

Lecture 2

Auction Formats

There are many different types of auctions. We explain some more common formats and investigate how to bid optimally. Aside from the auction format, optimal bidding depends on how correlated valuations are for the object up for auction.

There are different types of auctions

- ◆ In a **first price** sealed bid auction, each bidder submits his/her bid without knowing what the others are bidding, and the auctioneer sells the good to the highest bidder at the price he submitted.
- ◆ In an **English** auction bidders compete against each other by raising the price until everyone but one bidder drops out of the bidding.
- ◆ In a **Dutch** auction, the auctioneer reduces the price until a bidder indicates he/she is willing to take the object.
- ◆ In a **second priced** sealed bid auction, players simultaneously submit their bids, the highest bidder wins the auction, and pays the second highest bid.

Bidding strategies

- ◆ Does it matter what form the auction takes?
- ◆ Returning to basics, from SCM (45-970) a **strategy** is a complete description of instructions to be played throughout the game
- ◆ The **strategic form** of a game is the set of alternative strategies to each player and their corresponding expected payoffs from following them.
- ◆ Two games are **strategically equivalent** if they share the same strategic form.
- ◆ In strategically equivalent auctions, the set of bidding strategies that each potential bidder receives, and the mapping to the bidder's payoffs, are the same.

Common value auction: Oil field tract

- ◆ Consider a new oil field tract that drillers bid for after conducting seismic their individual explorations.
- ◆ The value of the oil field is the same to each bidder, but unknown. The n^{th} bidder receives a signal s_n which is distributed about the **common value** v , where

