

Price Discovery

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Trade and Investment Strategy

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Introduction to Portfolio Investment

The building blocks of modern investment analysis

MORE NEEDED ON DERIVATION OF EQUILIBRIUM AND WALRAS LAW

- APT provides some guidance about how assets closely related to each other are priced.
- But how do buyers and sellers form their individual valuations that ultimately determine the terms of trade?
- The notion of competitive equilibrium is the economist's standard tool for reconciling demand and supply in almost every market imaginable.
- Therefore it should come as no surprise that portfolio investment strategy is based on models of competitive equilibrium.
- Modern portfolio theory is based on three fundamental components:
 - ① competitive equilibrium (or perfect liquidity).
 - ② investor attitudes towards uncertainty and risk.
 - ③ the rate at which investors are willing to sacrifice current consumption for the certain gain of future consumption.

Liquidity

What is liquidity?

- ◆ There are many definitions of **liquidity**. They all relate to the idea of how easy or hard it is to buy and sell an asset, or how volatile the market is for idiosyncratic reasons.
- ◆ I adopt the following definition: An asset is **perfectly liquid** if and only if individual traders can buy and sell as many units as they wish at the same price they all anticipate.
- ◆ There is a close relationship between perfect liquidity and a **competitive equilibrium**. In a model of competitive equilibrium traders act as if all stocks are perfectly liquid.
- ◆ This is a common assumption in the analysis of portfolio investment, a topic to be addressed later in the course.

Liquidity

The law of one price

- Competitive equilibrium is sometimes referred to as the law of one price.
 - ◆ Can the concept of competitive equilibrium be adapted to limit order markets?
 - ◆ Suppose there are many fully informed potential buyers and sellers, and no one believes he or she can influence transaction prices.
 - ◆ Then:
 1. all trades transact at the same price.
 2. the constant transaction price is the bid or the ask.
 3. the market allocates resources perfectly . . . to investors who value the asset the most.

Liquidity

The spread is a poor measure of liquidity

- ◆ Because every (buy and sell) limit order is transacted at one price,
no trading occurs at any other price.
- ◆ That is limit order sell (buy) orders placed above (below) the
constant transaction price are never filled.
- ◆ Therefore, there is no reason to place:
 1. a limit sell order if the bid is at the transaction price
 2. a limit buy order if the ask is at the transaction price
- ◆ Hence the spread could well be infinite.
- ◆ Although not a perfect analogy, in some taxicab ranks:
 - ❑ sometimes there are unfilled orders (as passengers queue on the curbside)
 - ❑ sometimes there is excess inventory (as cabs queue in the rank)*... but the fare does not change!*
- ◆ This point also demonstrates that the size of the spread is a very
misleading measure of market liquidity.

Competitive Equilibrium

Exhausting the gains from trade

- ◆ There are two ways of making profitable trades in a limit order market . . .
 - ◆ **Create liquidity** by placing limit orders (adding to the book):
 - with a quote that straddles your valuation.
 - ◆ **Dissolve liquidity** by filling limit orders (enlarging the spread):
 - when the bid is less than your valuation (placing market buy orders)
 - or the ask is greater than your valuation (placing market sell orders).
 - ◆ Suppose all bidders place a profitable:
 1. market order if that is feasible.
 2. limit order if not.
- ⇒ if the limit orders are placed sufficiently close to the valuation (so the personal bid and ask are nearly equal) then a **competitive equilibrium allocation** is iteratively attained.

Competitive Equilibrium

Trading off immediacy against price

- Why does the law of one price break down when there are many traders on both sides of the market?

- ◆ Now suppose new orders infrequently.
- ◆ If individuals don't care when their orders transact, then the **law of one price** could be applied here too.
- ◆ But if they care how quickly their order is filled, they might pay a premium to transact earlier (and jump the "queue").
- ◆ This breaks the law of one price.
- ◆ When investors have preferences over "immediacy", we could interpret an LOM for a single security as a mechanism for trading **multiple products differentiated by their placement and withdrawal time window**.

