# Lecture 3
# Revenue Equivalence

The previous lecture introduced several auction formats, emphasized the difference between private and common values, and defined the concept of strategic equivalence. This lecture analyzes a weaker relation, called revenue equivalence. It is based on the notion of equal expected net benefit to each bidder. Revenue equivalence only applies to private value auctions. Our discussions of auctions (versus contracting) conclude with an example of what happens when there is differential information in a common value setting.

# Ascending and second price auctions

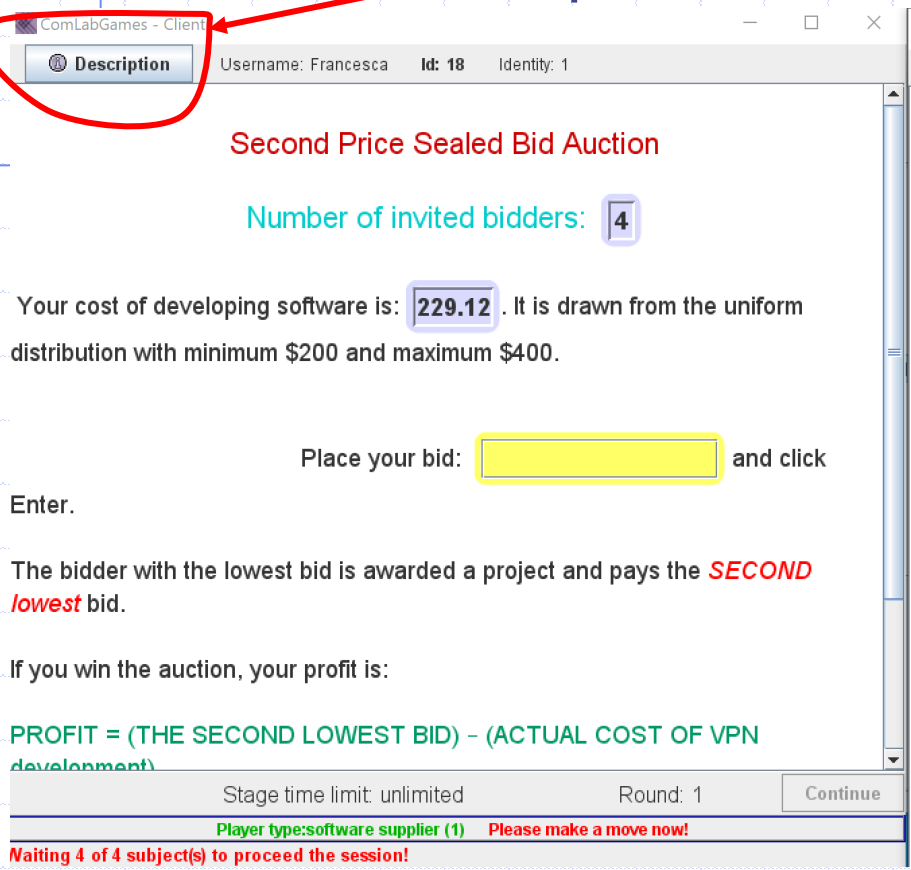
- To illustrate concepts introduced in the previous lecture we experimented with Dutch (descending) and first price auctions.
- We begin this lecture with an experiment involving English (ascending) and second price auctions.
- Recall that:
  - in an English auction, the auctioneer continues to raise the price until only one bidder is willing to pay it. (See some rules.)
  - in a second price (sealed bid) auction the highest bidder pays the second highest bid.

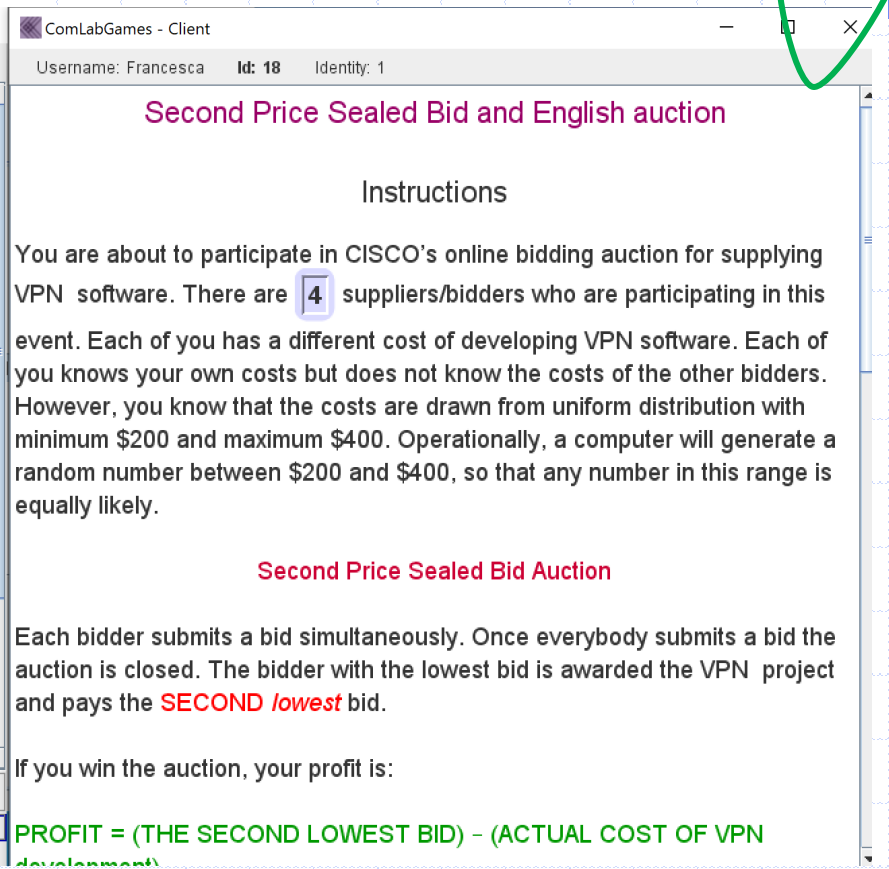
# A procurement auction

- In a procurement auction, the auctioneer seeks to buy some goods and services.
- Thus in a second price procurement auction bidders submit prices, the lowest price wins, and the winning bidder pays the second lowest price.
- In an English procurement auction, bidders have repeated opportunities to lower the price until only one remains, the winner, who pays the lowest price s/he bid.