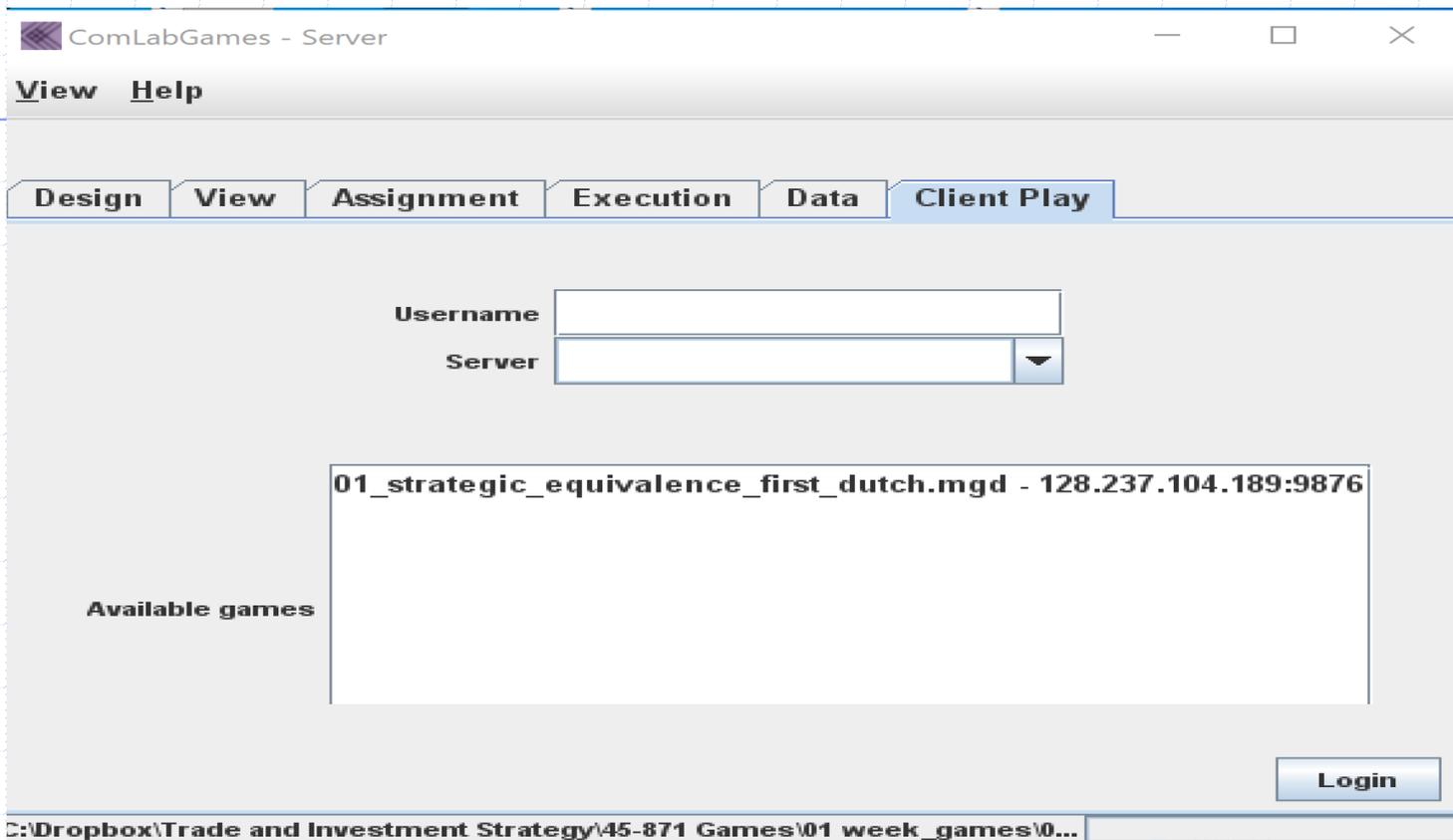
$$s_n = v + \varepsilon_n$$

and $\varepsilon_n \equiv E[v | s_n] - v$ is independently distributed across bidders.

- ◆ Notice that each drilling company would have more precise estimates of the common valuation from reviewing the geological survey results of their rivals.

Login Instructions

1. Click on Client Play
2. Click on Available games: "01_strategic_equivalence..."



The screenshot shows a window titled "ComLabGames - Server" with a menu bar containing "View" and "Help". Below the menu bar is a tabbed interface with tabs for "Design", "View", "Assignment", "Execution", "Data", and "Client Play". The "Client Play" tab is selected. In the "Client Play" tab, there are two input fields: "Username" and "Server". Below these fields is a list of available games, with the first entry being "01_strategic_equivalence_first_dutch.mgd - 128.237.104.189:9876". A "Login" button is located at the bottom right of the window. The status bar at the bottom of the window shows the file path: "C:\Dropbox\Trade and Investment Strategy\45-871 Games\01 week_games\0..."

ComLabGames - Server

View Help

Design View Assignment Execution Data Client Play

Username

Server

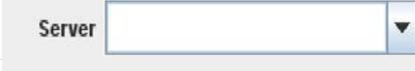
Available games

01_strategic_equivalence_first_dutch.mgd - 128.237.104.189:9876

Login

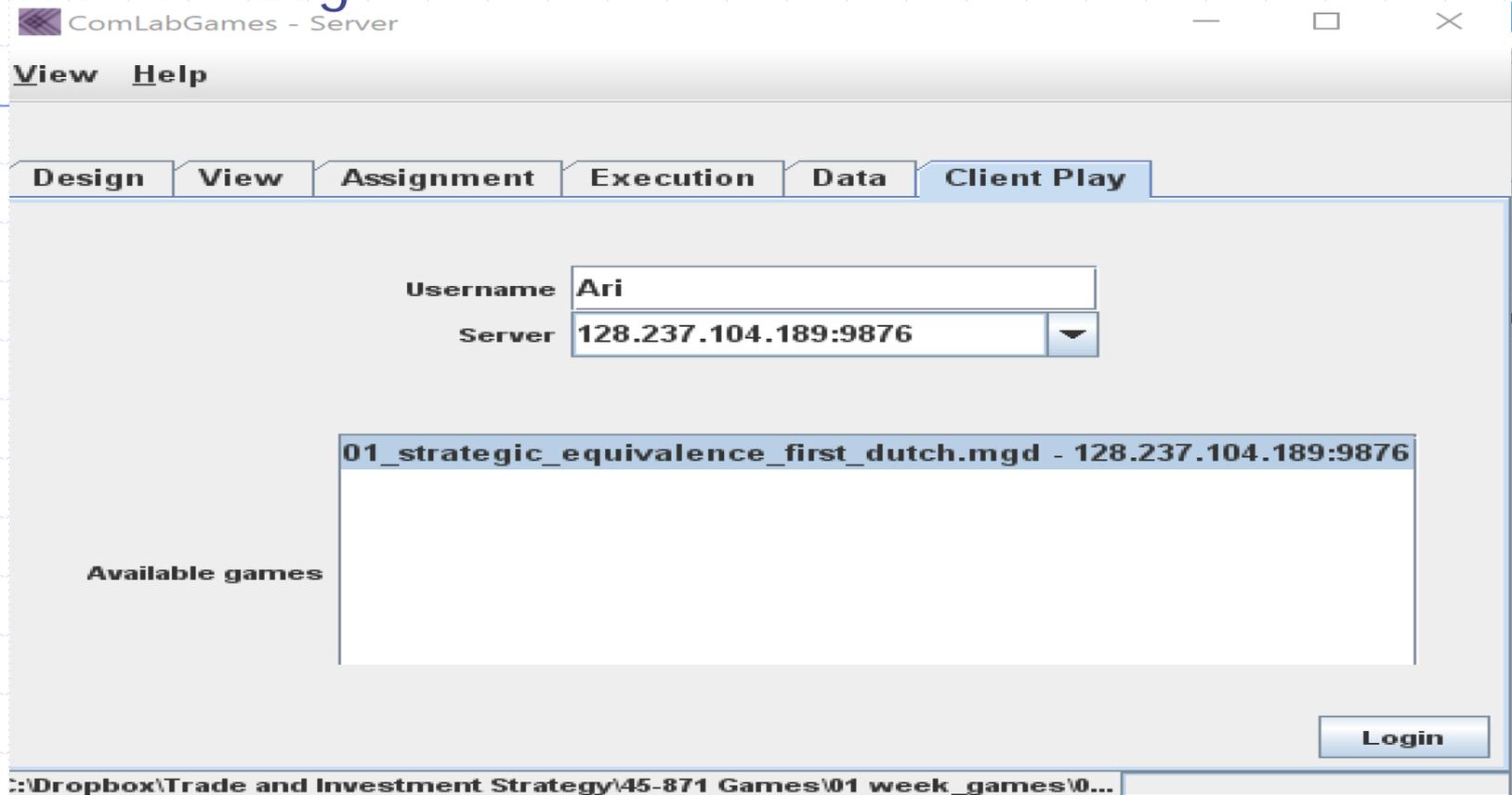
C:\Dropbox\Trade and Investment Strategy\45-871 Games\01 week_games\0...

