Dynamic Considerations

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Trade and Investment Strategy

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Introduction

What does valuation mean?

- The preceding lectures have summarized what matters about an asset by its *valuation*.
- The remaining lectures address the questions:
  - How is a valuation calculated?
  - Why do consumers, investors and workers hold assets?
- How an asset is priced depends on:
  - the trading mechanism
  - the asset's intrinsic value or value-in-use
- Intrinsic value should account for:
  1. *uncertainty of* benefits (and costs).
  2. benefits accruing at *different points in time*.

Expected Value

A summary measure

- The *expected net present value* (EPV) of an asset captures the first two features within one summary measure. Benefits received:
  - at different times are perfect substitutes priced using the *interest rate*.
  - for different uncertain outcomes are priced by their *probabilities*.
- Intuitively the asset sheds payouts, over time and with designated probabilities that translate to *generalized purchasing power* consumers intrinsically value.
- Measuring value by EPV is *simple*, *internally consistent* and therefore.
- But EPV is essentially a price aggregator, not an intrinsic value.
- Given a trading mechanism, how is intrinsic values related to price?
- These next three lectures analyze:
  1. *intertemporal substitution* (timing of consumption benefits)
  2. *risk premiums* (cost of uncertainty)
  3. *modern portfolio theory* (which synthesizes these two factors).

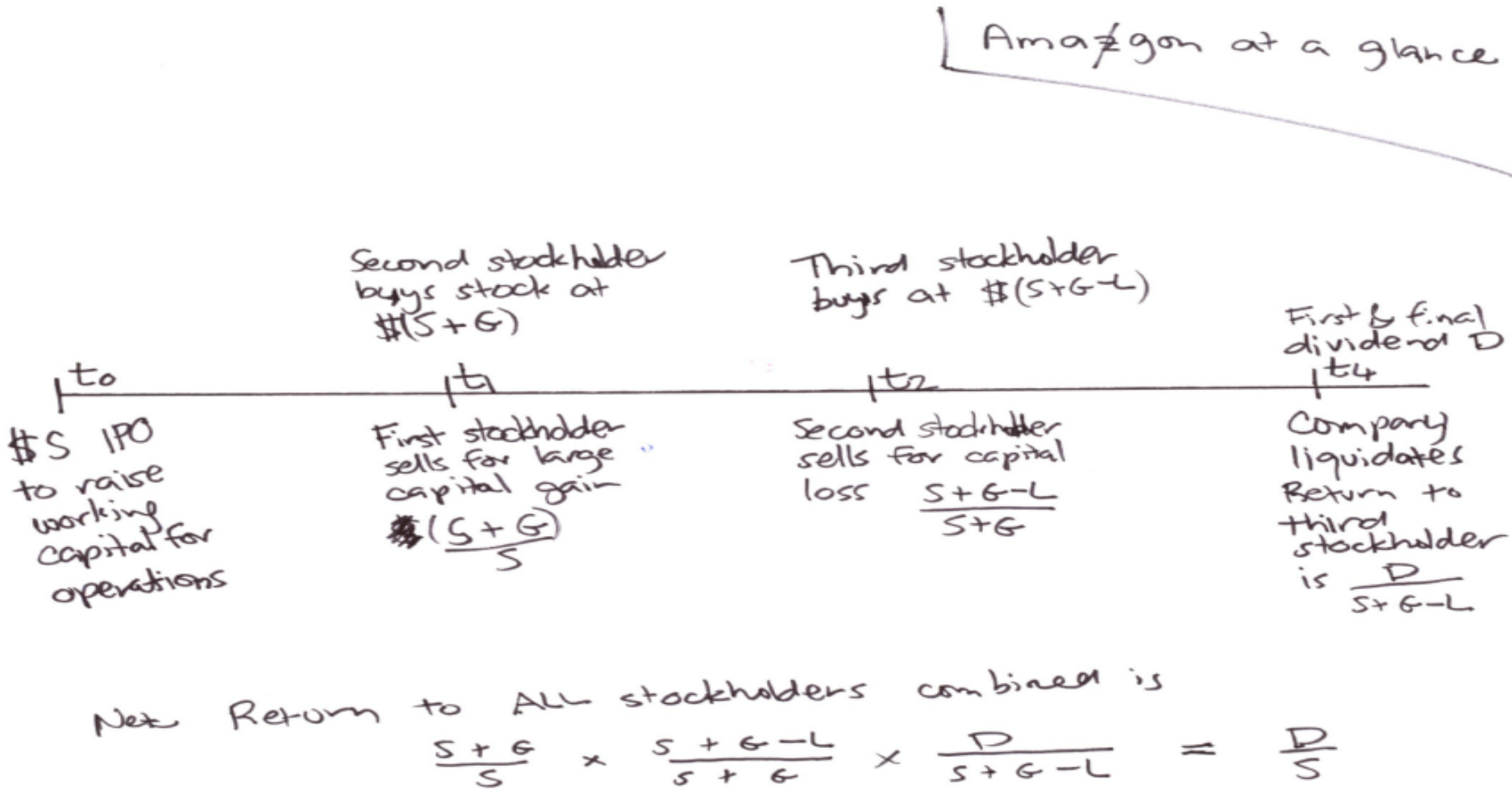
Investment in Firms

Corporations as an intermediary between current and future consumption

- The demand for shares in firm ownership comes from those wishing to defer consumption.
- The primary reason for stock issues is to finance the firm's operations.
- A compact between investor (or shareholder) and asset manager (or executive officer) embodies the trade off between present and future.
- The stock's dividend stream ultimately determines its value.
- Not all financial securities pay dividends in aggregate:
  - Many financial derivatives do not pay dividends directly.
  - Ponzi schemes swindle investors by paying some but ultimately sacrificing others.
- However:
  - derivatives are worthless unless the underlying stocks pay dividends.
  - Even Ponzi schemes exist only because some buyers believe the so-called securities they buy will pay dividends.

Investment in Firms

Why share value is (roughly) the discounted sum of the expected dividend stream



Allocating Resources over Time

Financing lifecycle consumption and investment (including human capital)

- Demand for investment vehicles and banking services arise in response to the lifetime goals of individuals and households who:
  1. first educate themselves rather than working.
  2. then form households, working less while raising young children.
  3. purchase first home when family and employment situations stabilize.
  4. work harder in prime age years.
  5. ramp down work schedule with declining mental and physical capacity.
- Their financial decisions mirror these lifecycle patterns, as they:
  1. borrow to pay for education and current consumption.
  2. repay loans, more slowly when children are young.
  3. take on homeownership, a risky asset.
  4. increase home equity, wealth and social security benefits.
  5. live rent free off assets upon retirement.