# A second price procurement auction bidding window and instruction window

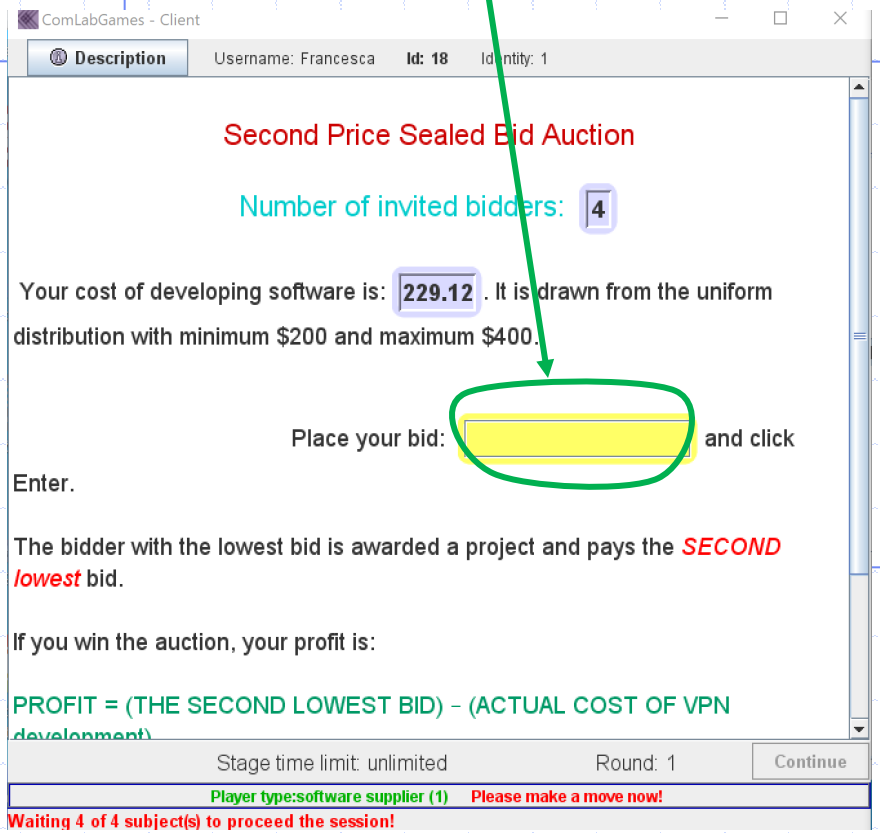
1. Instruction and auction window appears on your screen.
2. To close the instruction window click on "x". To retrieve it click on "Description".

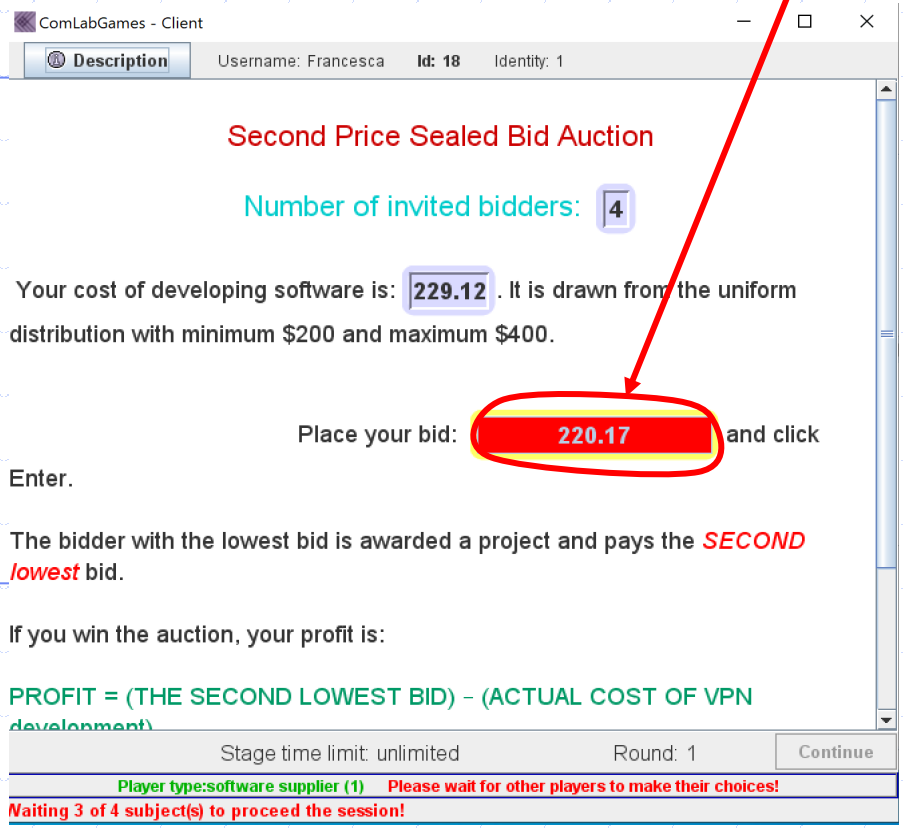


# Write a bid in a second price auction

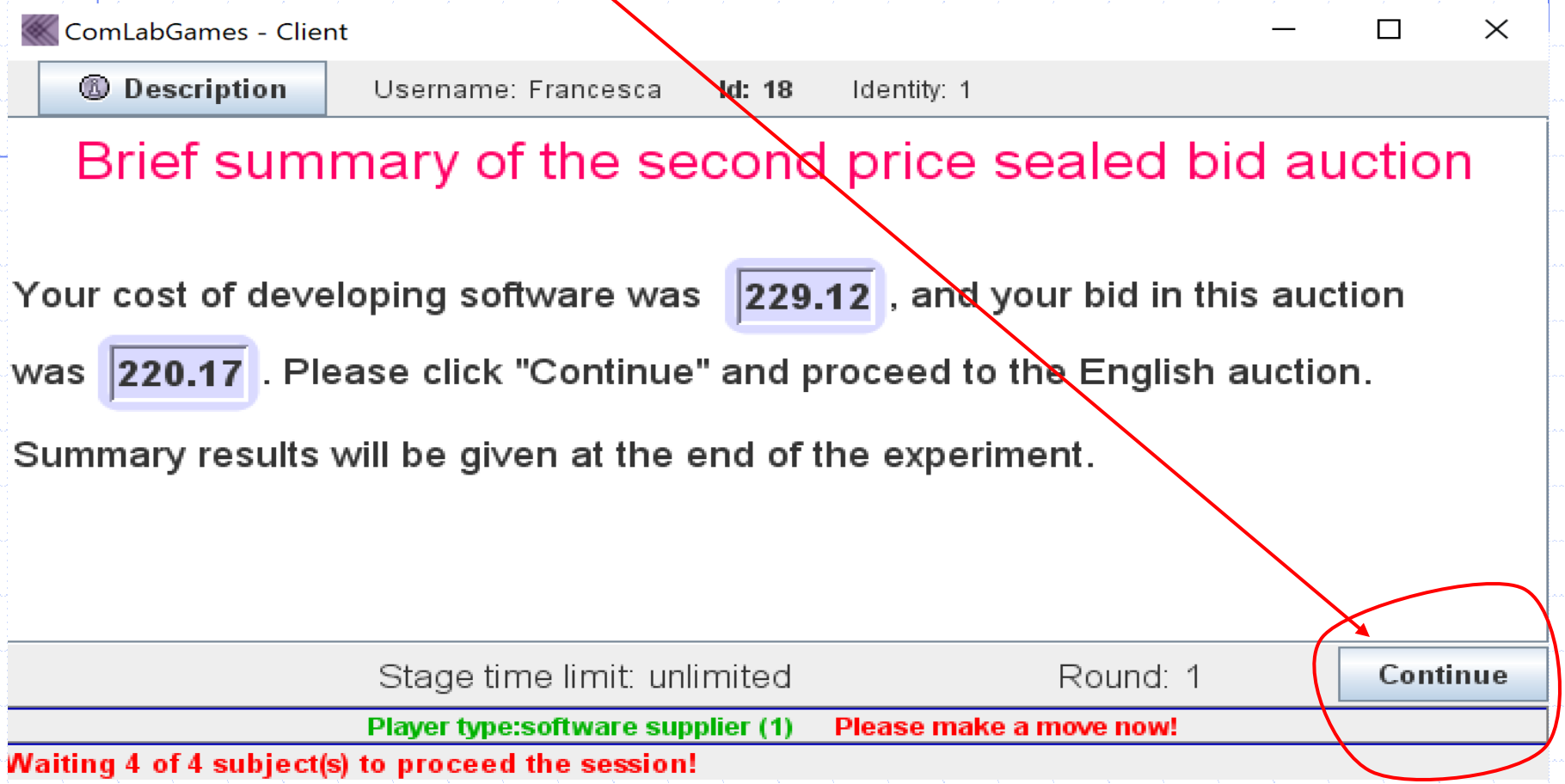
1. Write a number and click Enter (you can enter decimal number).
2. After submitting the bid, your bid should be colored in red.
3. Wait for all players in your session to submit bids.