Competitive Equilibrium

Equilibrium in the landing gates game

- price clearing

◆ Denote by:

- d_k demand for landing gate by airline k
- s_k supply of landing gates by airline k
- $s = s_1 + s_2 + \dots + s_6$ total supply of gates
- $v_d = \min\{v_k : d_k > 0\}$ the lowest valuation airline getting a gate
- $v_s = \max\{v_k : d_k = 0\}$ the highest valuation airline not getting a gate

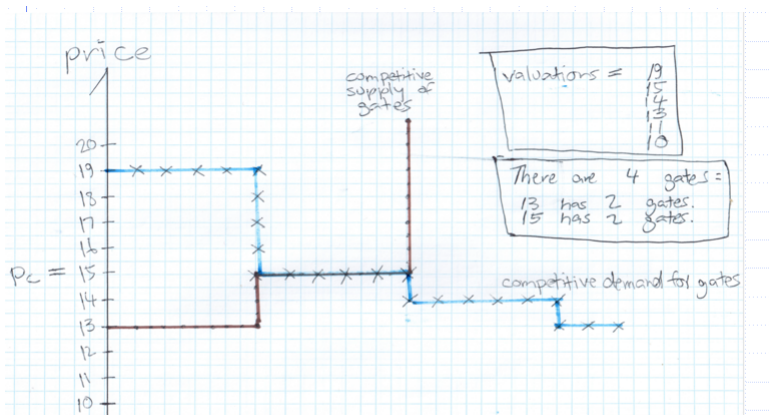
◆ A competitive equilibrium is a price p_c satisfying:

1. $v_d \geq p_c \geq v_s$
2. If $v_k > p_c$ then airline k is allocated 2 gates
3. If $v_k < p_c$ then airline k is allocated 0 gates
4. The total number of allocated gates is s .

Competitive Equilibrium

Exhausting the gains from trade

- If price was higher than 15 then suppliers would offer 4 units
- If price was lower than 13 supply would dry up
- price clearing occurs between 13 and 15 with 2 units sold from trader with valuation 10 to trader with valuation 19



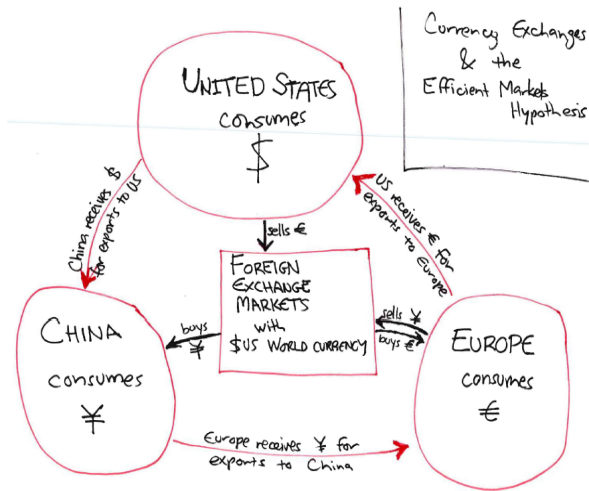
Currency Exchange

A hypothetical world

- Suppose US export companies sporadically receive euro injections from sales in the EU.
- Similarly European (Chinese) exporters earn yuan (dollars) for sales in China (the US).
- Export firms can also on the foreign exchange market between date 0 and T , but at date T all export companies are liquidated and no further value is placed on holding foreign currency.
- We assume the US dollar is a dominant currency, meaning all currency prices are quoted in dollars.
- We assume each export firm maximizes its expected accumulated domestic reserves before the liquidation date T .

Currency Exchange

A flow chart



Currency Exchange

Trading window

1. To trade in a specific market like "Euro", click on that market.
2. The selected market is highlighted in blue in "Summary data".
3. US export company's wealth is cash (i.e. US \$), for European company wealth is Euro, and for Chinese wealth is Yuan.

ComLabGames Username: Todd Id: 1 Identity: 1

History

Name	20:43:49
wealth	0

Limit order book

Price	Quant...	Cumulat...	Player...	Player ty...
-------	----------	------------	-----------	--------------

View more history

Cash 0 Duration 6000 Price 0 Quantity 1 Sell Buy

Summary data

Asset na...	Valuat...	Endowm...	St...	Buy and...	Bid (Quant...	Ask (Quant...	Trading price (Quant...
Euro	100						
Yuan	50						

Subject's own limit orders

Price	Quant...	Revenue/C...	Duration
-------	----------	--------------	----------

Transaction history

Price	Quant...	Buyer	Seller	Seller	Time
-------	----------	-------	--------	--------	------

Player type: US export Company (1)

Round: 1

History shows US\$ amount for US company which is Cash. Currently ID= 3 has 0 cash and therefore 0 wealth.