Allocating Resources over Time

The demand for future consumption

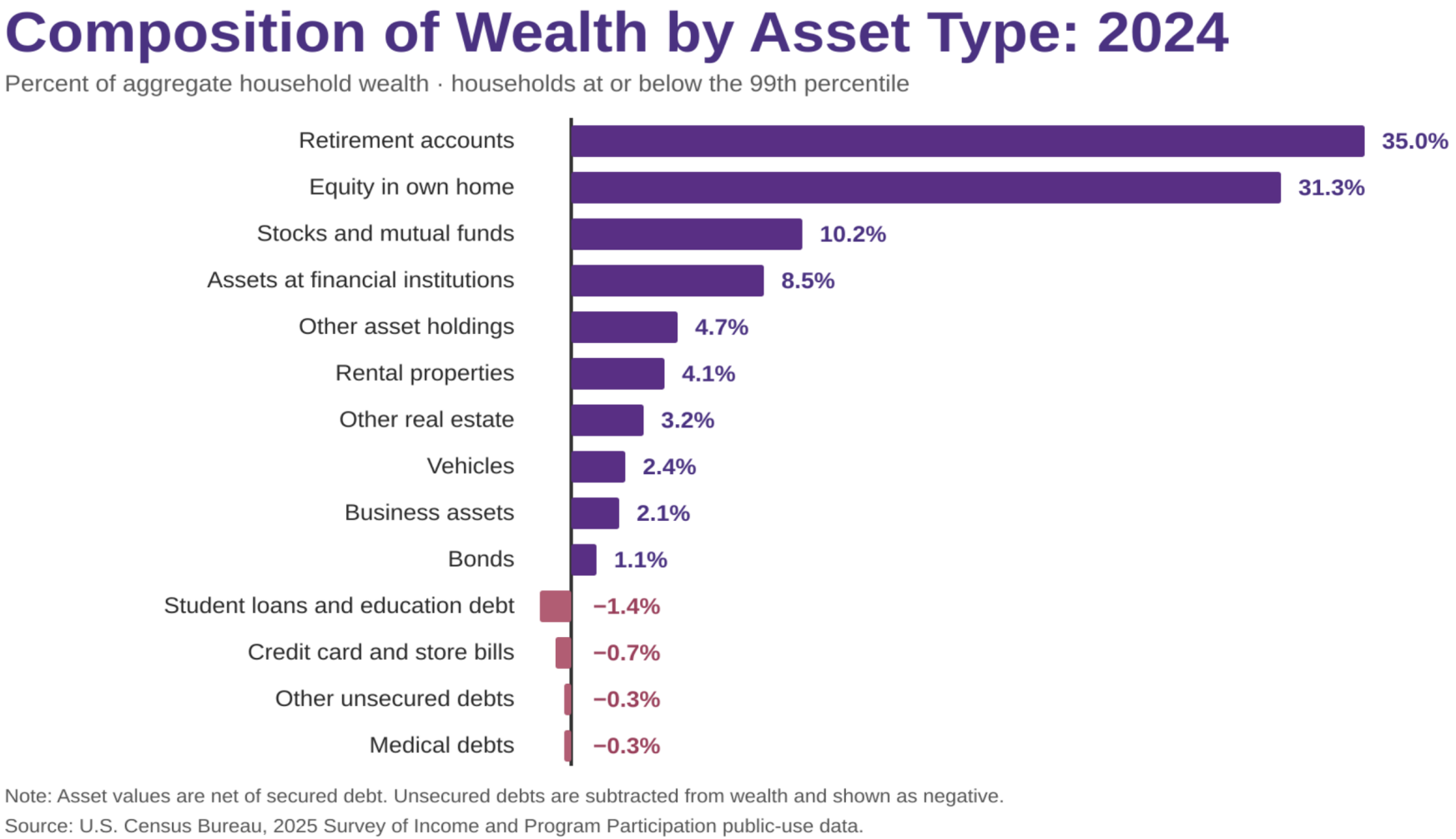
- In this country people save through their:
  1. contributions to social security
  2. investment in mutual funds
  3. home purchase and mortgage repayments
- One way of gauging the savings propensity for households is to survey their wealth:

The Value of Household Wealth by Percentile: 2024

Household net wealth (assets minus debts), in 2024 dollars

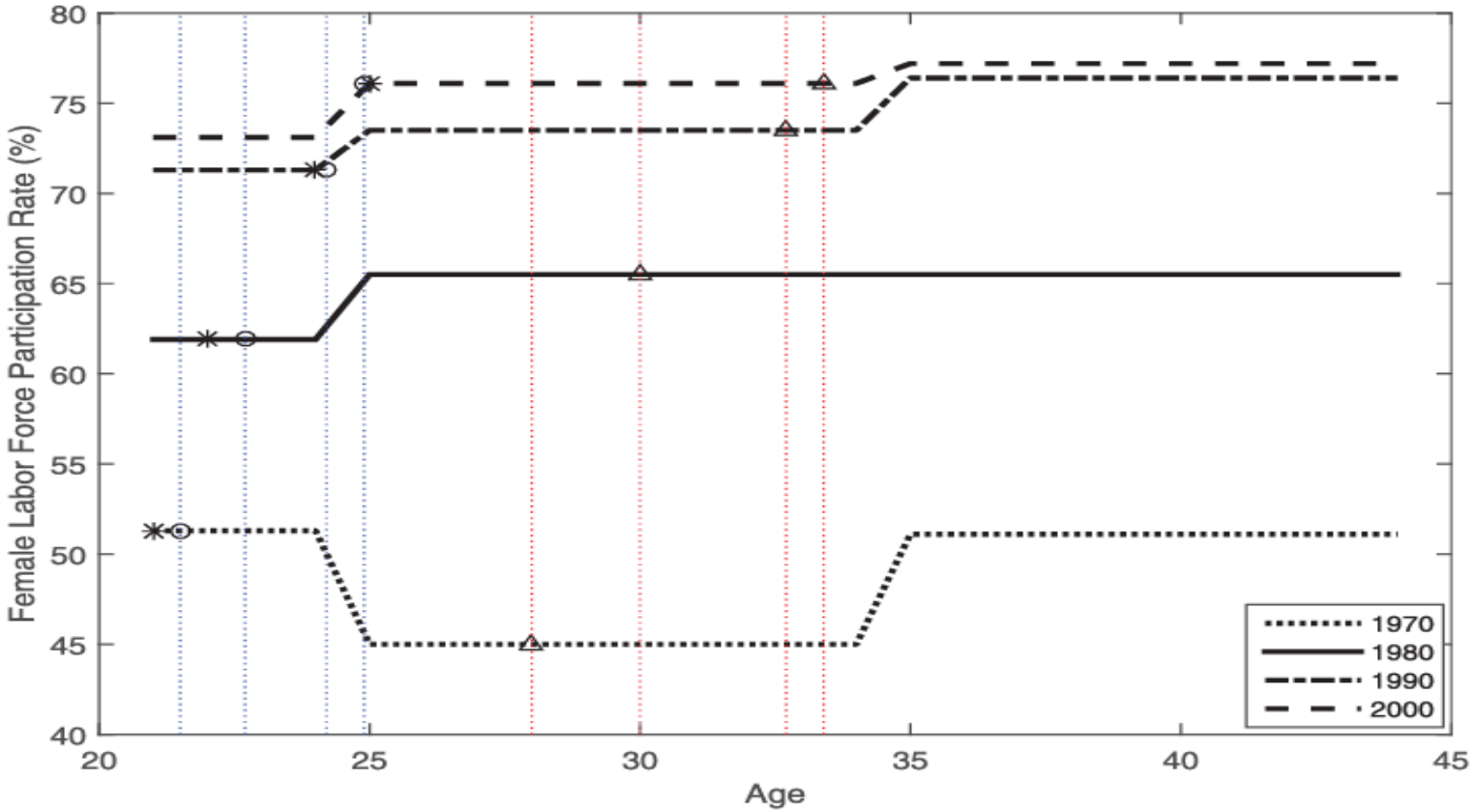
Percentile	2024 dollars
10th	$0
25th	$20,760
50th (median)	**$204,900**
75th	$672,700
90th	$1,810,000

Source: U.S. Census Bureau, 2025 Survey of Income and Program Participation public-use data.
Note: Wealth excludes equity in pension plans and the value of home furnishings. Estimates include households with negative wealth.


Composition of Wealth by Asset Type: 2024
Percent of aggregate household wealth · households at or below the 99th percentile
Retirement accounts
35.0%
Equity in own home
31.3%
Stocks and mutual funds
10.2%
Assets at financial institutions
8.5%
Other asset holdings
4.7%
Rental properties
4.1%
Other real estate
3.2%
Vehicles
2.4%
Business assets
2.1%
Bonds
1.1%
Student loans and education debt
−1.4%
Credit card and store bills
−0.7%
Other unsecured debts
−0.3%
Medical debts
−0.3%
Note: Asset values are net of secured debt. Unsecured debts are subtracted from wealth and shown as negative.
Source: U.S. Census Bureau, 2025 Survey of Income and Program Participation public-use data.