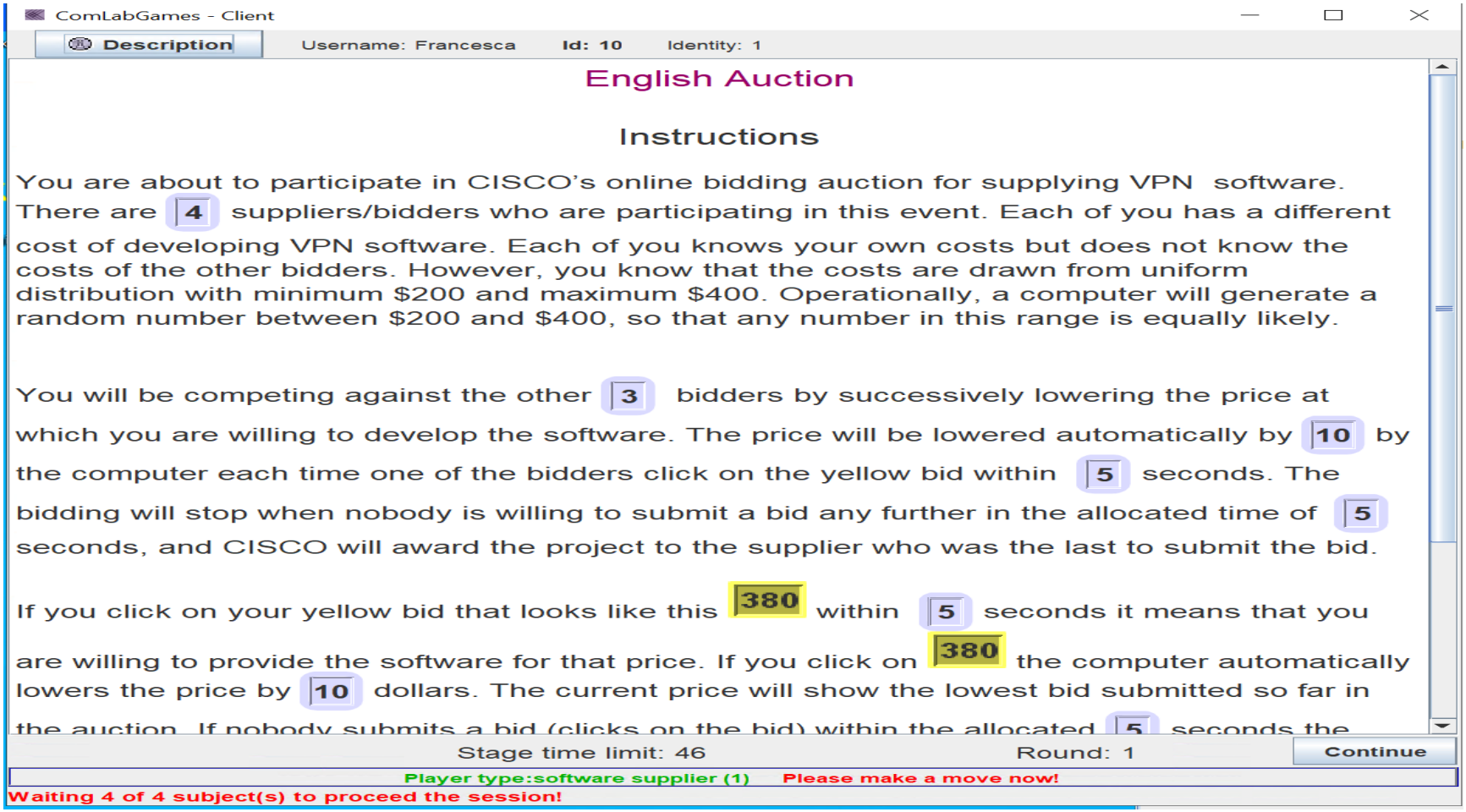



# Summary page for sealed bid auction

1. Click "Continue" to move to English auction instructions.
2. Wait for all players in your session to click "Continue".

