Price Discovery

Robert A. Miller

Trade and Investment Strategy

November 2026

Introduction to Portfolio Investment

The building blocks of modern investment analysis

- APT provides some guidance about how assets closely related to each other are priced.
- But how do buyers and sellers form their individual valuations that ultimately determine the terms of trade?
- The notion of competitive equilibrium is the economist's standard tool for reconciling demand and supply in almost every market imaginable.
- Therefore it should come as no surprise that portfolio investment strategy is based on models of competitive equilibrium.
- Modern portfolio theory is based on three fundamental components:
  1. competitive equilibrium (or perfect liquidity).
  2. investor attitudes towards uncertainty and risk.
  3. the rate at which investors are willing to sacrifice current consumption for the certain gain of future consumption.

Liquidity

What is liquidity?

- There are many definitions of liquidity. They all relate to the idea of how easy or hard it is to buy and sell an asset, or how volatile the market is for idiosyncratic reasons.
- I adopt the following definition: An asset is perfectly liquid if and only if individual traders can buy and sell as many units as they wish at the same price they all anticipate.
- There is a close relationship between perfect liquidity and a competitive equilibrium. In a model of competitive equilibrium traders act as if all stocks are perfectly liquid.
- This is a common assumption in the analysis of portfolio investment, a topic to be addressed later in the course.

Liquidity

The law of one price

- Competitive equilibrium is sometimes referred to as . . .

. . . *the law of one price*.

- Can the concept of competitive equilibrium be adapted to limit order markets?
- Suppose there are many fully informed potential buyers and sellers, and no one believes he or she can influence transaction prices.
- Then:
  1. all trades transact at the same price.
  2. the constant transaction price is the bid or the ask.
  3. the market allocates resources perfectly . . . to investors who value the asset the most.

Liquidity

The spread is a poor measure of liquidity

- Because every (buy and sell) limit order is transacted at one price, no trading occurs at any other price.
- That is limit order sell (buy) orders placed above (below) the constant transaction price are never filled.
- Therefore, there is no reason to place:
  1. a limit sell order if the bid is at the transaction price
  2. a limit buy order if the ask is at the transaction price
- Hence the spread could well be infinite.
- Although not a perfect analogy, in some taxicab ranks:
  - sometimes there are unfilled orders (as passengers queue on the curbside)
  - sometimes there is excess inventory (as cabs queue in the rank)

  . . . but the fare does not change!
- This point also demonstrates that the size of the spread is a very misleading measure of market liquidity.

Competitive Equilibrium

Exhausting the gains from trade

- There are two ways of making profitable trades in a limit order market . . .
- Create liquidity by placing limit orders (adding to the book):
  - with a quote that straddles your valuation.
- Dissolve liquidity by filling limit orders (enlarging the spread):
  - when the bid is less than your valuation (placing market buy orders)
  - or the ask is greater than your valuation (placing market sell orders).
- Suppose all bidders place a profitable:
  1. market order if that is feasible.
  2. limit order if not.

$\Longrightarrow$ if the limit orders are placed sufficiently close to the valuation (so the personal bid and ask are nearly equal) then a competitive equilibrium allocation is iteratively attained.

Competitive Equilibrium

Trading off immediacy against price

- Why does the law of one price break down when there are many traders on both sides of the market?

  - Now suppose new orders infrequently.
  - If individuals don't care when their orders transact, then the law of one price could be applied here too.
  - But if they care how quickly their order is filled, they might pay a premium to transact earlier (and jump the "queue").
  - This breaks the law of one price.
  - When investors have preferences over "immediacy", we could interpret an LOM for a single security as a mechanism for trading multiple products differentiated by their placement and withdrawal time window.

Competitive Equilibrium

Large numbers of traders on both sides of the market

- From our previous work on first price sealed bid auctions we know that the more bidders there are closer each one places his or her order to her true valuation.
- Intuitively, as the chance of winning the auction declines with the number of bidders, each one compensates a little by reducing his or her mark-up (padding).
- The same thing is true of limit order markets. As the number of competitors increase traders trim their mark-up on limit buy orders:
  - raising their personal limit order buy bids
  - reducing their personal limit order offers sell
  - closing their personal spreads around their valuations
- The limit of this process (as the number of traders increases) is a competitive equilibrium.