Euro market is selected. Endowment is 100 Euros for US company. Type is shown

ComLabGames Username: Julie Id: 4 Identity: 2

History

Name	20:43:49
wealth	0

Limit order book

Price	Quant...	Cumulat...	Player...	Player L...
-------	----------	------------	-----------	-------------

View more history

Cash 100 Duration 6000 Price 0 Quantity 1 Sell Buy

Summary data

Asset na...	Valuat...	Endowm...	St...	Buy and...	Bid (Quant...	Ask (Quant...	Trading price (Quant...
Euro	0						
Yuan	350						

Subject's own limit orders

Price	Quant...	Revenue/C...	Duration
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Transaction history

Price	Quant...	Buyer	Seller	Seller	Time
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Player type: European export Company (1)

Round: 1

History shows European company wealth is Euro endowment. Currently ID= 4 has 0 Euros and zero wealth

Assigned company is shown at the bottom.

Currency Exchange

Trading window

1. To trade in a specific market like "Euro", click on that market.
2. The selected market is highlighted in blue in "Summary data".
3. US export company's wealth is cash (i.e. US \$), for European company wealth is Euro, and for Chinese wealth is Yuan.

ComLabGames Username: Todd Id: 3 Identity: 1

History

Name	20:43:49
wealth	0

Limit order book

Price	Quan...	Cumulati...	Player...	Player ty...
-------	---------	-------------	-----------	--------------

Summary data

Asset na...	Valued...	Endowm...	St...	Buy ord...	Sell ord...	Buy Quant...	Sell Quant...	Trading price (Quant...
Euro		100						
Yuan		50						

Subject's own limit orders

Price	Quantity	Revenue/C...	Duration
-------	----------	--------------	----------

Transaction history

Price	Quantity	Buyer	Buyer	Seller	Seller	Time
-------	----------	-------	-------	--------	--------	------

Player type: US export Company (1)

Round: 1

History shows US\$ amount for US company which is Cash. Currently ID= 3 has 0 cash and therefore 0 wealth.

Euro market is selected. Endowment is 100 Euros for US company. Type is shown

ComLabGames Username: Julie Id: 4 Identity: 2

History

Name	20:43:49
wealth	0

Limit order book

Price	Quan...	Cumulati...	Player...	Player ty...
-------	---------	-------------	-----------	--------------

Summary data

Asset na...	Valued...	Endowm...	St...	Buy ord...	Sell ord...	Buy Quant...	Sell Quant...	Trading price (Quant...
Euro		6						
Yuan		350						

Subject's own limit orders

Price	Quantity	Revenue/C...	Duration
-------	----------	--------------	----------

Transaction history

Price	Quantity	Buyer	Buyer	Seller	Seller	Time
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Player type: European export Company (1)

Round: 1

History shows European company wealth is Euro endowment. Currently ID= 4 has 0 Euros and zero wealth

Assigned company is shown at the bottom.

Currency Exchange

Submitting a price & quantity in a specific market

1. Select the market you want to trade in.
2. Type a price and quantity, and then click **Sell** to sell and **Buy** to buy.
3. The limit order book shows outstanding limit orders for a specific market

Left Screenshot (Yuan Market):

- Limit order book:** Shows Price 30, Quantity 20, Cumulative 0, Player 3, Player type US export...
- Cash:** 0
- Duration:** 6,000
- Price:** 30
- Quantity:** 20
- Buttons:** Sell, Buy
- Summary data:** Asset name, Valuation, Endowment, Sell orders, Buy orders, Bid Quantity, Ask Quantity, Trading price (Quantity).
- Subjects own limit orders:** Price 30, Quantity 20, Revenue/Cost 0/0, Duration 6000, X.
- Transaction history:** Price, Quantity, Buyer ID, Buyer Ty, Seller ID, Seller Ty, Time.
- Player type:** US export Company (1)
- Round:** 1