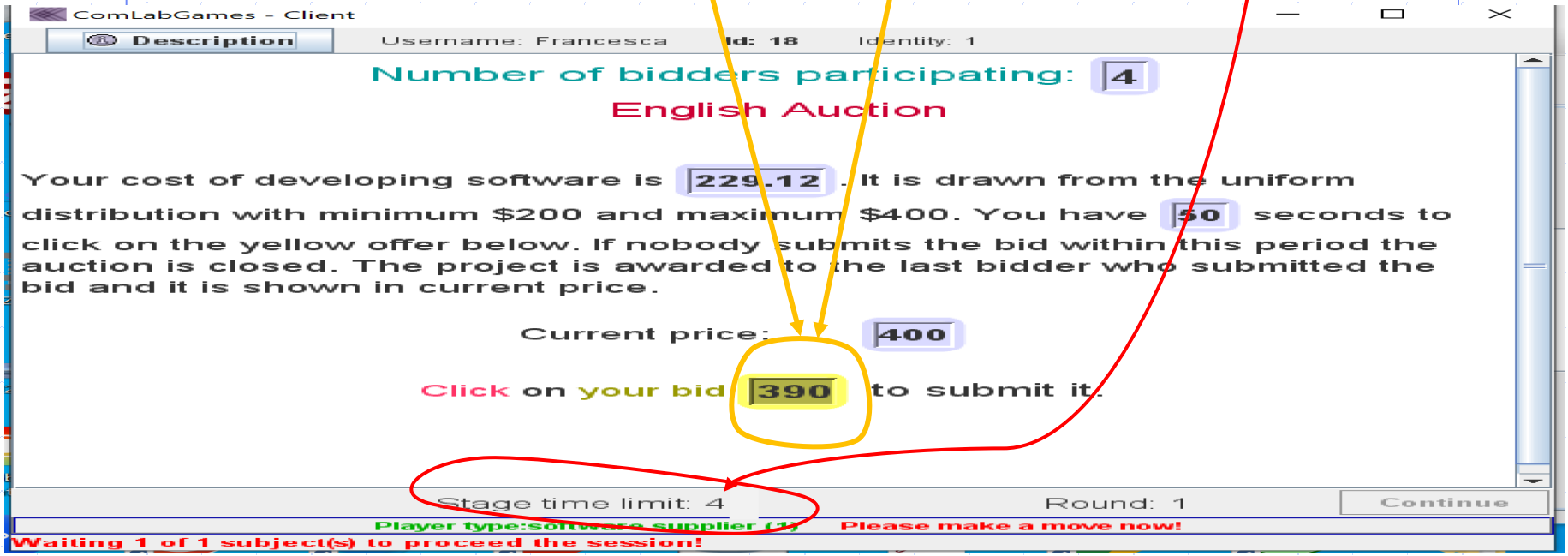


# English auction instructions



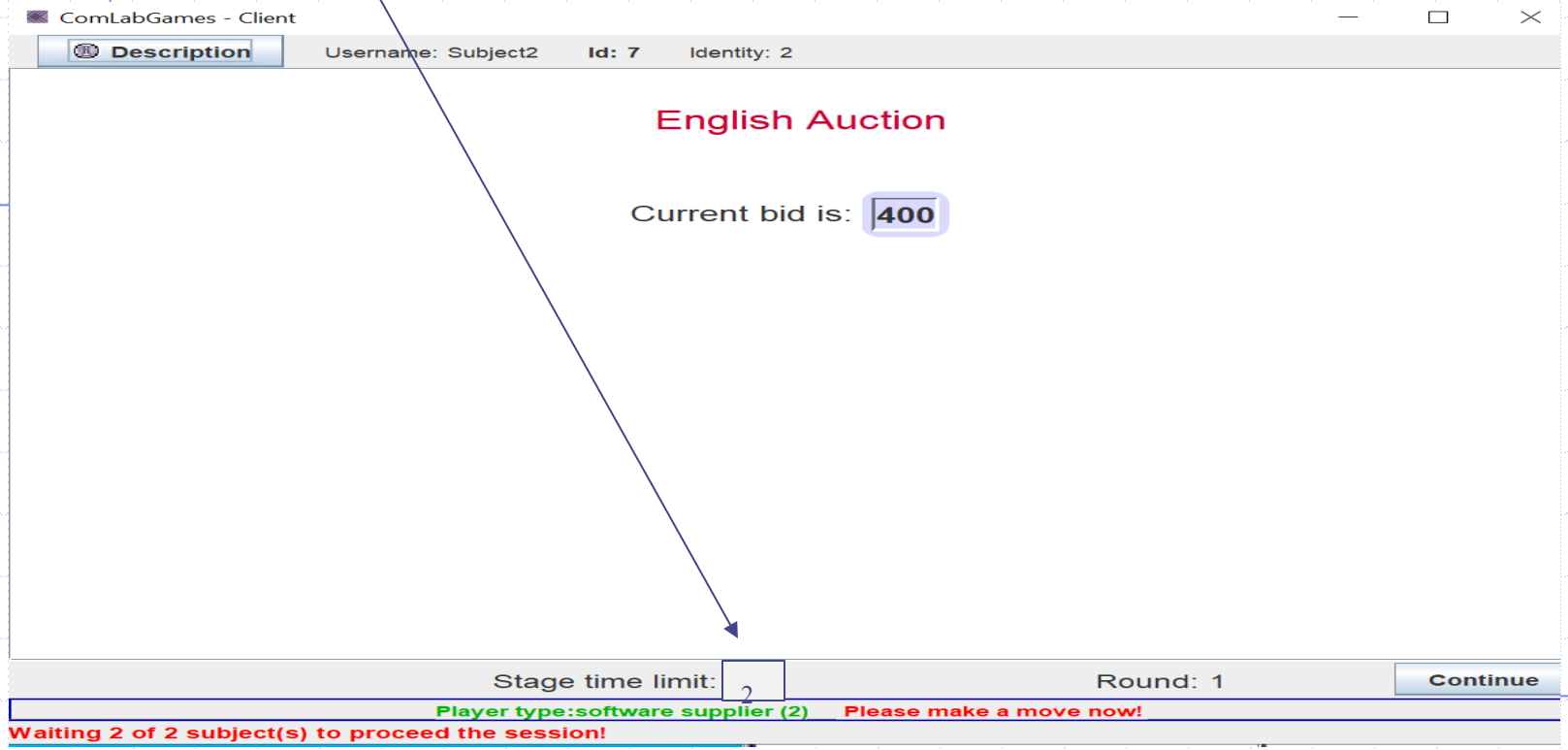
# English auction page

1. The price will be lowered automatically by $10 each time one of the bidders click on the yellow bid within 5 seconds.
2. The bidding will stop when nobody is willing to submit a bid any further in the allocated time of 5 seconds.
3. If you click on your yellow bid within 5 seconds it means that you are willing to provide the software for that price.

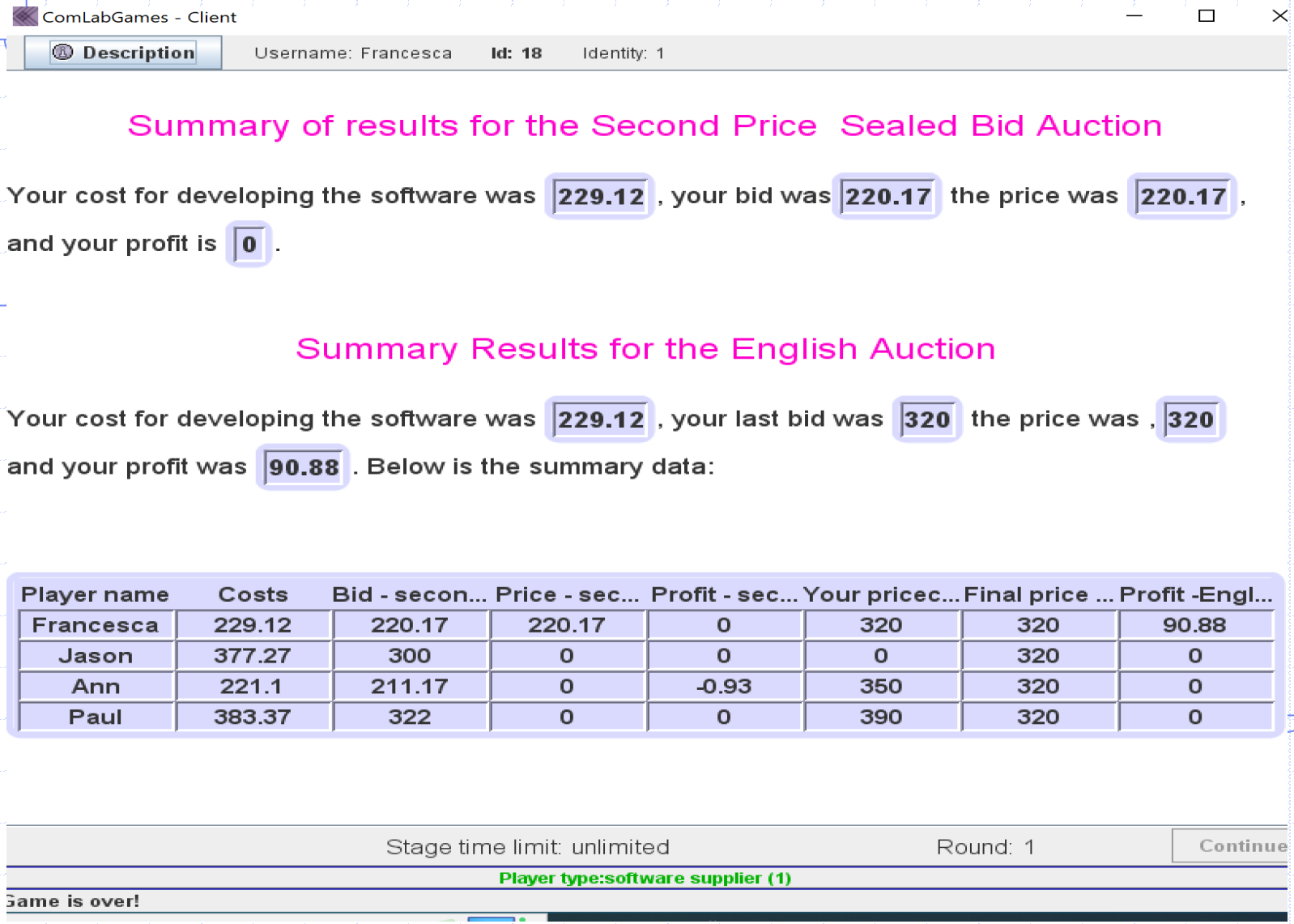


# Current bid page for English auction

1. For 2 seconds a window with the current bid is shown before the next round of bidding.



# Summary page



# Strategic equivalence between second-price and ascending auctions

- In private value second-price and in ascending auctions:
  - nobody learns anything about their own valuation during the auction.
  - the winner learns nothing about the highest losing bid until the auction ends.
  - the highest bidder pays (roughly) the second highest price.
- These features also hold when there are only 2 bidders, regardless of whether the valuations are private or not.
- In both cases an ascending auction is strategically equivalent to a second price sealed bid auction.
- In common value auctions these two auction mechanisms are not strategically equivalent if there are more than 2 players.