Competitive Equilibrium

Equilibrium in the landing gates game

Denote by:

- d_k demand for landing gates by airline k
- s_k supply of landing gates by airline k
- $s = s_1 + s_2 + ... + s_6$ total supply of gates
- $v_d = \min\{v_k : d_k > 0\}$ the lowest valuation airline getting a gate
- $v_s = \max\{v_k : d_k = 0\}$ the highest valuation airline not getting a gate

---

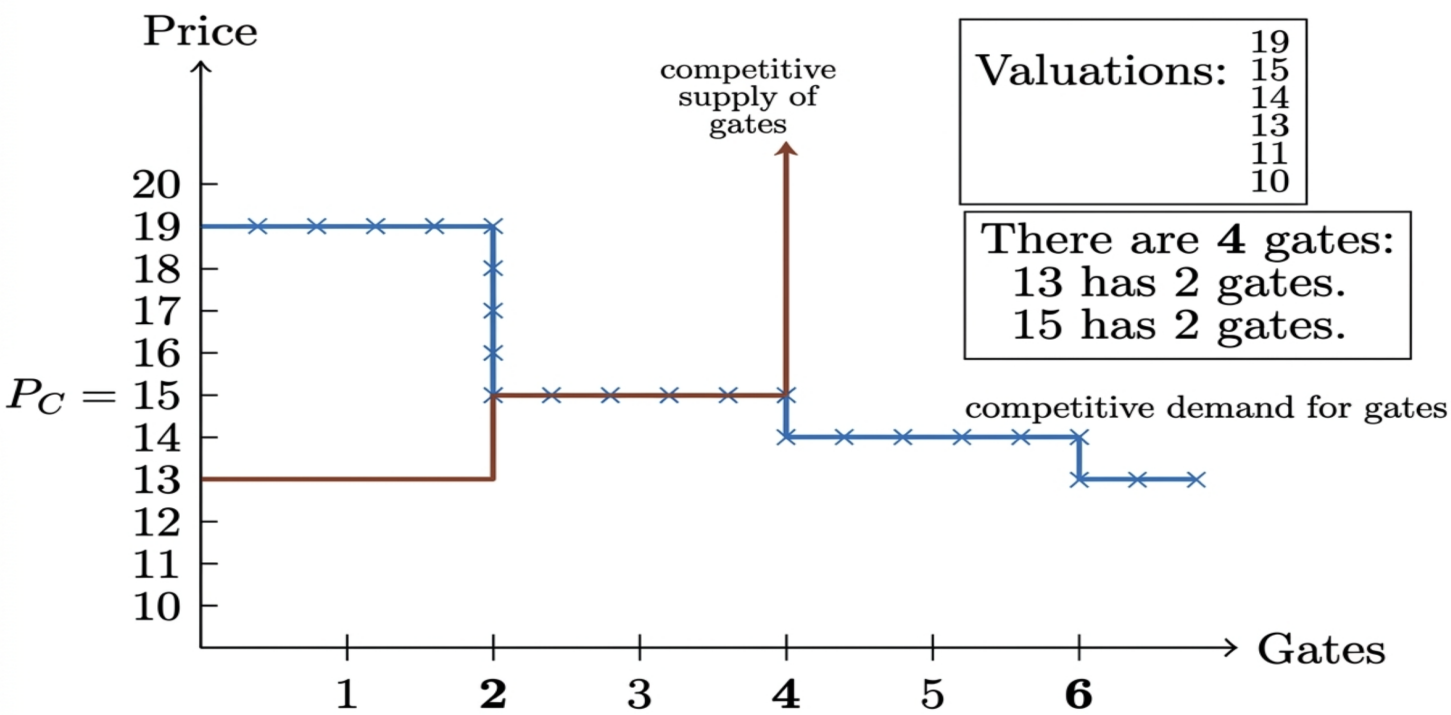
A competitive equilibrium is a price p_c satisfying:

1. $v_d \geq p_c \geq v_s$
2. If $v_k > p_c$ then airline k is allocated 2 gates
3. If $v_k < p_c$ then airline k is allocated 0 gates
4. The total number of allocated gates is s.