Continuing Login Instructions

1. Server address: 128.237.104.189 will appear automatically after selecting "01_strategic...". If not type 128.237.104.189 in .

2. Write a username.

3. Click on "Login".



The screenshot shows a window titled "ComLabGames - Server" with a menu bar containing "View" and "Help". Below the menu bar are several tabs: "Design", "View", "Assignment", "Execution", "Data", and "Client Play". The "Client Play" tab is selected. In the main area, there are two input fields: "Username" with the text "Ari" and "Server" with the text "128.237.104.189:9876". Below these fields is a list box titled "Available games" containing one entry: "01_strategic_equivalence_first_dutch.mgd - 128.237.104.189:9876". A "Login" button is located at the bottom right of the window. The status bar at the bottom shows the file path: "C:\Dropbox\Trade and Investment Strategy\45-871 Games\01 week_games\0..."

Auction window and Instruction window

1. Instruction and auction window appears on your screen.
2. To close the instruction window click on "x". To retrieve it click on "Description".

ComLabGames - Client

Description Username: ari Id: 12 Identity: 1

First Price Sealed Bid Common Value Auction

Number of companies participating: 4



Your signal is: 806.95. This signal is randomly drawn from the interval $(v - 100)$ to $(v + 100)$. Each signal is equally likely. Each of you gets a different signal.

Place your bid and click Enter.

The highest bid wins the auction!

Stage time limit: unlimited Round: 1 Continue

Player type: consortium (1) Please make a move now!

Waiting 4 of 4 subject(s) to proceed the session!

ComLabGames - Client

Instructions for First price sealed bid auction and for Dutch auction Username: ari Id: 12 Identity: 1



You are about to participate in an auction for oil-drilling rights in Gulf of Mexico. You do not know exactly what the project would generate in terms of financial value if you win this contract. You know that the common value, denoted by v , would be between \$500 and \$1,000 (millions). Operationally, a computer will generate a random number between \$500 and \$1000, so that any number in this range is equally likely.

During the auction you will not know what the exact common value. Instead, you will be receive a signal. This will narrow down the range of possible values for determining your bid, denoted by b . There are 4 companies, counting your company, involved in this auction. Each of you gets a different signal. We generate the value of a signal from the common value, and a random variable, epsilon (ϵ). So, the signal that you get will be somewhere between $(v - \epsilon)$ and $(v + \epsilon)$. Any number in this interval has an equal chance of being drawn. The value of ϵ is \$100. If you win the auction, your profit is: Common value minus price. This can be a gain or a loss.