***Rule 2***: *If there are only two bidders, and/or valuations are private, choose the same reservation price in English and second price auctions.*

# Bidding in a second-price auction

- If you know your own valuation, there is a general result about how to bid in a second price sealed bid auction, or where to stop bidding in an ascending auction.
- Bidding should not depend on what you know about the valuations of the other players, nor on what they know about their own valuations.
- It is a dominant strategy to bid your own valuation.
- A corollary of this result is that if every bidder knows his own valuation, then the object will be sold for the second highest valuation.

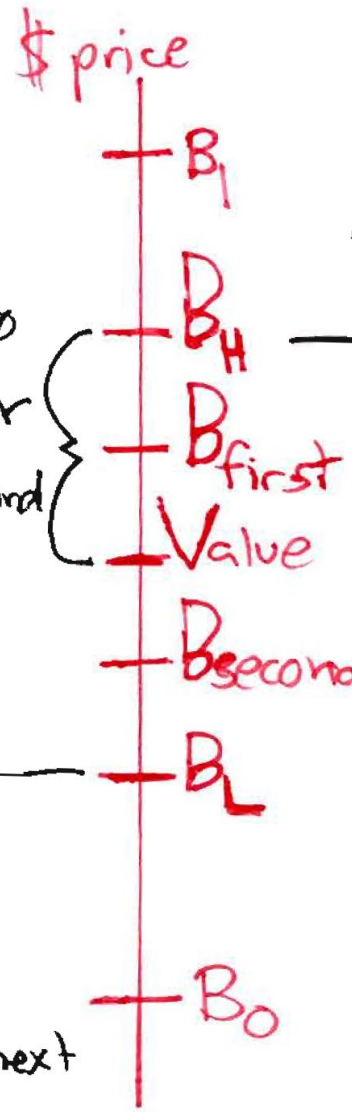
***Rule 3*** : *In a second price sealed bid auction, bid your valuation if you know it.*

# Proving the third rule

- Suppose you bid above your valuation, win the auction, and the second highest bid also exceeds your valuation. In this case you make a loss. If you had bid your valuation then you would not have won the auction in this case. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding above it.
- Suppose you bid below your valuation, and the winning bidder places a bid between your bid and your valuation. If you had bid your valuation, you would have won the auction and profited. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding below it.
- The proof is completed by combining the two parts.

Bidding in an English or a Second Price Sealed Bid Auction

SHOULD YOU BID
Value or $B_H$?




Bid $B_H$, Value or $B_L$ and lose auction if highest bid is $B_1$ and make zero.

Bid Value, lose auction to $B_{first}$, and make zero, or bid $B_H$, win auction and lose $(B_{first} - Value)$.

Bid Value, win auction, and make $(Value - B_{second})$, or bid $B_L$ and lose auction.

Bid $B_H$, Value or $B_L$ and win auction to make $(Value - B_0)$ if next highest bid is $B_0$.

SHOULD YOU BID
Value or $B_L$?

# Revenue versus Strategic Equivalence

- In strategically equivalent auctions, the strategic form solution strategies of the bidders, and the payoffs to all them, are identical. Are bidders ever indifferent to auctions that lack strategic equivalence?
- Two auction mechanisms are revenue equivalent if, given a set of players their valuations, and their information sets, the expected surplus to each bidder and the expected revenue to the auctioneer is the same.
- Revenue equivalence is a less stringent condition than strategic equivalence. Thus two strategic equivalent auctions are invariably revenue equivalent, but not all revenue equivalent auctions are strategic equivalent.

# Revenue equivalence theorem

- The revenue equivalence theorem states that in private value auctions, the expected surplus to each bidder does not depend on the auction mechanism itself providing the following conditions are satisfied:

1. Every bidder is risk-neutral.
2. Valuations are independent and identically distributed.
3. In equilibrium the bidder with highest valuation wins.
4. The lowest possible valuation has zero expected value.

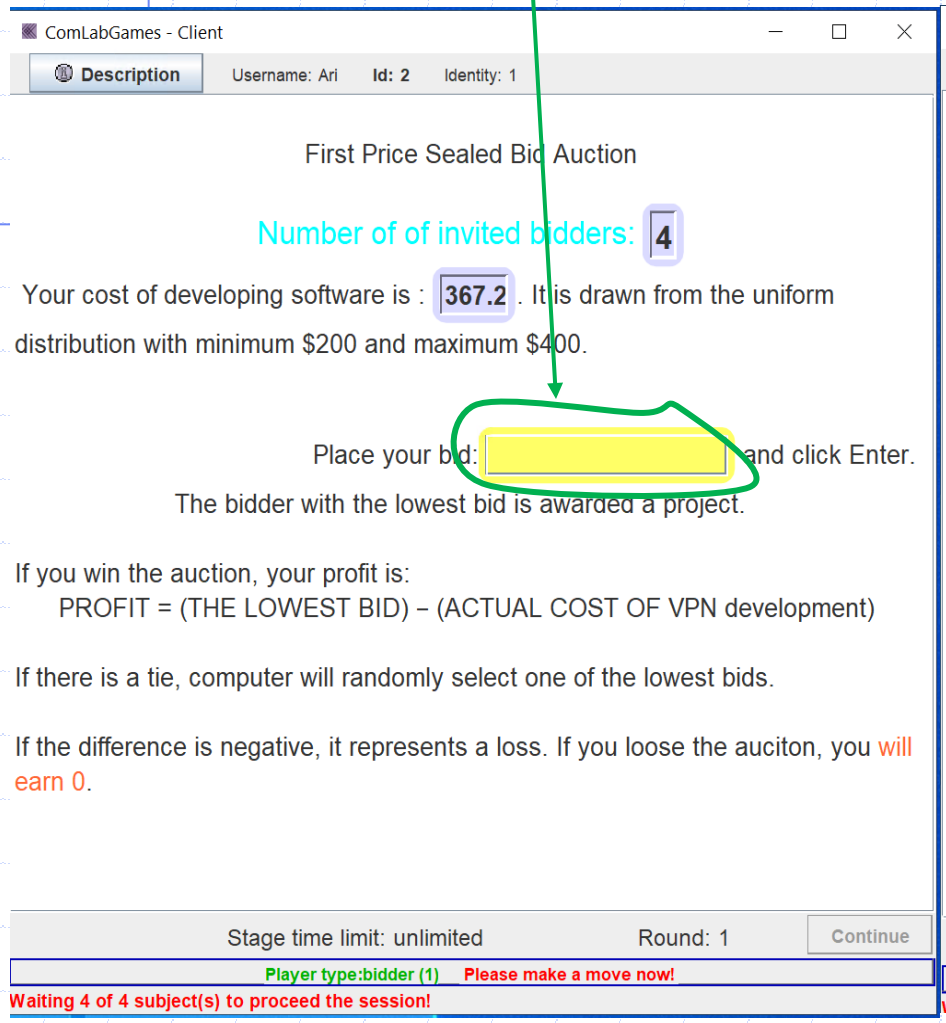
- Note that if all bidders obtain the same expected surplus, the auctioneer obtains the same expected revenue too.