Lifecycle Trends over the Years

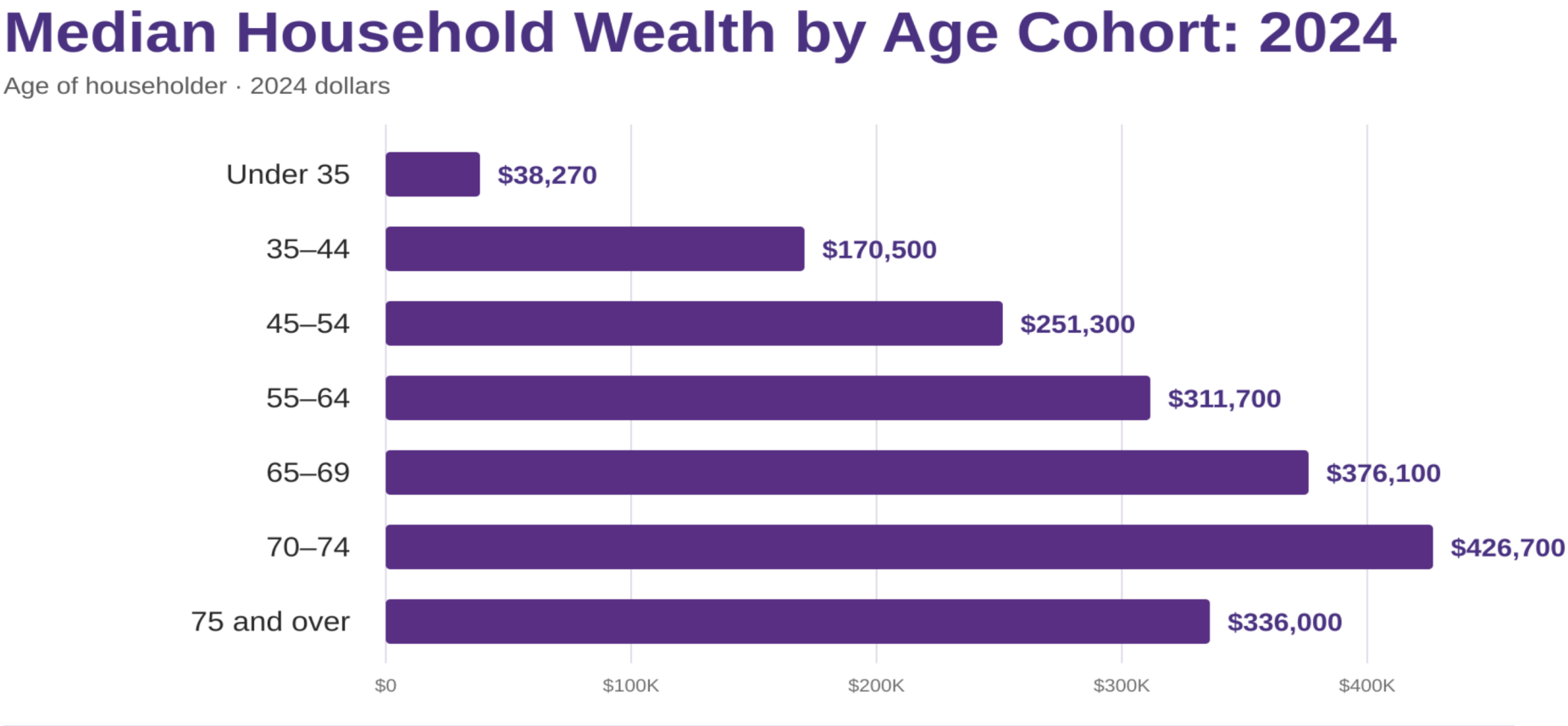
Female labor supply, marriage, first birth & homeownership (Khorunzhina and Miller, 2022)



NOTES: "star" denotes median age at first marriage, "circle" denotes average age at first birth, "triangle" denotes average age at first homeownership Age at first marriage is taken from the U.S. Census Bureau, age at first birth is taken from the National Vital Statistical Reports (Mathews and Hamilton, 2002), age at first homeownership is computed from the PSID, whereas labor force participation rates are taken from publications of the U.S. Bureau of Labor Statistics (Toossi, 2002, 2012).

Lifecycle Trends over the Years

People accumulate human capital, then physical capital and then lose it all

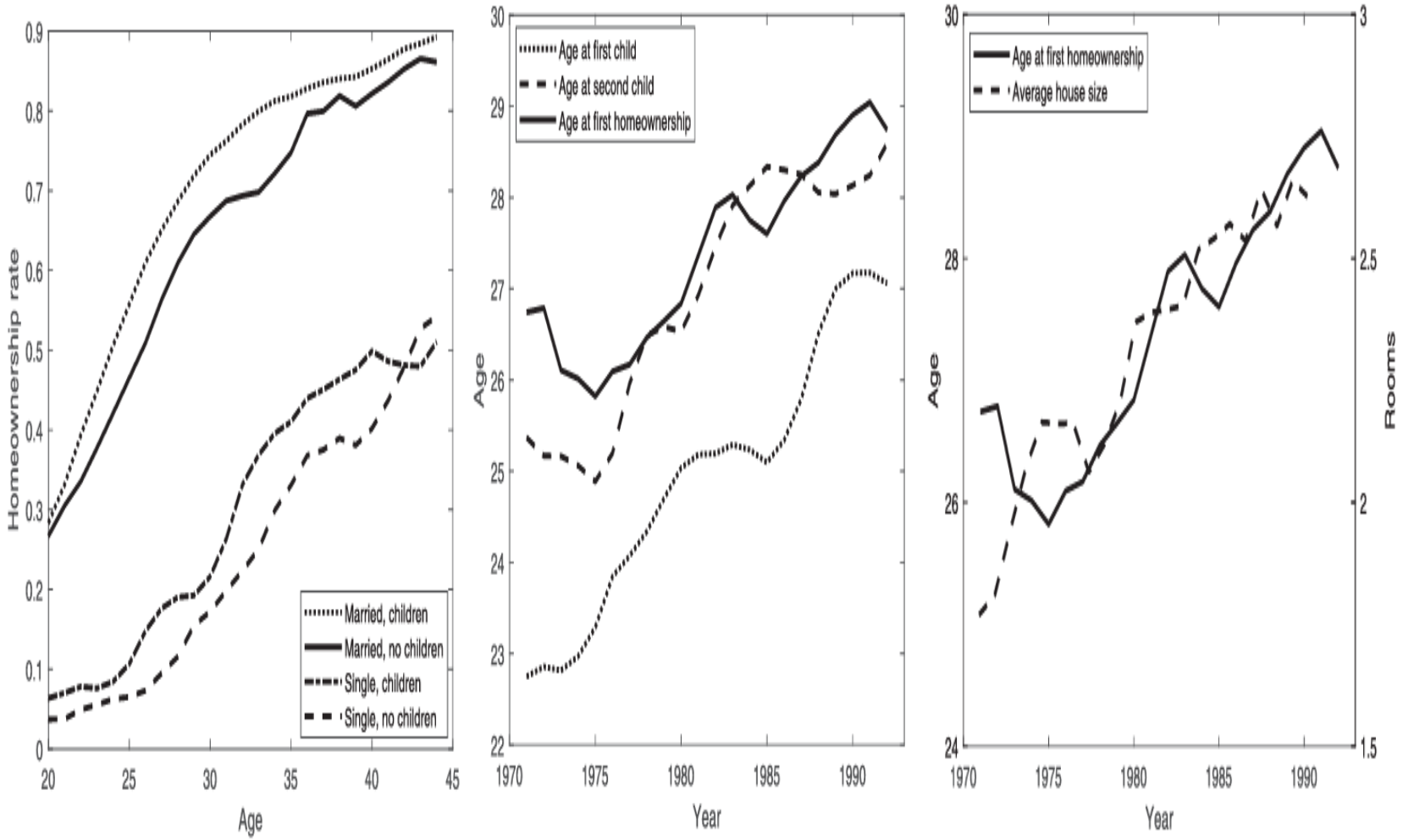


Note: The householder is the person who owns or rents the housing unit. Values are weighted interpolated medians.

