive for all $v > w$ and negative for all $v < w$, and zero at $v = w$. Therefore bidding $\beta(v)$ is optimal for the informed bidder with valuation v.

The derivative

Noting:

$$P(w)\beta(w) = P(w)E[V|V \le w] = \int_{v_0}^{w} tP'(t)dt$$

it follows from the fundamental theorem of calculus that:

$$\frac{d}{dw}[P(w)\beta(w)] = P'(w)w$$

and so the derivative of P(w)[v - β(w)] with respect to w is:

$$P'(w)v - \frac{d}{dw}[P(w)\beta(w)] = P'(w)v - P'(w)w$$

Lecture Summary

- Auctions are amongst the simplest of trading mechanisms.
- We analyzed strategic equivalence.
- We proved what to bid for second price auctions and ascending auctions when there are private values.
- Our proof of the optimal bidding rule for auctions with differential information illustrates how complicated the formulas for optimal bidding can become even in relatively simple settings.