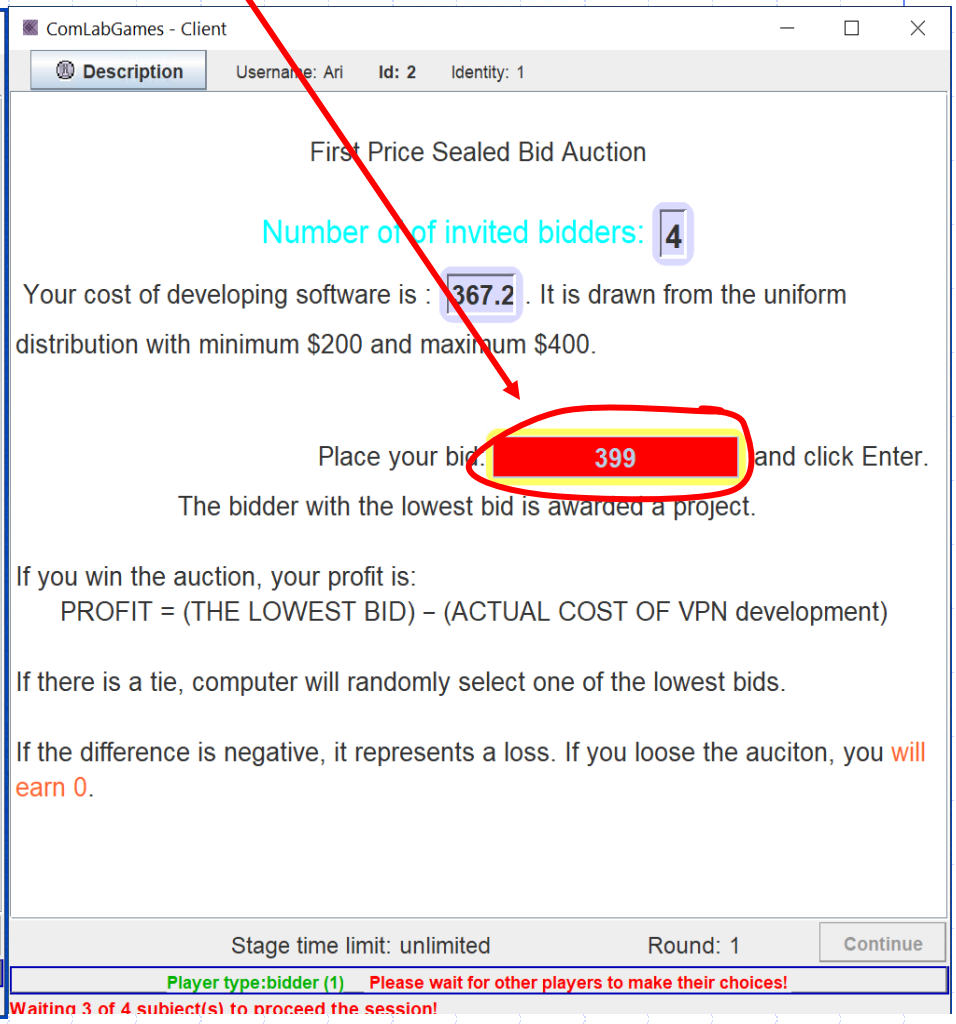
# Intuition from revenue equivalence

- Calibrate your bid to your valuation only to the extent that it affects your beliefs about the highest valuation of the all the other bids.
- Working from the assumption that yours is the highest valuation, bid high enough to induce the next highest bidder to make a small expected loss in order to beat your bid.
- The theorem implies that the auctioneer's expected revenue is also the same.

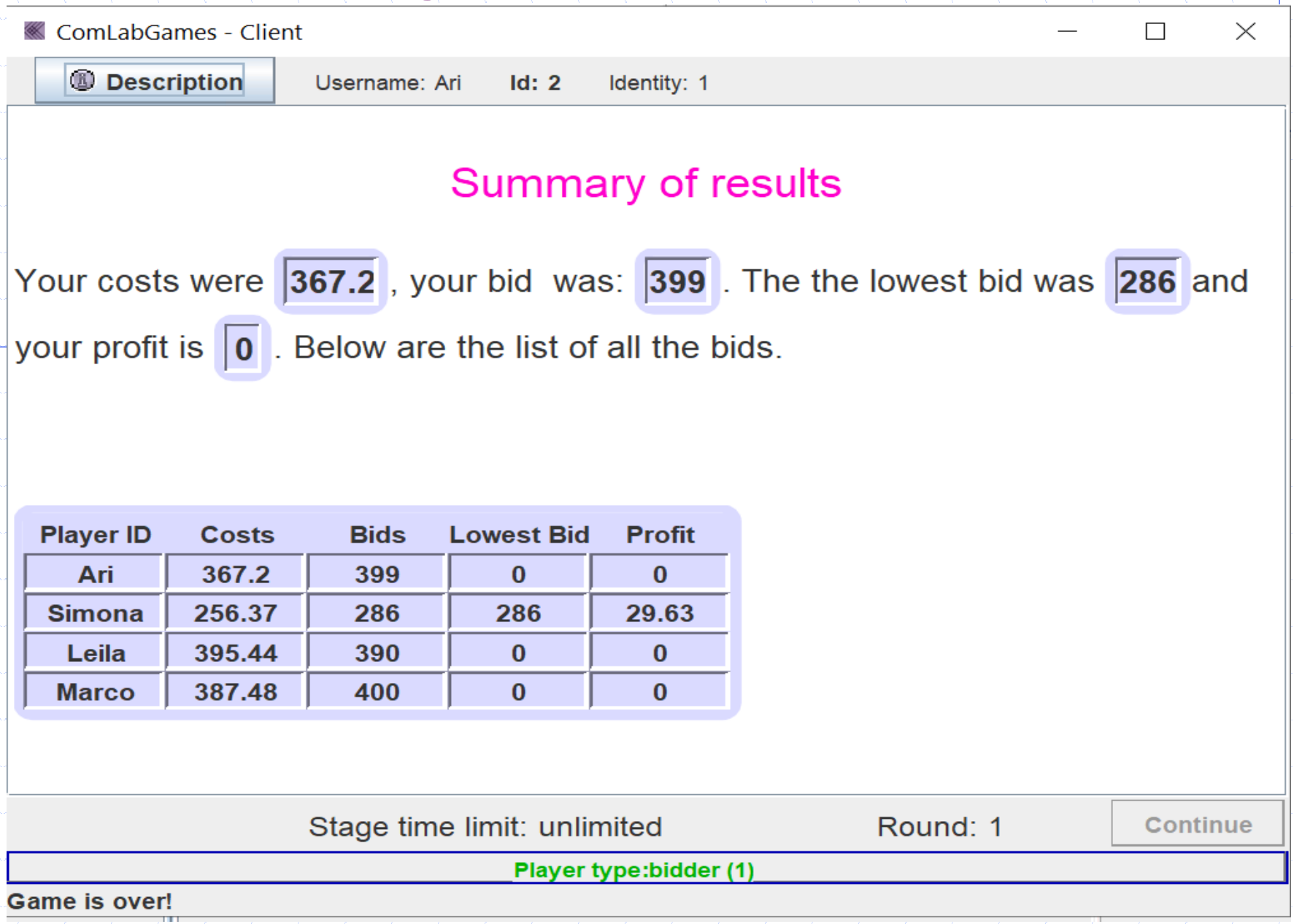
# Write a bid in First price sealed bid auction

1. Write a number and click Enter.
2. Your bid should be colored in red.
3. Wait for other players in your session to submit a bid.