Source: U.S. Census Bureau, 2025 Survey of Income and Program Participation public-use data.

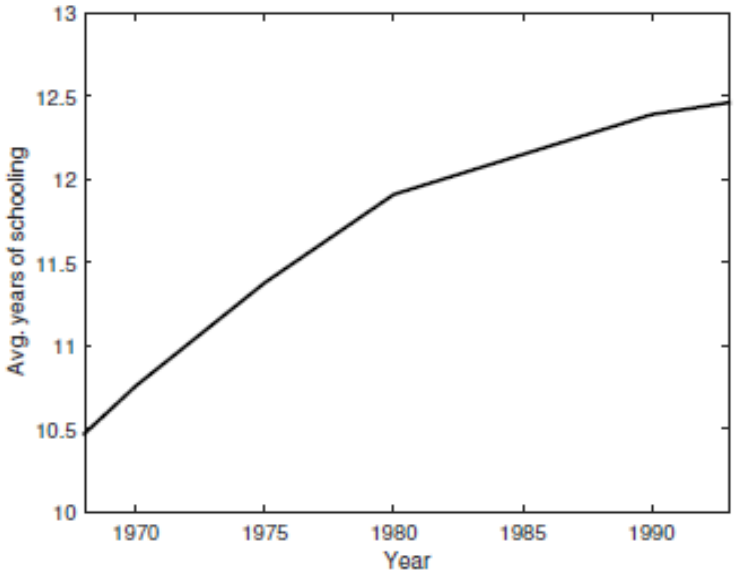
Lifecycle Trends over the Years

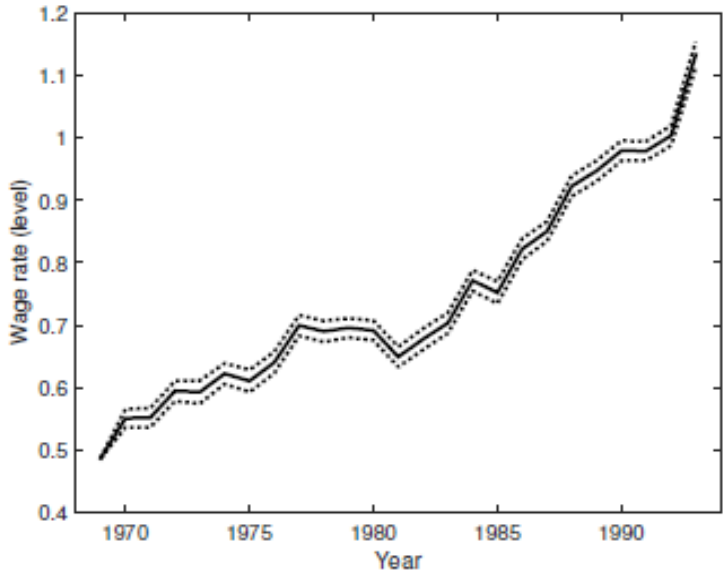
Timing of children, homeownership and house size (Khorunzhina and Miller, 2022)

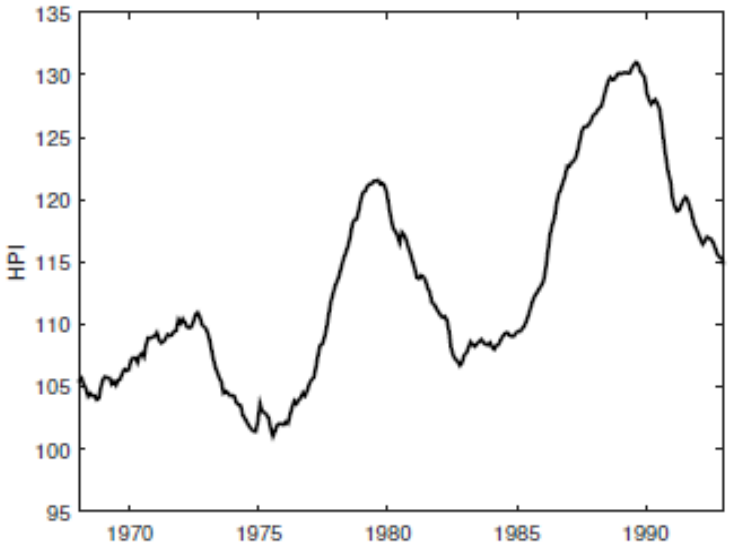


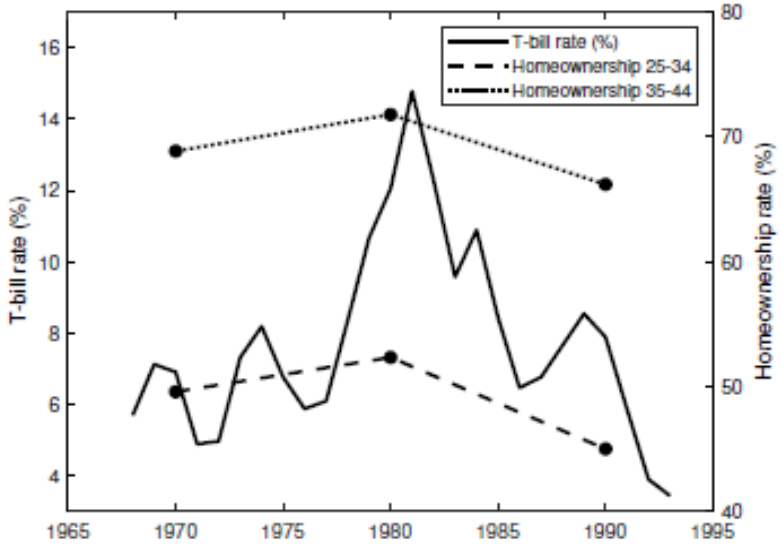
Lifecycle Trends over the Years

The driving factors: education, female wages, housing prices and interest rates









Lifecycle Trends over the Years

The effects of the driving factors (Khorunzhina and Miller, 2022)

- Two main factors have driven the trend to the decline in homeownership:
  1. rising house prices
  2. rising female wages, medical advances and increased education, .
- The second factor is significant because higher wages:
  - encourage women to increase their labor force participation.
  - postpone and reduce childbearing
  - home ownership is more valued by families with children.
- Two factors mitigate this trend:
  - Higher educated women are more likely to be homeowners.
  - Higher real interest rates increase home ownership: households save more when the interest rate increase, and home ownership is a savings vehicle.