Competitive Equilibrium

Exhausting the gains from trade

- If the price is higher than 15 then suppliers would offer 4 units.
- If the price is lower than 13 then supply would dry up.
- The market clears at 15 with 2 units sold from trader 10 to trader 19.
- Only traders 13 and 19 benefit from the market.



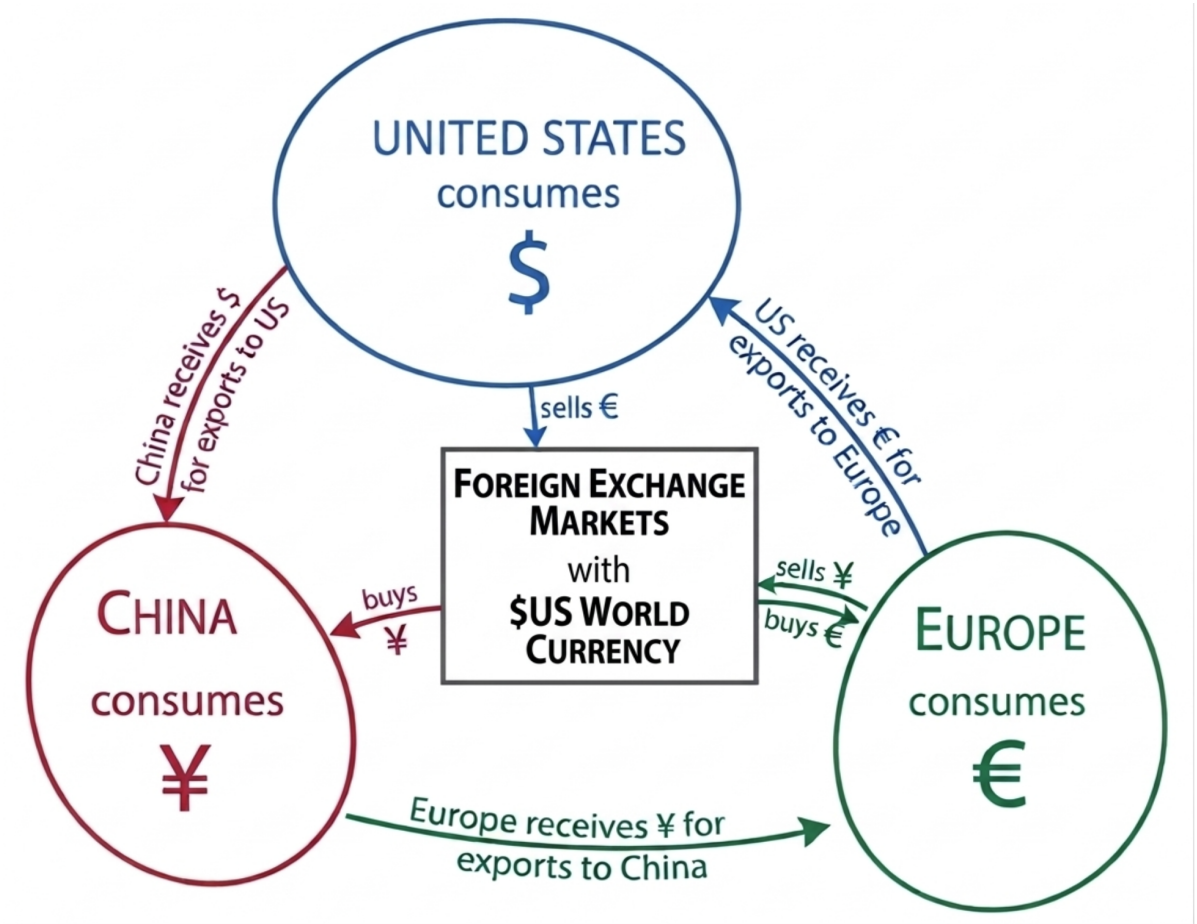
Currency Exchange

Three markets

- Suppose US export companies sporadically receive euro injections from sales in the EU.
- Similarly European (Chinese) exporters earn yuan (dollars) for sales in China (the US).
- Export firms can also on the foreign exchange market between date 0 and T, but at date T all export companies are liquidated and no further value is placed on holding foreign currency.
- We assume the US dollar is a dominant currency, meaning all currency prices are quoted in dollars.
- We assume each export firm maximizes its expected accumulated domestic reserves before the liquidation date T.