# Summary page for sealed bid auction



# Comparison of bidding strategies

- The bidding strategies in the first and second price auctions markedly differ.
- In a second price auction bidders should submit their valuation regardless of the number of players bidding on the object.
- In the first price auction bidders should shave their valuations, by an amount depending on the number of bidders.

# Using the revenue equivalence theorem to derive optimal bidding functions

- We can also derive the solution bidding strategies for auctions that are revenue equivalent to the second price sealed bid auction.
- Consider, for example a first price sealed bid auctions with independent and identically distributed valuations.
- The revenue equivalence theorem implies that each bidder will bid the expected value of the next highest bidder conditional upon his valuation being the highest.

# An example: the uniform distribution

- Suppose valuations are uniformly distributed within a closed interval, with probability distribution:

$$P(v) = (v - v_0)/(\bar{v} - v_0)$$

- Then in equilibrium, a player with valuation v bids a weighted average of the lowest possible valuation and his own, where the weights are 1/N and (N-1)/N:

$$b(v_n) = v_n - P(v_n)^{1-N} \int_{v_0}^{v_n} P(v)^{N-1} dv$$

$$= v_n - (v_n - v_0)^{1-N} \int_{v_0}^{v_n} (v - v_0)^{N-1} dv$$

$$= v_0 / N + v_n (N-1)/N$$

# Differential information in common value auctions

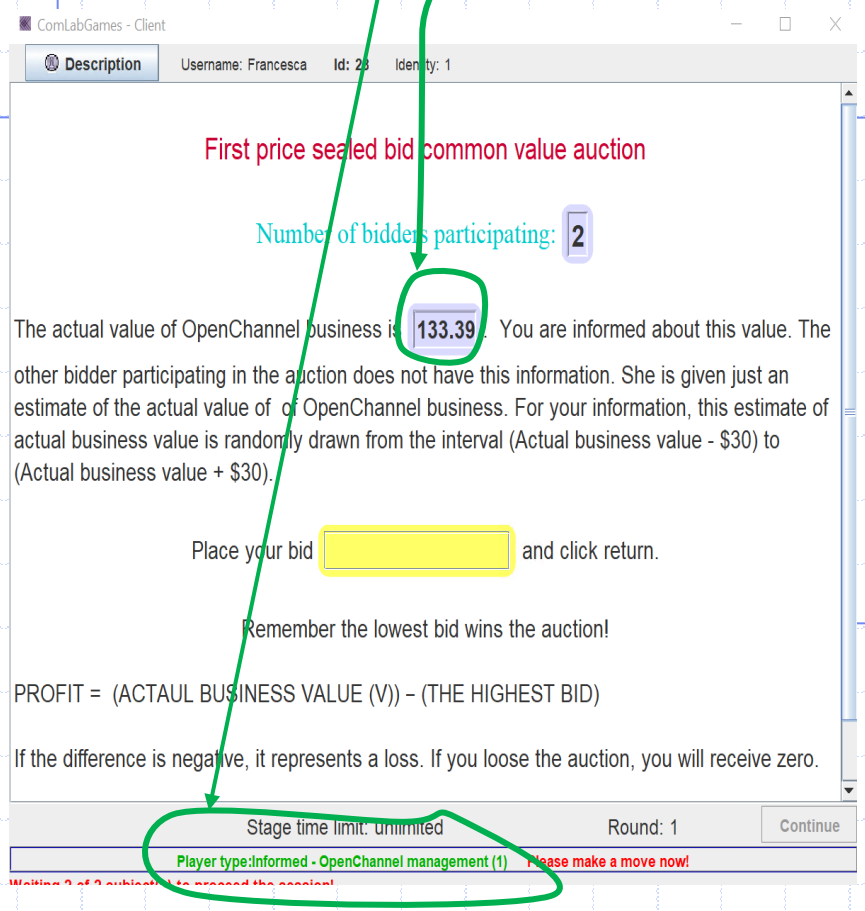
- In common value auctions the second lecture highlighted, everyone was equally informed. Each bidder knew their own signal was as good as every other.
- What happens if bidders are differentially informed about a common value?
- For example, one bidder might know more about the value of the object being auctioned than the others.
- An extreme form of dependent signals occurs when one bidder knows the common value exactly and the others do not:
  - How should the informed player bid?
  - What about an uninformed player?

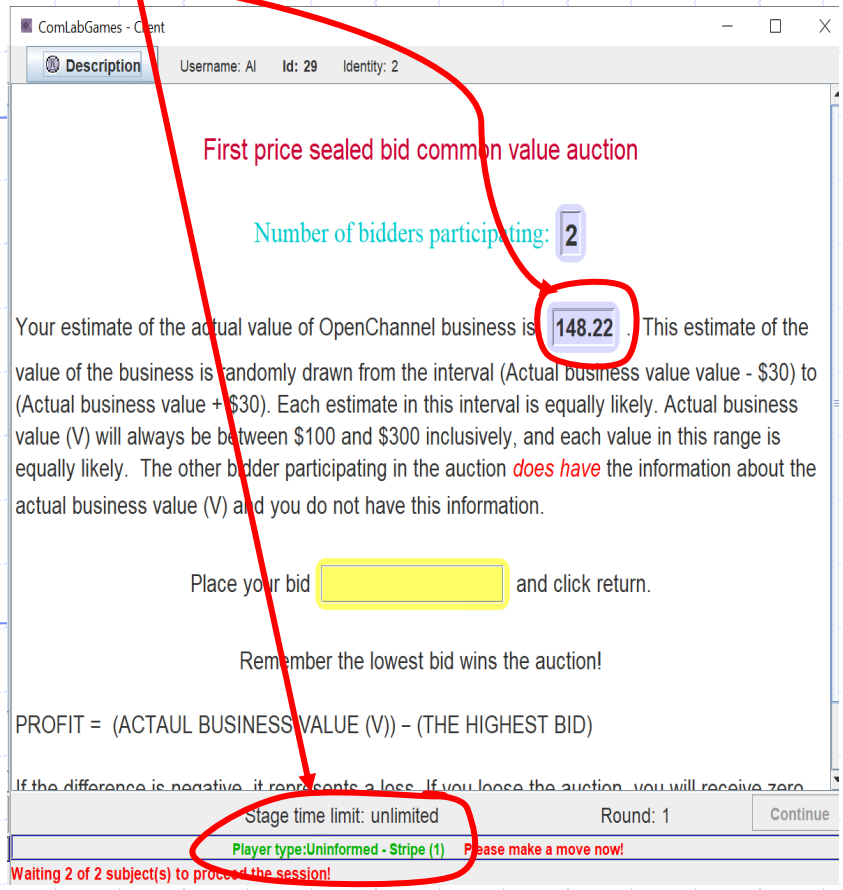
# Second price sealed bid auctions

- In a SPSB auction, Rule 3 implies the informed player optimally bids his true value.
- The uninformed player bids any pure or mixed distribution:
  - If she wins the auction she pays the common value.
  - If she loses she pays nothing.
  - She neither gains or loses on any bid.
- This implies the revenue from the auction is indeterminate, that is anywhere between zero the common value.
- For example, if the uninformed player bids:
  - zero, the auction nets zero.
  - more than the item could ever be worth, the auctioneer nets the common value.
- What about a FBSB auction?