Consumption and Saving over the Lifecyle

Linear utility and extremal consumption in a two period model

- Denote by c_t individual consumption in period $t \in \{1, 2\}$.
- Let $U(c_1, c_2)$ denote her lifetime utility, where:

$$U(c_1, c_2) = c_1 + \beta c_2$$

  and $\beta \in (0, 1)$ is the rate at which she discounts future consumption.
- The interest rate is i and her income in period $t \in \{1, 2\}$ is y_t.
- She chooses (c_1, c_2) to maximize $U(c_1, c_2)$ subject to her lifetime budget constraint:

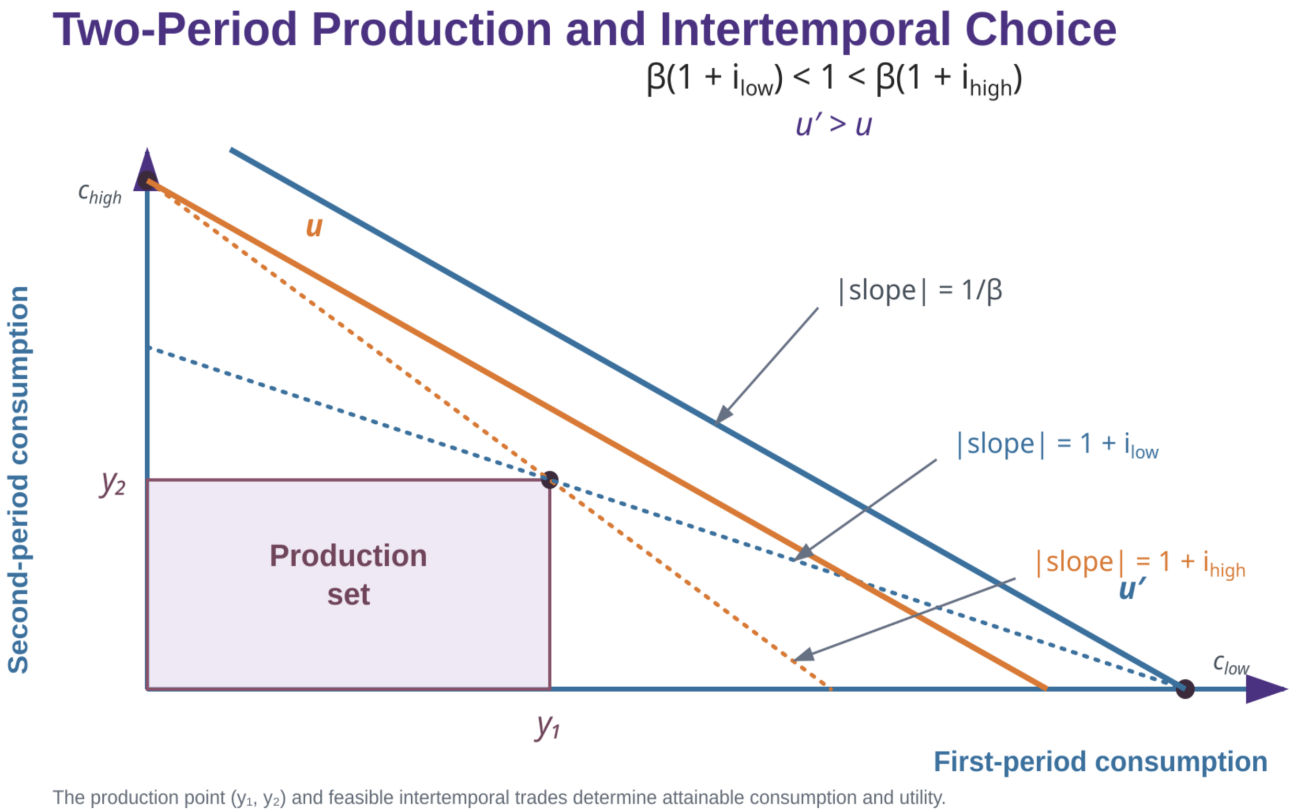
$$c_1 + c_2 / (1 + i) \leq y_1 + y_2 / (1 + i) \equiv Y$$

- Let c_t^o denote her optimal consumption in period $t \in \{1, 2\}$. Then:

$$(c_1^o, c_2^o) = \begin{cases} (Y, 0) & \text{if } \beta (1 + i) < 1 \\ (0, (1 + i) Y) & \text{if } \beta (1 + i) > 1 \end{cases}$$

Consumption and Saving over the Lifecyle

Future connsmption is not a perfect substitute for current consumption



- At the high interest rate i_{high} all consumption takes place in the second period at c_{high} yielding utility u.
- At the low interest rate i_{low} all consumption takes place in the second period at c_{low} yielding utility u'.

Preferences over Time

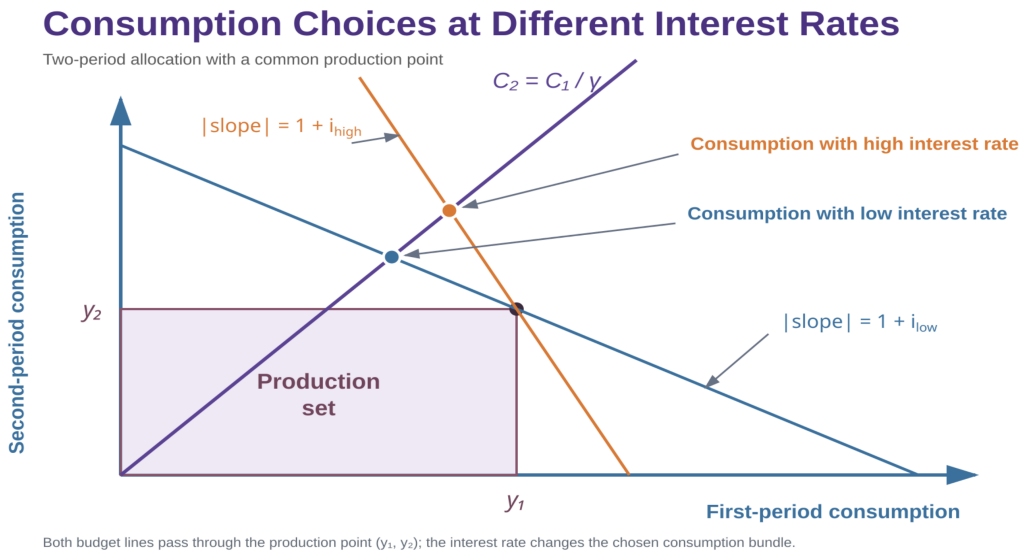
Future consumption is not a perfect complement to current consumption either

- The other extreme is that c_1 and c_2 are perfect complements, consumed in fixed proportions:

$$U(c_1, c_2) = \min\{c_1, \gamma c_2\}$$

- The optimal lifetime consumption is to set $c_1^o = \gamma c_2^o$. Solving for c_2^o:

$$c_2^o = Y \Big/ \left[(1+i)^{-1} + \gamma\right]$$



Preferences over Time

Generalizing to many periods

- More generally suppose an individual receives a *lifetime utility* of:

$$U(c_1, \ldots, c_T) = \sum_{t=1}^{T} \beta^t u(c_t)$$

where c_t is consumption in period t and $u(c_t)$ is concave increasing, meaning $u'(c) > 0$ and $u''(c) \leq 0$. Two key features:

1. The *marginal rate of substitution* between c_t and c_{t+1}:

$$MRS(c_{t+1}, c_t) = \frac{\partial U(c_1, \ldots, c_T) / \partial c_{t+1}}{\partial U(c_1, \ldots, c_T) / \partial c_t} = \frac{\beta u'(c_{t+1})}{u'(c_t)}$$

measures the amount of c_t a person is willing to sacrifice for an extra unit of c_{t+1} to attain the same level of utility, and comes from:

$$\frac{\partial U}{\partial c_{t+1}} dc_{t+1} + \frac{\partial U}{\partial c_t} dc_t = 0 \Longrightarrow MRS(c_{t+1}, c_t)\, dc_{t+1} + dc_t = 0$$

2. Noting $MRS(c, c) = \beta$, the *subjective discount factor* β measures the individual's impatience. Typically $0 < \beta < 1$.