First Price auction

Each bidder submits a bid simultaneously. Once everybody submits a bid the auction is closed. The bidder with the highest bid wins the license for drilling rights and pays the amount she bid. The other bidders neither pay nor receive anything.

PROFIT = COMMON VALUE - THE HIGHEST BID

If the difference is negative, it represents a loss. If you lose the auction, you will receive zero.

Dutch auction

Each 3 seconds the auctioneer places consecutive limit order order that lowers the price of the auctioned item by 20 dollars until the first bidder clicks on Buy button. At that time the auction concludes. A company who submitted a market order to Buy buys the rights to drill at the current (most attractive) limit order price. The winning company's profit is the common value minus the current limit order price. The other companies neither pay nor receive anything.

Write a bid in First price sealed bid auction

1. Write a number and click Enter
2. Your bid should be colored in red.
3. Wait for all players in your session to submit bids.

ComLabGames - Client

Description Username: ari Id: 12 Identity: 1

First Price Sealed Bid Common Value Auction

Number of companies participating: 4



Your signal is: **806.95**. This signal is randomly drawn from the interval $(v - 100)$ to $(v + 100)$. Each signal is equally likely. Each of you gets a different signal.

Place your bid and click Enter.

The highest bid wins the auction!

Stage time limit: unlimited Round: 1

Player type: consortium (1) Please make a move now!

ComLabGames - Client

Description Username: ari Id: 12 Identity: 1

First Price Sealed Bid Common Value Auction

Number of companies participating: 4



Your signal is: **806.95**. This signal is randomly drawn from the interval $(v - 100)$ to $(v + 100)$. Each signal is equally likely. Each of you gets a different signal.

Place your bid and click Enter.

The highest bid wins the auction!

Stage time limit: unlimited Round: 1

Player type: consortium (1) Please wait for other players to make their choices!

Waiting 3 of 4 subject(s) to proceed the session!

Summary page for sealed bid auction

1. Click "Continue" to move to Dutch auction.
2. Wait for all players in your session to click "Continue".

ComLabGames - Client

Description Username: ari Id: 12 Identity: 1

Brief summary of the sealed bid auction

Your signal was: **806.95**, and your bid in this auction was **800**. Please click "Continue" and proceed to the next auction. Summary results will be given at the end of the experiment.

Stage time limit: unlimited Round: 1 **Continue**

Player type: consortium (1) Please make a move now!

Waiting 4 of 4 subject(s) to proceed the session!

Dutch auction page

1. Every 3 seconds limit order price is lowered by \$20.
2. The first subject in a session who click on **Buy** buys the drilling rights.

ComLabGames - Client

Description Username: ari Id: 12 Identity: 1

Dutch Auction

Number of companies participating: **4**

Your signal is: **806.95**. This signal is randomly drawn from the interval $(v - 100)$ to $(v + 100)$. Each estimate is equally likely. Each of you gets a different value estimate of a project.

Current limit order price: **940**

Select **Buy** if you want to buy the drilling rights.

Each **3** seconds the auctioneer places consecutive limit order order that lower the price of the auctioned item by **20** dollars until the first bidder clicks on **Buy** button. At that time the aution concludes. The bidder who submitted a market order to **Buy** buys the drilling rights at the current (most attractive) limit order price.

Stage time limit: 1 Round: 1 **Continue**

Player type: consortium (1) **Please make a move now!**

Summary page

ComLabGames - Client

Description

Username: ari Id: 12 Identity: 1

Summary

For both auctions, your signal was **806.95**, and common value (v) was **791.33**.

Sealed bid auction: The winning bid was **800**, your bid was **800** and your profit is **0**.

Dutch auction: The current limit order price at which the drilling rights were bought was **740**. You clicked on **Buy** and your profit is **51.33**. Below is the list of all the decisions, valuations, prices, and profits for all sessions.

Player's us...	s	v	b first price	Winning bi...	Profit first ...	Dutch Buy ...	Winning bi...	Profit Dutch
ari	806.95	791.33	800	800	0	<input checked="" type="checkbox"/>	740	51.33
ari1	704.03	791.33	760	800	0	<input type="checkbox"/>	740	0
ari2	839.16	791.33	800	800	-8.67	<input type="checkbox"/>	740	0
ari3	774.25	791.33	750	800	0	<input type="checkbox"/>	740	0

Stage time limit: unlimited

Round: 1

Continue

Game is over!