Right Screenshot (Euro Market):

- Limit order book:** Shows Price 10, Quantity 5, Cumulative 0, Player 4, Player type European...
- Cash:** 50
- Duration:** 6,000
- Price:** 10
- Quantity:** 5
- Buttons:** Sell, Buy
- Summary data:** Asset name, Valuation, Endowment, Sell orders, Buy orders, Bid Quantity, Ask Quantity, Trading price (Quantity).
- Subjects own limit orders:** Price 10, Quantity 5, Revenue/Cost 0/0, Duration 6000, X.
- Transaction history:** Price, Quantity, Buyer ID, Buyer Ty, Seller ID, Seller Ty, Time.
- Player type:** European export Company (1)
- Round:** 1

Annotations:

- Red:** At the beginning US company can only submit sell orders in Euro and Yuan markets (i.e. no cash/wealth available).
- Green:** Submitted buy order in Euro market.
- Green:** European company can buy Euros with Cash available and sell Yuan.

Currency Exchange

Transactions

1. To buy/sell directly select the best order by clicking on the price of the order and click **Buy** / **Sell**.
2. Summary data and Transaction history show the executed order.

ContLabGames

Description Username: Todd Id: 3 Identity: 1

History

Name	09:49:53 (L... 07:45:03 (T...
wealth	120 3

Two units of Euros at price 60 was sold and US wealth increased to 120.

View more history

Asset name Valuation Endowment Sell orders Buy orders Bid (Quantity) Ask (Quantity) Trading price (Quantity)

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Euro	60	8 (480)	8 (5)	10 (5)	99 (8)	60 (2)	
Yuan	20	29 (595)					

Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration
60	8	480	4840

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time
60	2	5	Chinese	3	US expo...	09:48:53

Player type: US export Company (1)

Round: 1

ContLabGames

Description Username: Dawn Id: 5 Identity: 3

History

Name	07:45:03 (B...
wealth	0

Chinese company bought two Euros for at price = 60. Cash decreased to 80. Wealth stayed at 0. Endowment of Euros increased to 102

View more history

Asset name Valuation Endowment Sell orders Buy orders Bid (Quantity) Ask (Quantity) Trading price (Quantity)

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Euro	102	0 (0)	0 (0)	10 (4)	49 (8)	60 (2)	
Yuan	0	0 (0)					

Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration
60	2	120	120

Transaction history

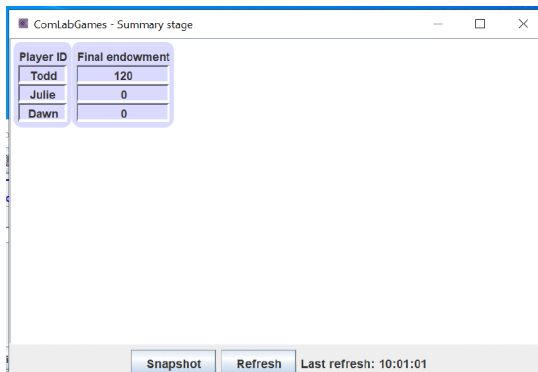
Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time
60	2	5	Chinese...	3	US expo...	09:48:53

Player type: Chinese export Company (1)

Round: 1

Currency Exchange

Summary page



The screenshot shows a window titled "ComLabGames - Summary stage". Inside, there is a table with two columns: "Player ID" and "Final endowment". The table lists three players: Todd with an endowment of 120, Julie with 0, and Dawn with 0. At the bottom of the window, there are two buttons labeled "Snapshot" and "Refresh", followed by the text "Last refresh: 10:01:01".

Player ID	Final endowment
Todd	120
Julie	0
Dawn	0

Snapshot Refresh Last refresh: 10:01:01

Currency Exchange

Competitive equilibrium as a tool for price discovery

- In a multimarket setting, it is quite problematic to intuit where prices will settle.
- The concept of *competitive equilibrium* is a useful tool for predicting what prices will emerge.
- From your courses in microeconomics, a competitive equilibrium (CE):
 - is a *price vector*, one for every market but one (the numeraire)
 - where *suppliers and demanders optimally* pick quantities to trade given the price vector
 - so that *markets clear*.
- That is aggregate supply equals aggregate demand in each market:
 - There are no unanticipated *orders* unfilled or unwanted *inventory*
- In a CE all potential gains from trade are realized:
 - a CE allocates resources efficiently.