Currency Exchange

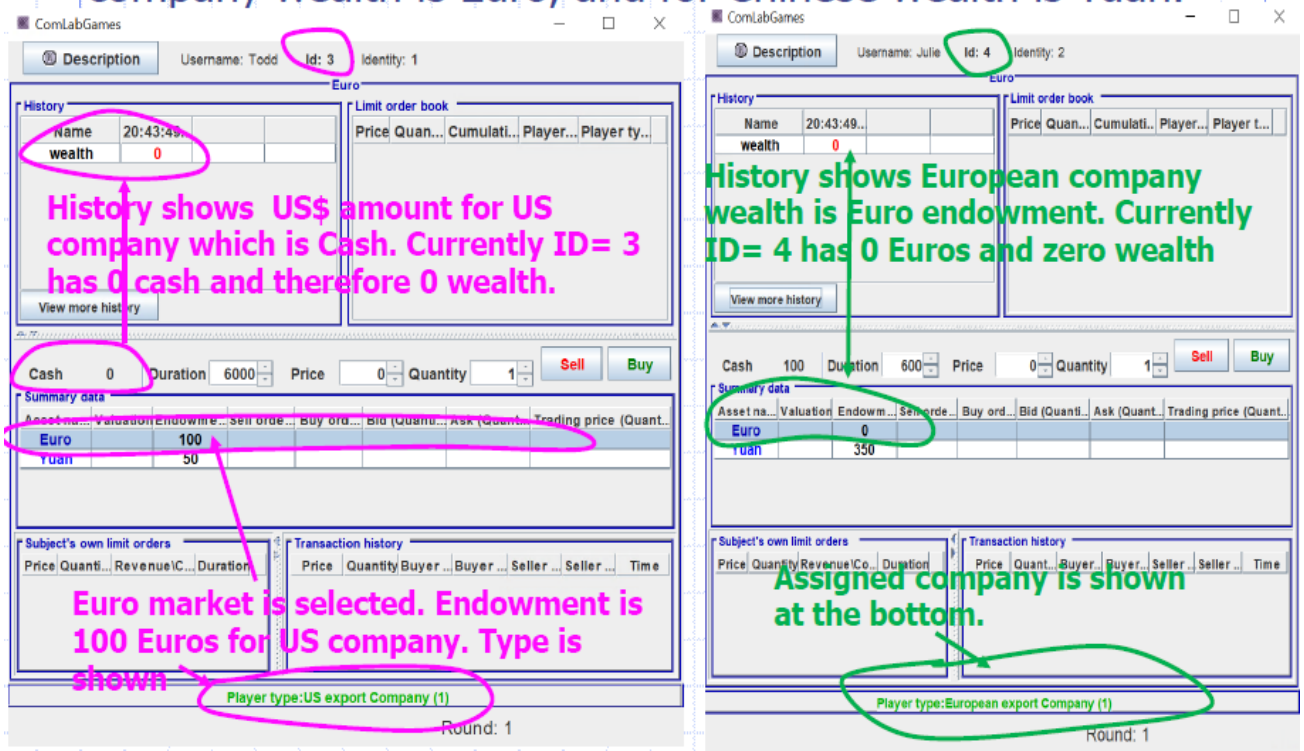
A flow chart



Currency Exchange

Trading window

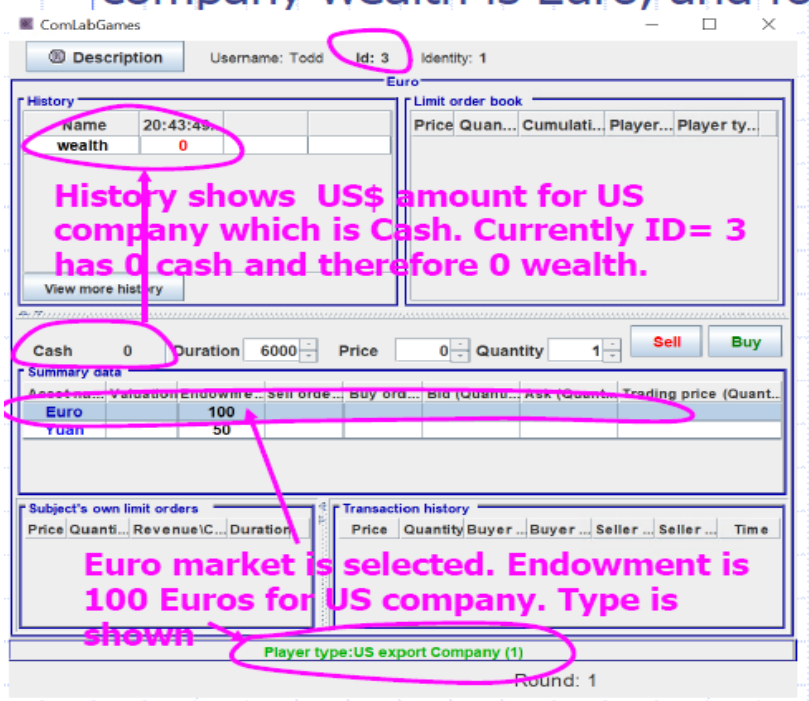
1. To trade in a specific market like "Euro", click on that market .
2. The selected market is highlighted in blue in "Summary data".
3. US export company's wealth is cash (i.e. US $), for European company wealth is Euro, and for Chinese wealth is Yuan.