Consumption and Saving over the Lifecyle

The risk free rate, wealth adjustment, and the intertemporal consumption tradeoff

- Denote by:
  - i_t denote the *real interest rate* in period t
  - $r_{t+1} = i_t + 1$ denote the *risk free return*, the extra consumption available in period $t+1$ by reducing consumption in period t by one unit.
  - W_t denote wealth at t, and y_t income received in period t, implying :

$$W_{t+1} = (W_t - c_t + w_t)\, r_{t+1}$$

- Reduce c_t by dc and increase c_{t+1} by $r_{t+1} dc$.
  - Lifetime expenditure is unchanged, but lifetime utility changes by:

$$\begin{aligned} & \beta^{t+1} \left[u\left(c_{t+1} + r_{t+1} dc\right) - u\left(c_{t+1}\right) \right] + \beta^t \left[u\left(c_t\right) - u\left(c_t - dc\right) \right] \\ = \quad & \beta^{t+1} u'\left(c_{t+1}\right) r_{t+1} dc - \beta^t u'\left(c_t\right) dc \end{aligned}$$

  If this expression is:
  - *positive*, then increase c_{t+1} and reduce c_t.
  - *negative*, then increase c_t and reduce c_{t+1}.
  - *zero*, then utility can't be increased by reallocating between t and $t+1$.

Consumption and Saving over the Lifecyle

The marginal rate of substitution function

- Denoting c_t^o the optimal level of c_t, it now follows that:

$$-\beta^t u'\left(c_t^o\right) + \beta^{t+1} u'\left(c_{t+1}^o\right) r_{t+1} = 0$$

or:

$$1 = r_{t+1} \frac{\beta u'\left(c_{t+1}^o\right)}{u'\left(c_t^o\right)} \equiv r_{t+1} MRS_{t+1}\left(c_{t+1}, c_t\right)$$

- For example, assuming that:

$$u(c) = \frac{1}{1-\alpha} c^{1-\alpha} \text{ for some } \alpha > 0 \text{ which implies } u'(c) = c^{-\alpha}$$

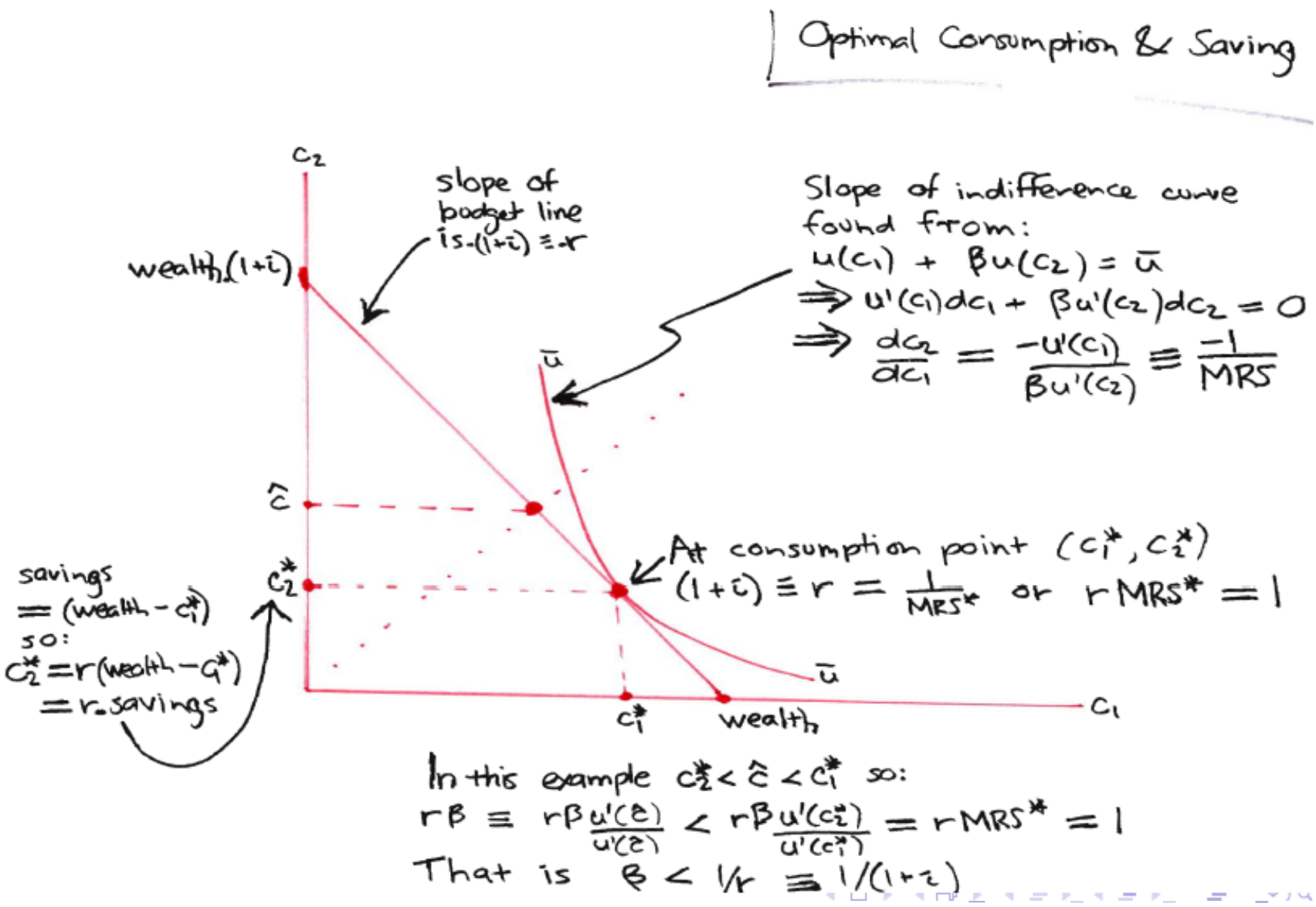
then:

$$1 = r_{t+1} MRS_{t+1}\left(c_{t+1}, c_t\right) = r_{t+1} \frac{\beta u'\left(c_{t+1}^o\right)}{u'\left(c_t^o\right)} = r_{t+1} \beta \left(\frac{c_t^o}{c_{t+1}^o}\right)^{\alpha}$$

- Notice that:
  - if $\beta \uparrow$ then $c_{t+1}^o \uparrow$ and $c_t^o \downarrow$
  - similarly if $r_{t+1} \uparrow$ then $c_{t+1}^o \uparrow$ and $c_t^o \downarrow$