Player type: consortium (1)

The expected value of the item upon winning the auction

- ◆ If the n^{th} bidder wins the auction, he realizes his signal exceeded the signals of everybody else, that is

$$s_n \equiv \max\{s_1, \dots, s_N\}$$

so he should condition the expected value of the item on this new information.

- ◆ His expected value is now the expected value of v_n conditional upon observing the maximum signal:

$$E[v_n | s_n \equiv \max\{s_1, \dots, s_N\}]$$

- ◆ This is the value that the bidder should use in the auction, because he should recognize that unless his signal is the maximum he will receive a payoff of zero.

The Winner's Curse

- ◆ Conditional on the signal, but before the bidding starts, the expectation of the common value is:

- ◆ We define the **winner's curse** as:

$$\begin{aligned} & E[v|s_n] - E[v|s_n = \max\{s_1, \dots, s_N\}] \\ & = s_n - E[v|s_n = \max\{s_1, \dots, s_N\}] > 0 \end{aligned}$$

- ◆ Although bidders should make due allowance for the fact that their valuation will typically overstate the true value of the object if they win the auction, novice bidders typically do not take it into account when placing a bid.

Descending auctions are strategically equivalent to first-price auctions

- ◆ During the course of a descending auction no information is received by bidders.
- ◆ Each bidder sets his reservation price before the auction, and submits a market order to buy if and when the limit auctioneer's limit order to sell falls to that point.
- ◆ Dutch auctions and first price sealed bid auctions share strategic form, and hence yield the same realized payoffs if the initial valuation draws are the same.

Rule 1: Pick the same reservation price in Dutch auction that you would submit in a first price auction

Second-price versus ascending auctions

- ◆ When there are only 2 bidders, an ascending auction mechanism is strategically equivalent to the second price sealed bid auction (because no information is received during the auction).
- ◆ More generally, both auctions are (almost) strategically equivalent if all bidders have independently distributed valuations (because the information conveyed by the other bidders has no effect on a bidder's valuation).
- ◆ In common value auctions the two mechanisms are not strategically equivalent if there are more than 2 players.

Rule 2: If there are only two bidders, or if valuations are independently distributed, choose the same reservation price in English and second price auctions.

Bidding in a second-price auction

- ◆ If you know your own valuation, there is a general result about how to bid in a second price sealed bid auction, or where to stop bidding in an ascending auction.
- ◆ Bidding should not depend on what you know about the valuations of the other players, nor on what they know about their own valuations.
- ◆ It is a dominant strategy to bid your own valuation.
- ◆ A corollary of this result is that if every bidder knows his own valuation, then the object will be sold for the second highest valuation.

Rule 3 : In a second price sealed bid auction, bid your valuation if you know it.

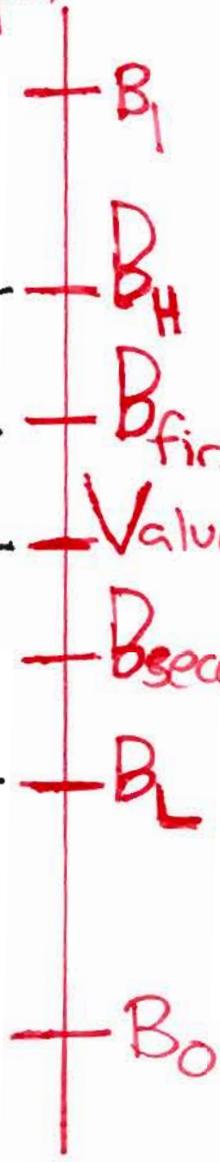
Proving the third rule

- ◆ Suppose you bid above your valuation, win the auction, and the second highest bid also exceeds your valuation. In this case you make a loss. If you had bid your valuation then you would not have won the auction in this case. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding above it.
- ◆ Suppose you bid below your valuation, and the winning bidder places a bid between your bid and your valuation. If you had bid your valuation, you would have won the auction and profited. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding below it.
- ◆ The proof is completed by combining the two parts.

SHOULD YOU BID
Value or B_H ?

Bidding in an English or a
Second Price Sealed Bid Auction

\$ price



Bid B_H , Value or B_L and
lose auction if highest bid is B_1
and make zero.

Bid Value, lose auction to
 B_{first} , and make zero, or
bid B_H , win auction and
lose $(B_{first} - Value)$.

Bid Value, with auction,
and make $(Value - B_{second})$,
or bid B_L and lose
auction.

Bid B_H , Value or B_L
and win auction to
make $(Value - B_0)$ if next
highest bid is B_0 .