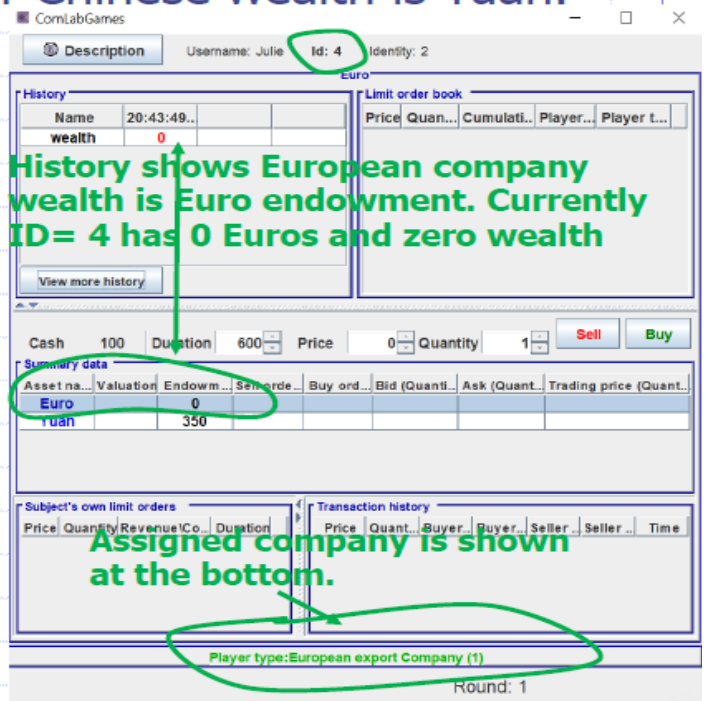


Currency Exchange

Trading window

1. To trade in a specific market like "Euro", click on that market.
2. The selected market is highlighted in blue in "Summary data".
3. US export company's wealth is cash (i.e. US $), for European company wealth is Euro, and for Chinese wealth is Yuan.