# Differential Information First price auction

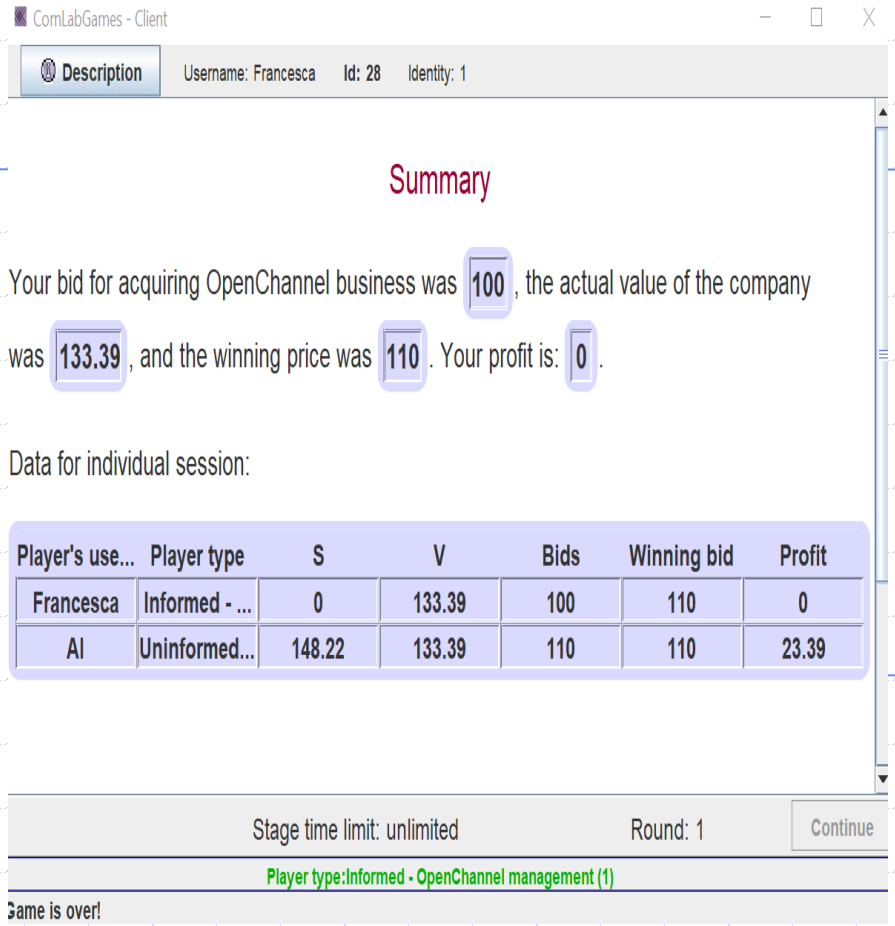
1. Bidding screen for the informed bidder, *OpenChannel management team*: the actual value of business shown.
2. Bidding screen for the *uninformed bidder, Stripe*: the business value estimate shown.



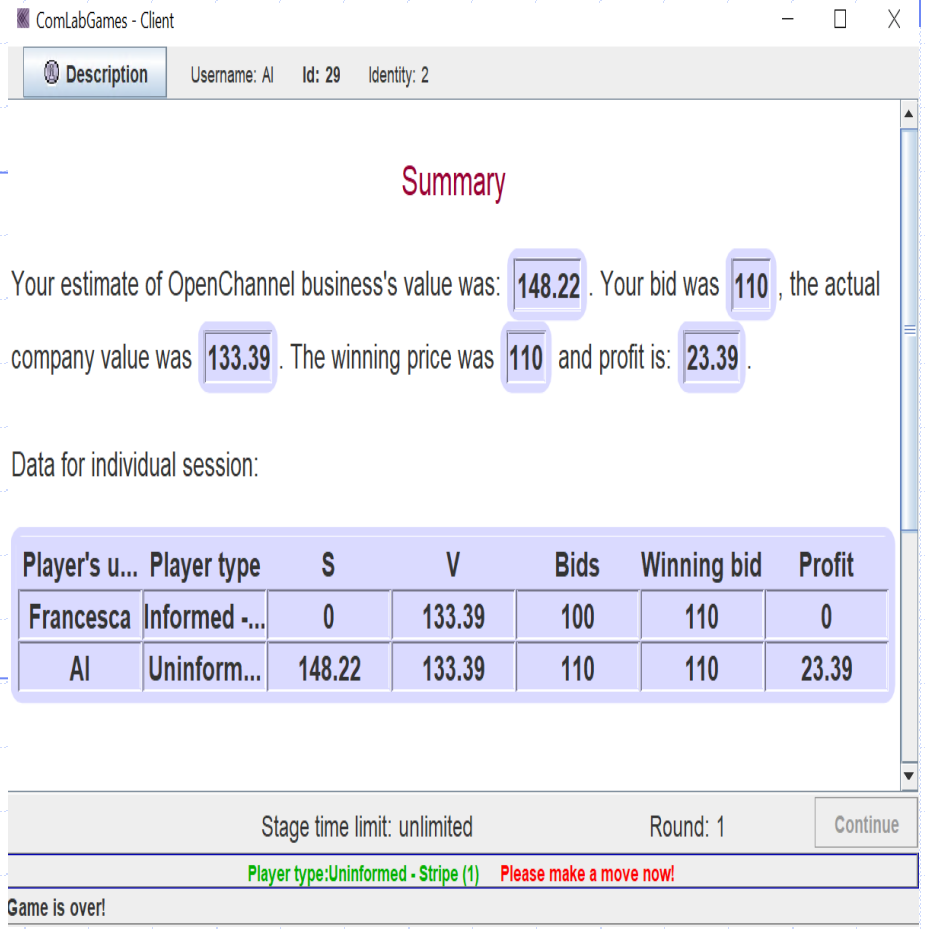



# Summary page for differential auction

## Informed bidder




## Uninformed bidder



# Uninformed bidders should not be predictable

- Suppose the uninformed bidder always makes the same positive bid, denoted $b_{fixed}$. This is an example of a pure strategy.
- Is this pure strategy part of a Nash equilibrium?
- The best response of the informed bidder is to bid a little more than $b_{fixed}$ when the value of the object v is worth more than $b_{fixed}$, and less than $b_{fixed}$ otherwise.
- Therefore the uninformed bidder loses or makes zero by playing a pure strategy in this auction. A better strategy would be to bid zero (Session 3.4).

# Principles for Bidding in Auctions

*Summarizing some general principles for bidding:*

1. Assume your own valuation is the high enough to win. Bid high enough to pay the break-even value of the highest losing bid.
2. Shave your bid to account for the winner's curse if you don't know your own valuation but only have a signal that is correlated with those of the other bidders.
3. Be wary of bidding with less knowledge than your rivals.
4. The greater the number of bidders the less opportunities there are for profit.
5. In a second price auction bid your valuation, and in an ascending auction, up to your valuation, if you know it.
6. Bid the same way in a first price sealed bid auction as a descending auction.