SHOULD YOU BID
Value or B_L ?

Asymmetric valuations

- ◆ In a private valuation auctions the bidders have different uses for the auctioned object, and this fact is common knowledge to every bidder.
- ◆ Each bidder knows that everyone else is drawing their valuations from the same probability distribution, and uses that information when making her bid.
- ◆ What happens if the private valuations of bidders are not drawn from the same probability distribution function?

Bidding with differential information

- ◆ For example one bidder might know more about the value of the object being auctioned than the others.
- ◆ What happens if they are asymmetrically informed about a common value?
- ◆ An extreme form of dependent signals occurs when one bidder know the signal and the others do not. How should an informed player bid? What about an uninformed player?

Second price sealed bid auctions

- ◆ In a second price sealed bid auction, Rule 3 implies the informed player optimally bids his true value.
- ◆ The uninformed player bids any pure or mixed distribution. If he wins the auction he pays the common value, if he loses he pays nothing, and therefore makes neither gains or losses on any bid.
- ◆ This implies the revenue from the auction is indeterminate.

Perspective of the less informed bidder in a first price auction

- ◆ Suppose the uninformed bidder always makes the same positive bid, denoted b_{fixed} . This is an example of a pure strategy.
- ◆ Is this pure strategy part of a Nash equilibrium?
- ◆ The best response of the informed bidder is to bid a little more than b_{fixed} when the value of the object v is worth more than b_{fixed} , and less than b_{fixed} otherwise.
- ◆ Therefore the uninformed bidder makes an expected loss by playing a pure strategy in this auction. A better strategy would be to bid nothing.

Equilibrium bidding in a FPSB auction

- ◆ The argument in the previous slide shows that the uninformed bidder plays a mixed strategy in this game.
- ◆ One can show that in equilibrium when the auctioned item is worth v the informed bidder bids:

$$\beta(v) = E[V|V \leq v]$$

- ◆ Furthermore the uninformed bidder chooses a bid at random from the interval $[0, E[V]]$ according to the probability distribution H defined by:

$$H(b) = \text{Prob}[\beta(v) \leq b]$$

Return to the uninformed bidder

- ◆ If the uninformed player bids more than $E[V]$, then his expected return is negative, since he would win the auction every time $v < E[V]$ but less frequently when $v > E[V]$.
- ◆ We now show that if his bid $b < E[V]$, his expected return is zero, and therefore any bid $b < E[V]$ is a best response to the informed player's bid.
- ◆ If the uninformed bids less than $E[V]$ and loses the auction, his return is zero. If he bids less than $E[V]$, and wins the auction, his return is:

$$\begin{aligned} E[V | \beta(V) < b] - b &= E[V | V < \beta^{-1}(b)] - b \\ &= \beta(\beta^{-1}(b)) - b \\ &= 0 \end{aligned}$$

Return to the informed bidder

- ◆ Since the uninformed player bids less than $E[v]$ with unit probability, so does the informed player.
- ◆ Noting that $\beta(w)$ varies from \underline{v} to $E[v]$, we prove it is better to bid $\beta(v)$ rather than $\beta(w)$. Given a valuation of v , the expected net benefit from bidding $\beta(w)$ is:

$$H(\beta(w))[v - \beta(w)] = \Pr\{V \leq w\}[v - \beta(w)] = P(w)[v - \beta(w)]$$

- ◆ Differentiating with respect to w , using derivations found on the next slide, yields $P'(w)[v - w]$ which is positive for all $v > w$ and negative for all $v < w$, and zero at $v = w$. Therefore bidding $\beta(v)$ is optimal for the informed bidder with valuation v .

The derivative

Noting:

$$P(w)\beta(w) = P(w)E[V|V \leq w] = \int_{v_0}^w tP'(t)dt$$

it follows from the fundamental theorem of calculus that:

$$\frac{d}{dw} [P(w)\beta(w)] = P'(w)w$$

and so the derivative of $P(w)[v - \beta(w)]$ with respect to w is:

$$P'(w)v - \frac{d}{dw} [P(w)\beta(w)] = P'(w)v - P'(w)w$$

Lecture Summary

- ◆ Auctions are amongst the simplest of trading mechanisms.
- ◆ We analyzed strategic equivalence.
- ◆ We proved what to bid for second price auctions and ascending auctions when there are private values.
- ◆ Our proof of the optimal bidding rule for auctions with differential information illustrates how complicated the formulas for optimal bidding can become even in relatively simple settings.