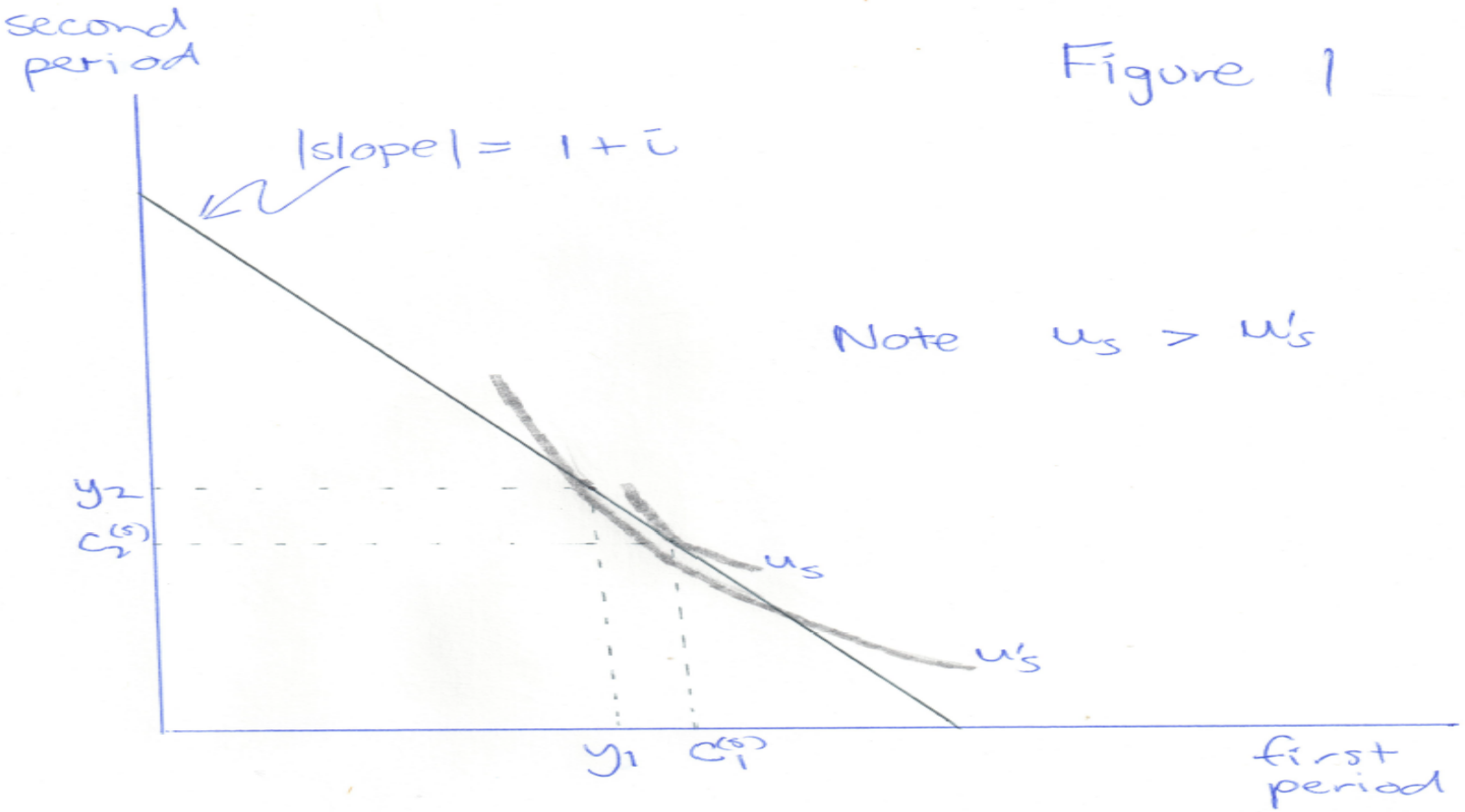
Consumption and Saving over the Lifecyle

Trading off current and future consumption in a two period lifecycle model



Consumption and Saving over the Lifecyle

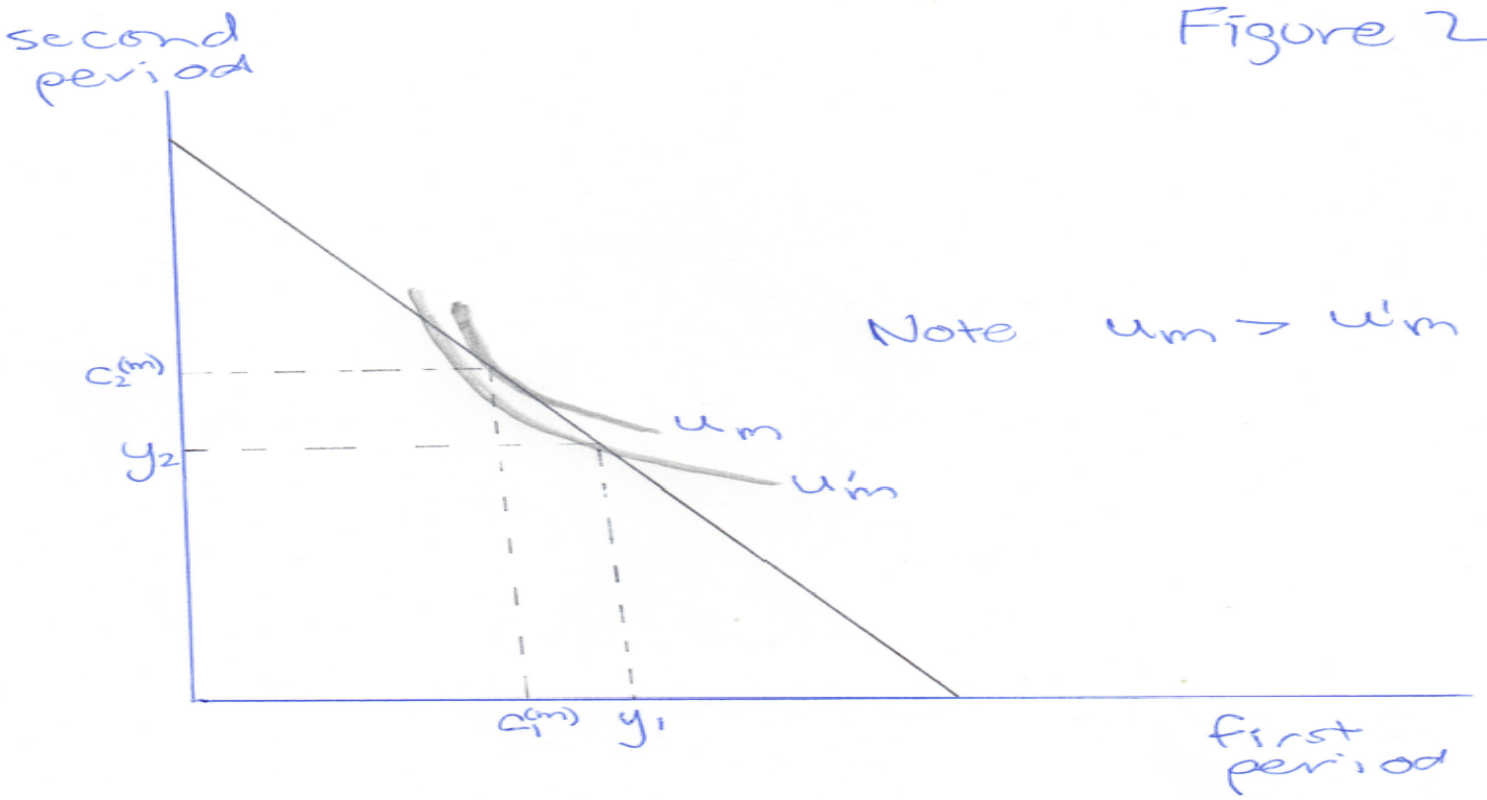
Borrowing in equilibrium



Spendthrift (s) borrows $(c_1^{(s)} - y_1)$ in first period and repays $(y_2 - c_2^{(s)}) = (c_1^{(s)} - y_1)(1+i)$ in the second.

