Trade and Investment Strategy 45871
Professor Robert A. Miller
Mini 2 December 2025

Take home examination (out of 20 points)

There are three equally weighted questions. You must take the test individually, but you are allowed to discuss the questions with each other beforehand. You are also allowed to "do the numbers" together in groups of up to 4, and this especially applies in Question 3. So don't copy and paste, don't divide and specialize, but it is OK to verbally discuss the answers in a group of up to 4, and then write up the whole exam by yourself without further discussions. Please indicate who is in your pretest discussion group, after your name.

Please submit your answers preferably via canvas, or if you experience technical difficulties, as an attached file (a PDF format is preferred) by email to my teaching assistant Pei Xue before 11:55PM Sunday December 14. I will grant extensions for a few days to those of you who want them; in that case you would get an incomplete, to be updated in January.

Question 1

Consider the following auctions. In the first three parts to this question suppose each bidder knows his or her own valuation, and all the valuations are all distributed uniformly (that is evenly) between $100 million and $400 million. For example, the probability that a typical bidder's valuation lies between $300 million and $400 million is one third.

1. Suppose you are one of 5 bidders, your valuation is $235 million, and you are bidding in a first price sealed bid auction. What should you bid? What are your expected profits from bidding in this auction (that is after you know your valuation, the number of bidders in the auction, but before you know the outcome of the auction)? Explain or justify your answers.

2. Suppose you are one of 5 bidders, your valuation is $235 million, and you are bidding in an ascending auction. How high should you be prepared to bid? What are your expected profits from bidding in this auction (that is after you know your valuation, the number of bidders in the auction, but before the bidding starts)? Explain or justify your answers.

3. Now suppose you are one of 2 bidders, your valuation is $235 million, and you are bidding in a first price sealed bid auction. What should you bid? What are your expected profits from bidding in this auction (that is after you know your valuation, the number of bidders in the auction, but before you know the outcome of the auction)? Intuitively explain why your answer differs from your answer in the first part of this question.

4. Suppose you are one of 5 bidders and your valuation is $235 million. In an all-pay auction (a little bit like a raffle) every bidder pays their bid, *whether they win or not*. (So if you bid $220 million and another bidder bids $300 million you lose $220

million.) The Revenue Equivalence Theorem says that the expected net benefit from bidding in this auction is the same as the expected net benefit from bidding from bidding in a first price sealed bid auction. Using that theorem, and your answer in the first part of this question, what should you bid in this all-pay auction. If the math looks too challenging, give me some intuition about roughly what you would do.

5. Now suppose there are 5 bidders, and you are bidding in a first price sealed bid auction. However in this new setup *every bidder has the same valuation* but no one knows what that valuation is. They all know that this common valuation is distributed uniformly between $100 million and $400 million. Each bidder gets a noisy signal about what the valuation is, that gives them some clue about the true valuation, but not an exact reading. Suppose the reading you get is $235 million. Should you bid more or less than what you bid the first part of this question? What if there were 10 bidders? In words, how would your answer change then, and why?

6. As in the previous question *every bidder has the same valuation.* There are only two bidders. You know exactly what the valuation is. The other bidder does not. The other bidder believes the valuation is distributed between valuation is distributed uniformly between $100 million and $400 million. He decides to bid cautiously, bidding $150 million, thinking that the expected value is $250 million, and so if he wins, he would profit on average $100 million. You know the actual value. (It is some number between $100 million and $400 million.) Given his bidding strategy, what should you bid? Assuming the valuation is distributed between valuation is indeed initially distributed uniformly between $100 million and $400 million, what are your expected profits, what are his expected losses, and what is the expected revenue of the auctioneer?

Question 2

We now expand the currency exchange model (of Lecture 9) to four, adding in Japanese yen (JPY) to the US dollar (USD), the Chinese yuan (CNY) and the European euro (EUR). The endowments, described export earnings in the lecture, are:

Supply of Currencies

	USD	CNY	EUR	JPY
US	0	700	90	$15,000$
$China$	300	0	200	$5,200$
$Europe$	650	850	0	450
$Japan$	790	2100	670	0

As before we assume that all demand for a currency comes from its home exporters. Thus US exporters supply their foreign currency to the exchange market (that is 700 CNY, 90 EUR and 15,000 JPY) to demand all the 300 + 650 + 790 = 1740 USD. Let p_{CNY} denote the exchange rate of CNY in terms of USD. In words p_{CNY} the dollar price of a yuan. Similarly let p_{EUR} denote the exchange rate of EUR in terms of USD, and p_{JPY} the exchange rate of JPY in terms of USD.

1. In a competitive equilibrium the foreign exchange rates are set so that all the foreign currency markets clear. Display what the matrix above will look like after all trading has taken place in a competitive equilibrium, by filling in its 16 elements. Your new matrix is called the **Demand for Currencies**.

2. Write down the equations that equate supply with demand in the yuan market, the euro market and the yen market.

3. Show that the market for dollars will clear, that is the supply of dollars equals the demand for dollars, if p_{CNY}, p_{EUR} and p_{JPY} are set so that the other three markets clear. (This is called *Walras' law*.)

4. Solve for the competitive equilibrium exchange rates p_{CNY}, p_{EUR} and p_{JPY}.

Question 3

The results from the recycling game are posted on the course webpage under the take home test heading. How well did the outcomes of the garbage disposal trading game perform as an allocation mechanism? In particular:

1. How much did the traders benefit collectively from trading rather than consuming their endowment? (This is the total value of the trading.)

2. Were the gains from trade fully exhausted or were there profitable opportunities for trades that were not exploited? If so, compute the loss from not exploiting all the gains from trade.

3. Find a competitive equilibrium prices for this market that could be traded to from the final allocation point the class reached in this session. That is, starting at the allocations already reached by the end of the game, pick a set of market clearing prices that will make some traders better off and no one worse off, and fully exhaust all the gains from trade.

4. Compare the gains made by the traders (your class colleagues) in this game with the extra gains they could have made if a competitive equilibrium had been reached.