Currency Exchange

Competitive equilibrium defined for this game

- In this model of international trade there are technically six markets
 - three export commodities plus three currencies.
- However matching supply to demand in this CE is trivial because:
 - aggregate demand for each export commodity comes from just one currency area.
 - each exporting firm only values its own currency.
- It follows that in CE:
 - US buys all of the 300 dollars supplied
 - EU buys all the 200 euros supplied
 - China buys all the 400 yuan supplied
- Therefore solving for the CE amounts to finding an US dollar exchange rate for both euros and yuan to equate the supply and demand in the currency markets.

Currency Exchange

Walras law

- Prove Walras law
- Two prices
- Two budget constraints and markets clear
- Derive equilibrium in more detail

Currency Exchange

Supply of foreign exchange

	\$	€	¥
US	—	100	50
CHINA	200	100	—
EUROPE	100	—	350
TOTAL	300	200	400

Currency Exchange

Competitive equilibrium

Using competitive equilibrium analysis:

$$D_{US} = P_{¥} 50 + P_{€} 100 = 300$$

$$P_{¥} D_C = 200 + P_{€} 100 = P_{¥} 400$$

$$P_{€} D_E = 100 + P_{¥} 350 = P_{€} 200$$

$$\textcircled{1} D_{US} - P_{¥} D_C =$$

$$P_{¥} 50 - 200 = 300 - P_{¥} 400$$

$$\Rightarrow P_{¥} = \frac{500}{450} = \frac{10}{9} \approx 1.1$$

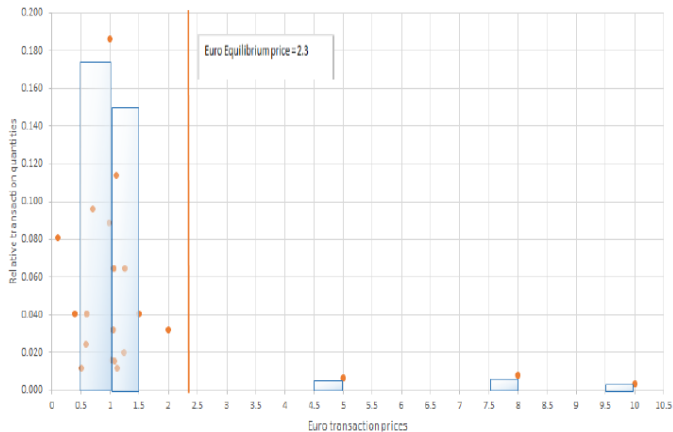
Using $\textcircled{1}$ and $P_{€} D_E$:

$$P_{€} = \frac{100 + P_{¥} 350}{200}$$

$$= \frac{1}{2} + \frac{7}{4} \cdot \frac{10}{9} \approx 2.3$$

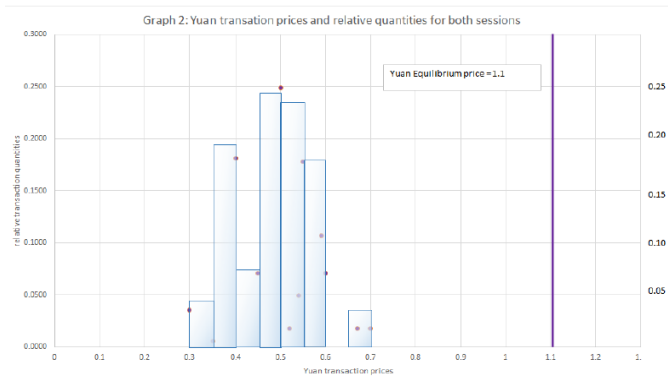
Currency Exchange

Transaction prices in Euro market



Currency Exchange

Transaction prices in Yuan market



Currency Exchange

Efficiency of Limit Order Markets

	US \$	EU	Yuan
US Export company	0.30	0.11	0.04
European Export Company	0.39	0.55	0.28
Chinese export company	0.31	0.34	0.68
	1	1	1