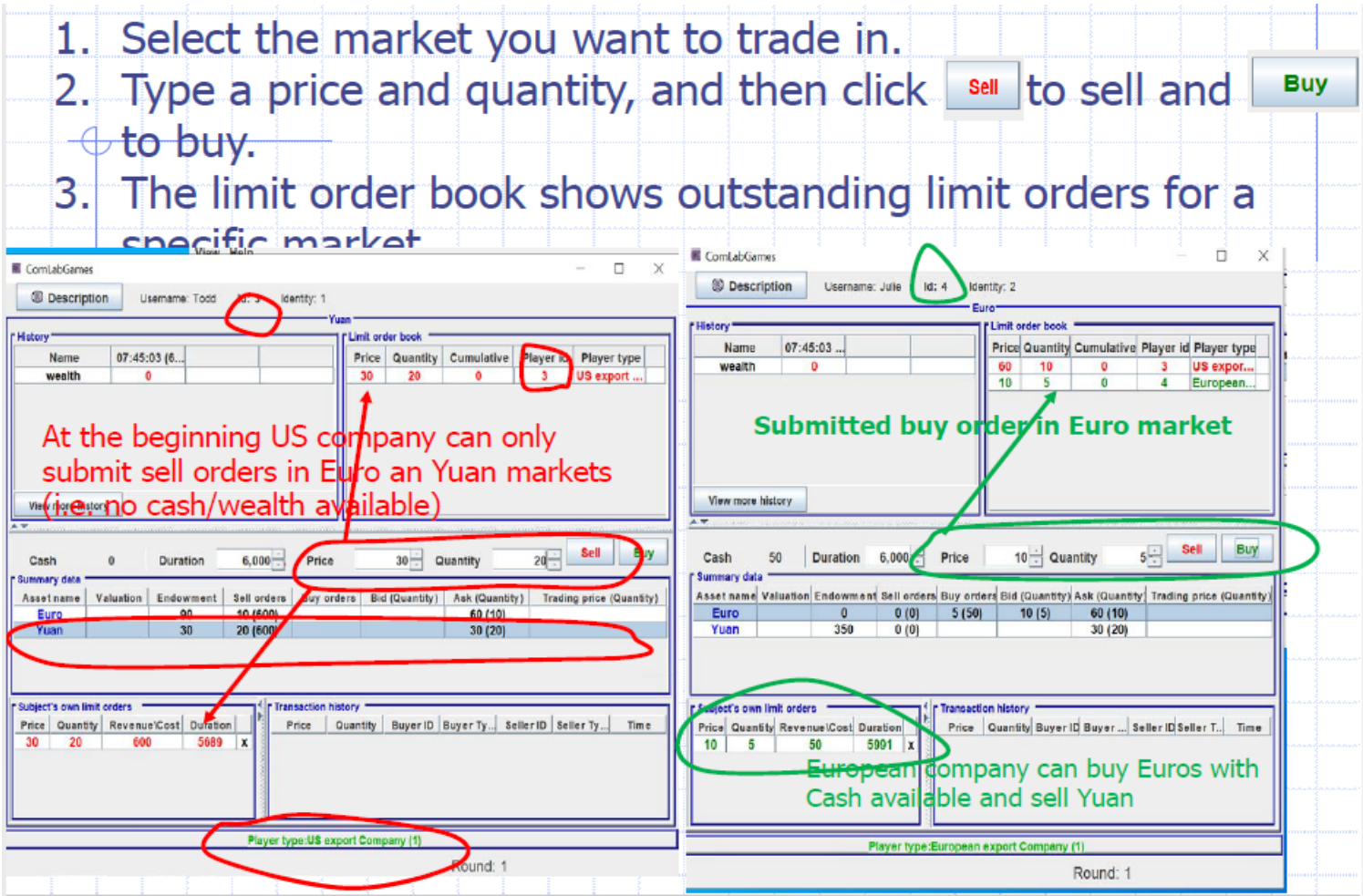




Currency Exchange

Submitting a price & quantity in a specific market

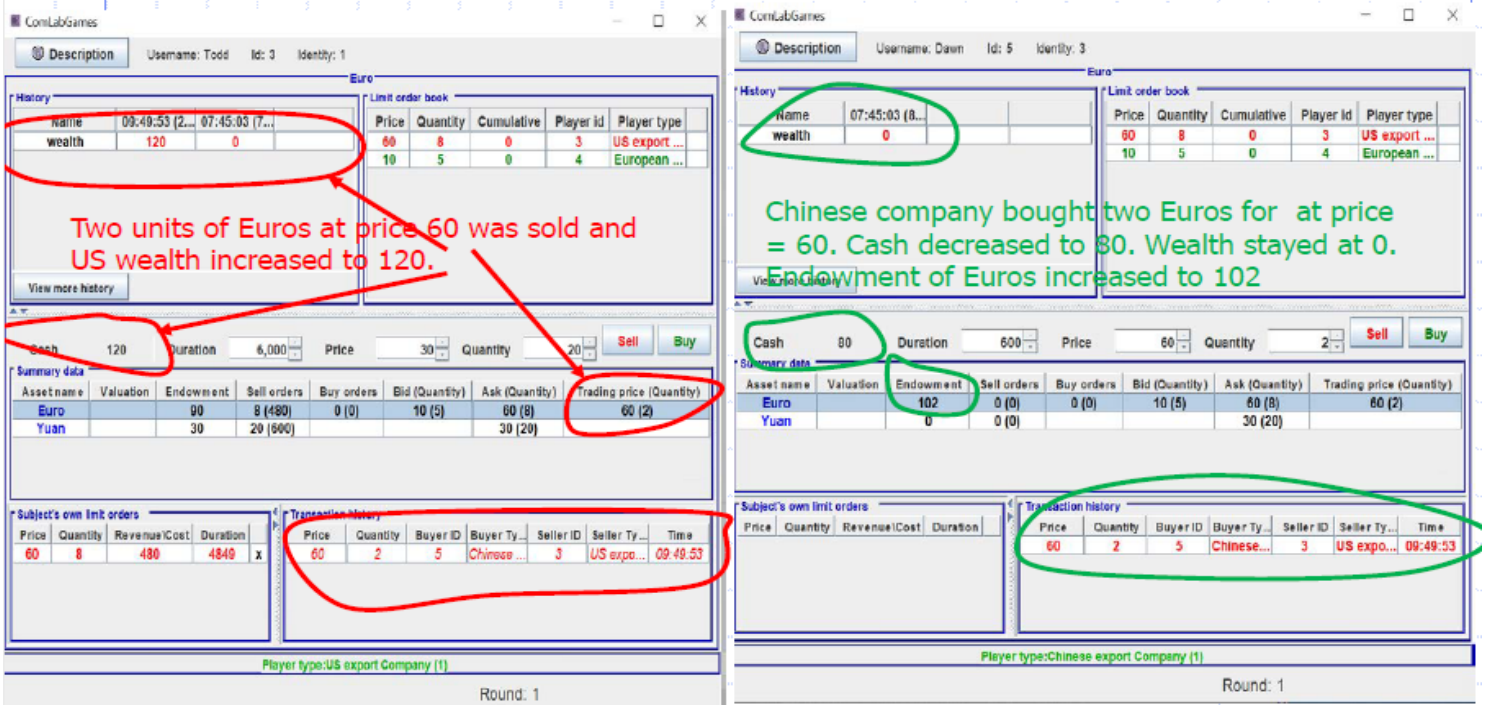
1. Select the market you want to trade in.
2. Type a price and quantity, and then click Sell to sell and Buy to buy.
3. The limit order book shows outstanding limit orders for a specific market