Consumption and Saving over the Lifecyle

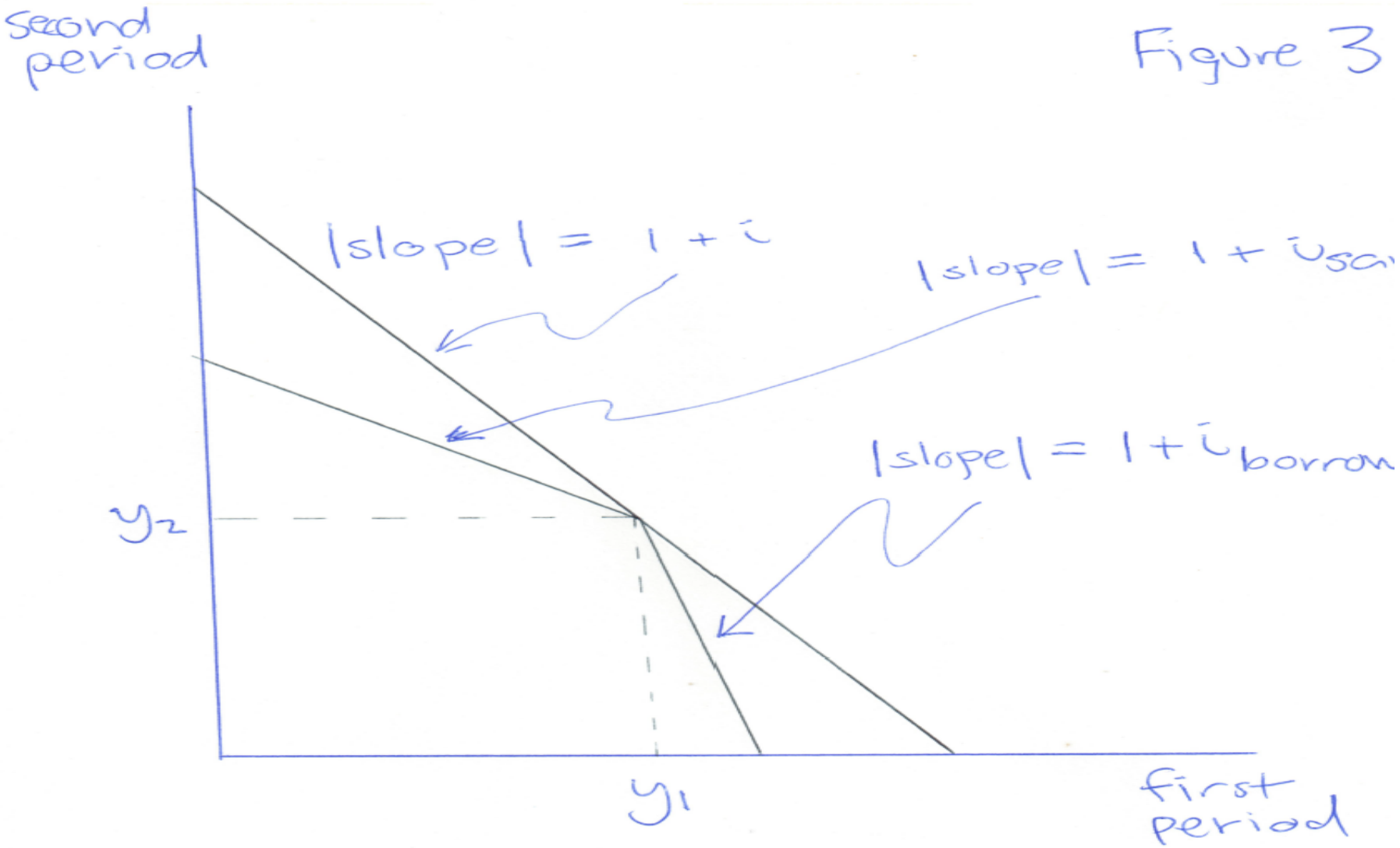
Saving in equilibrium



Miser (m) saves $(y_1^{(m)} - c_1^{(m)})$ in first period and spends an extra $(c_2^m - y_2) = (c_1^m - y_1)$ in the second.

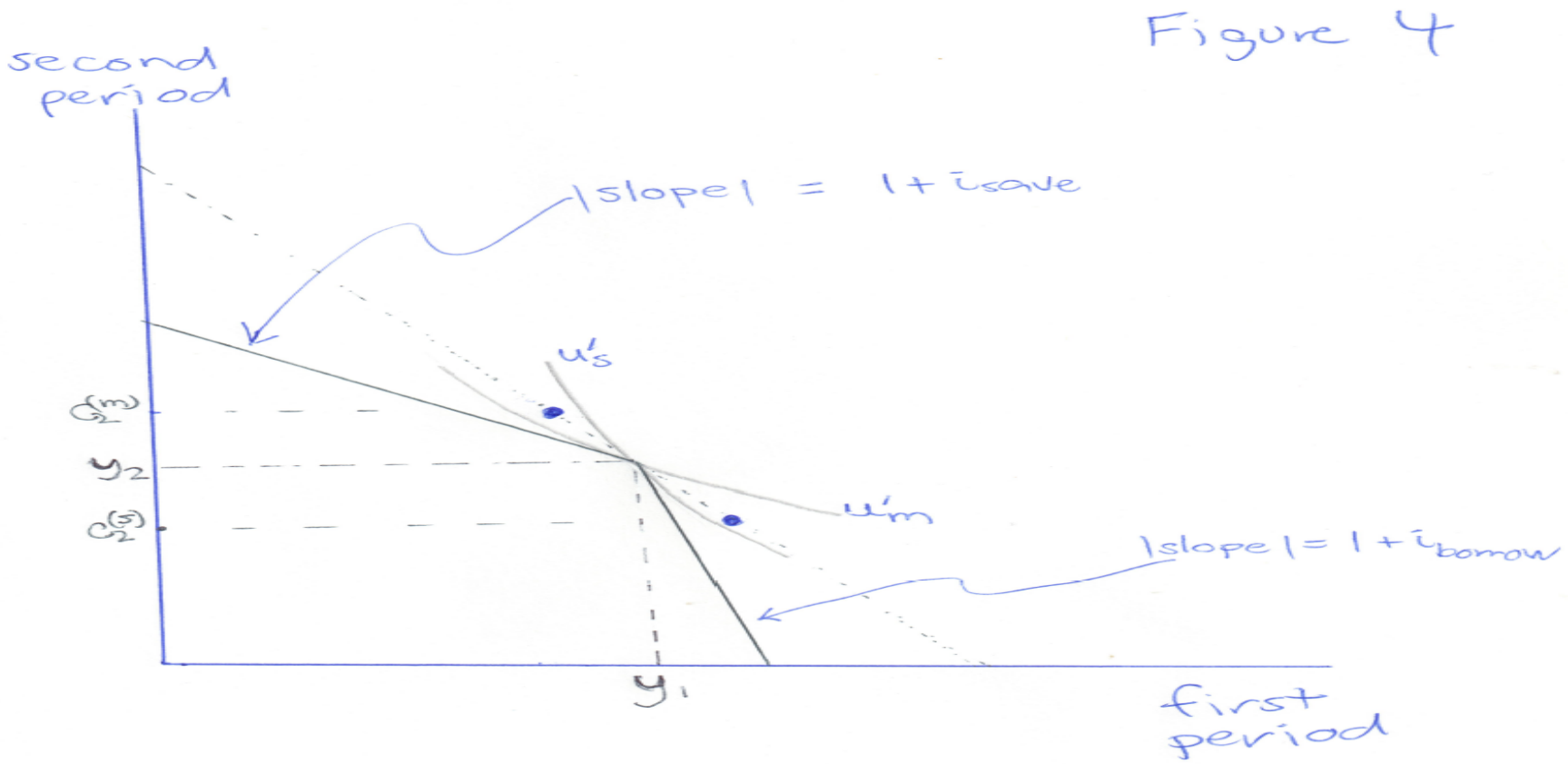
Consumption and Saving over the Lifecyle

A wedge is driven between the borrowing and lending rates



Consumption and Saving over the Lifecyle

Neother the miser nor the spendthrift save or borrow



Consumption and Saving over the Lifecyle

Can this wedge explain why Americans save so little?

- First of all they don't "save so little":
  - Social security is a form of forced savings.
  - Many contribute to defined benefit and defined retirement plans.
  - Many are eligible for health subsidies in their old age.
  - Households gradually acquire ownership of their own homes.
- Second of all these payments differ:
  - Social security payments and benefits differ by income.
  - Retirement plans are largely discretionary.
  - Health plans differ across households.
  - Homes differ vastly in their quality and amenities.
- Nevertheless people might spend their disposable income immediately because of the wedge between the borrowing and savings rates.
- Intertemporally sharing and managing wealth between borrower and lender is a costly activity.