Currency Exchange

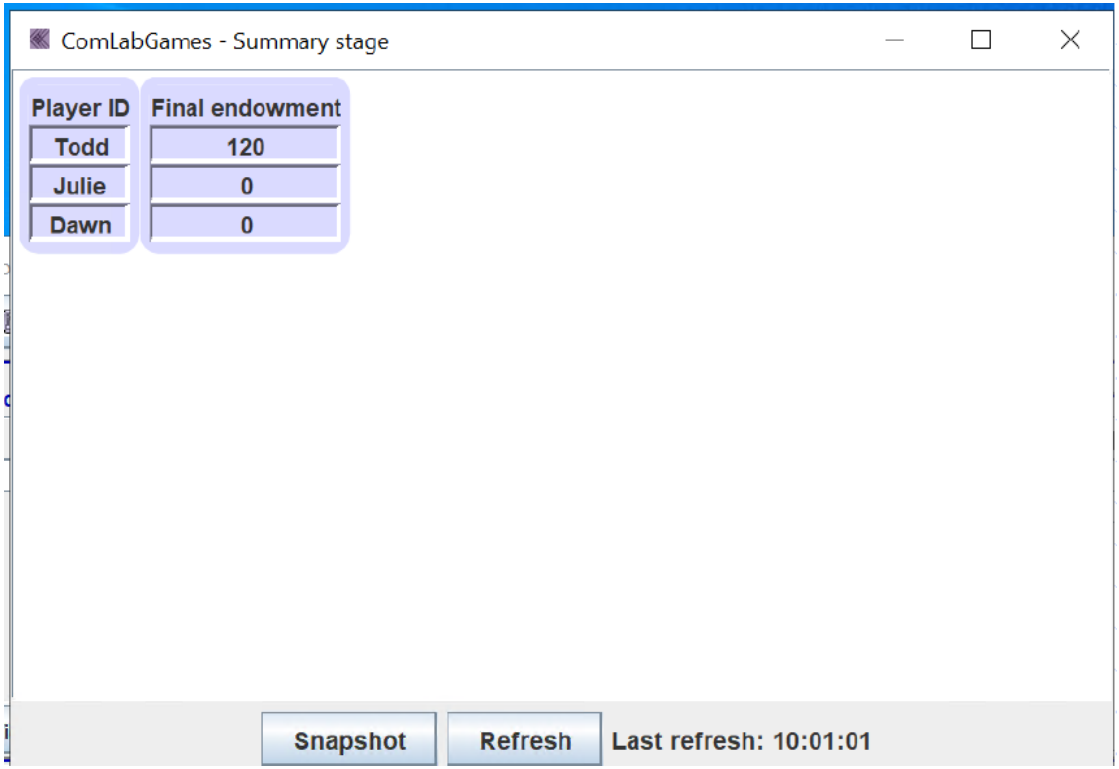
Transactions

1. To buy/sell directly select the best order by clicking on the price of the order and click Buy / Sell .
2. Summary data and Transaction history show the executed order.



Currency Exchange

Summary page



Currency Exchange

Competitive equilibrium as a tool for price discovery

- In a multimarket setting, it is hard to intuit where prices will settle.
- The concept of *competitive equilibrium* is a useful tool for predicting what prices will emerge.
- Technically, a competitive equilibrium (CE) for multiple markets:
  - is a (relative) *price for every market* but one (the numeraire)
  - where *suppliers and demanders* trade at those prices
  - so that *markets clear* (and supply equals demand)
- In a CE all potential gains from trade are realized:
  - a CE allocates resources efficiently.
  - those willing to pay the most buy the goods and services.

Currency Exchange

Competitive equilibrium defined for this game

- In this model of international trade there are technically six markets
  - three export commodities plus three currencies.
- However matching supply to demand in this CE is trivial because:
  - aggregate demand for each export commodity comes from just one currency area.
  - each exporting firm only values its own currency.
- It follows that in CE:
  - US buys all of the 300 dollars supplied
  - EU buys all the 200 euros supplied
  - China buys all the 400 yuan supplied
- Therefore solving for the CE amounts to finding an US dollar exchange rate for both euros and yuan to equate the supply and demand in the currency markets.

Currency Exchange

Supply of foreign exchange

	$	€	¥
US	–	100	50
CHINA	200	100	–
EUROPE	100	–	350
TOTAL	**300**	**200**	**400**

Currency Exchange

Solving for the competitive equilibrium

Using competitive equilibrium analysis:

$$\begin{aligned} D_{US} &= P_{¥}50 + P_{€}100 &&= 300 \\ P_{¥}D_C &= 200 + P_{€}100 &&= P_{¥}400 \\ P_{€}D_E &= 100 + P_{¥}350 &&= P_{€}200 \end{aligned}$$

---

(1) $D_{US} - P_{¥}D_C$:

$$P_{¥}50 - 200 = 300 - P_{¥}400$$

$$\Rightarrow P_{¥} = \frac{500}{450} = \frac{10}{9} \approx 1.1$$

Using (1) and D_{US}:

$$P_{€} = \frac{300 - P_{¥}50}{100}$$

$$= 3 - \frac{1}{2} \cdot \frac{10}{9} = \frac{22}{9} \approx \mathbf{2.4}$$

Currency Exchange

Walras law

- Notice that we only use the first two lines to obtain the prices.
- Walras law say that is good enough:
  - The third market will clear at those prices.
  - More generally if there are N markets we can just focus on any $(N-1)$ of them.
- Confirming the law in this case:

Substitute $P_{¥} = \frac{10}{9}$ and $P_{€} = \frac{22}{9}$ into the third market equation $(P_{€}D_E)$:

$$100 + P_{¥}350 = P_{€}200$$

$$100 + \frac{10}{9} \cdot 350 = \frac{22}{9} \cdot 200$$

Evaluate both sides:

$$\frac{900}{9} + \frac{3500}{9} = \frac{4400}{9}$$

$$\frac{4400}{9} = \frac{4400}{9} \quad (\approx 488.89)$$