Trade and Investment Strategy 45871
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Mini 2 December 2023

Take home examination (out of 30 points)

There are five equally weighted questions. You must take the test individually, but you are allowed to verbally discuss the questions with each other beforehand. So don't copy and paste, don't divide and specialize, but it is OK to verbally discuss the answers in a group of up to 4, and then write up the whole exam by yourself without further discussions. Please indicate who is in your pretest discussion group, after your name.

Question 1 (6 points: one for each part)

Consider the following auctions. In the first three parts to this question suppose each bidder knows his or her own valuation, and all the valuations are all distributed uniformly (that is evenly) between $100 million and $400 million. For example, the probability that a typical bidder's valuation lies between $300 million and $400 million is one third.

1. Suppose you are one of 5 bidders, your valuation is $235 million, and you are bidding in a first price sealed bid auction. What should you bid? What are your expected profits from bidding in this auction (that is after you know your valuation, the number of bidders in the auction, but before you know the outcome of the auction)? Explain or justify your answers.

2. Suppose you are one of 5 bidders, your valuation is $235 million, and you are bidding in an ascending auction. How high should you be prepared to bid? What are your expected profits from bidding in this auction (that is after you know your valuation, the number of bidders in the auction, but before the bidding starts)? Explain or justify your answers.

3. Now suppose you are one of 2 bidders, your valuation is $235 million, and you are bidding in a first price sealed bid auction. What should you bid? What are your expected profits from bidding in this auction (that is after you know your valuation, the number of bidders in the auction, but before you know the outcome of the auction)? Intuitively explain why your answer differs from your answer in the first part of this question.

4. Suppose you are one of 5 bidders and your valuation is $235 million. In an all-pay auction (a little bit like a raffle) every bidder pays their bid, *whether they win or not*. (So if you bid $220 million and another bidder bids $300 million you lose $220 million.) The Revenue Equivalence Theorem says that the expected net benefit from bidding in this auction is the same as the expected net benefit from bidding from bidding in a first price sealed bid auction. Using that theorem, and your answer in the first part of this question, what should you bid in this all-pay auction. If the math looks too challenging, give me some intuition about roughly what you would do.

5. Now suppose there are 5 bidders, and you are bidding in a first price sealed bid auction. However in this new setup *every bidder has the same valuation* but no one knows what that valuation is. They all know that this common valuation is distributed uniformly between \$100 million and \$400 million. Each bidder gets a noisy signal about what the valuation is, that gives them some clue about the true valuation, but not an exact reading. Suppose the reading you get is \$235 million. Should you bid more or less than what you bid the first part of this question? What if there were 10 bidders? In words, how would your answer change then, and why?

6. As in the previous question *every bidder has the same valuation.* There are only two bidders. You know exactly what the valuation is. The other bidder does not. The other bidder believes the valuation is distributed between valuation is distributed uniformly between \$100 million and \$400 million. He decides to bid cautiously, bidding \$150 million, thinking that the expected value is \$250 million, and so if he wins, he would profit on average \$100 million. You know the actual value. (It is some number between \$100 million and \$400 million.) Given his bidding strategy, what should you bid? Assuming the valuation is distributed between valuation is indeed initially distributed uniformly between \$100 million and \$400 million, what are your expected profits, what are his expected losses, and what is the expected revenue of the auctioneer?

Question 2 (6 points: 2 points for each part)

When specialists managed the market at the NYSE front running was deemed illegal.

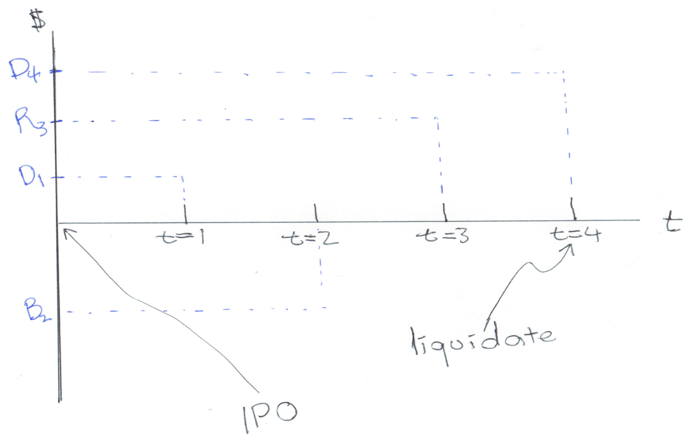
1. Suppose front running was legalized, how would the specialist set the price at which he buys a security and the price at which he sells it? It's not necessary to come up with formulas, but tell me intuitively how to set his profit margin. Use diagrams if they are helpful.

2. Given the high price that a seat for a specialist commanded, could front running have been more common than the prosecutions and convictions (which were few and far between) suggest? Explain a possible connection between the high price of a seat and the frequency of front running.

3. Does the exchange become so corrupted by front running that all trading ceases? Can the analysis we described for the Vancouver stock exchange discussed in lecture notes be related to this question? Could the replacement of specialists by electronic markets have anything to do with giving the market more integrity. (Again, is the Vancouver experience relevant?)

Question 3 (6 points: 1.5 point for each part)

Consider the diagram below, showing the life of a (very short-lived) firm, that goes public at time $t = 0$ and liquidates at time $t = 4$. At times $t = 1$ and $t = 4$ the firm distributes dividends of D_1 and D_4 respectively. At time $t = 2$ it raises capital with a bond B_2 to be repaid when the firm liquidates. At time $t = 3$ it buys back R_3 of the value of the firm from equity holders. Assume throughout this question that the *Efficient*



Markets Hypothesis holds. In the first three parts of this question also assume investors have perfect foresight: everything about the future of the firm is known with certainty in advance, so there is no uncertainty.

1. Suppose there is no discounting (so the risk free rate is one and the real interest rate is zero). What is the total value of the firm (debt plus equity) at times $t = 4$ just before it liquidates, at $t = 3$ just before it repurchases R_3 of its share value, at $t = 2$ just before takes on debt of B_2 in bonds, and at $t = 1$ just before it issues its first dividend?

2. What is the equity value of the firm at each of these times, and hence what is the value of the IPO (initial public offering)?

3. Now suppose each successive period is discounted by $(1+i)$ where i is the one-period interest rate. Putting everything in present value terms, how do your answers to the first two parts change?

4. Now we relax the assumption of perfect foresight. We now assume that at the beginning of time $t = 1$ there is a 50 percent chance the firm will pay a dividend of D_1, and a 50 percent chance the firm won't pay a dividend at all, and that at the beginning of at the beginning of period $t = 3$ there is a 40 percent chance the firm will buy back R_3 of the outstanding share value of the firm and a 60 percent chance it won't. Assume the two events are independent. How do your answers to the third part change?

Question 4 (6 points: 1.5 points for each part)

We now expand the currency exchange model (of Lecture 9) to four, adding in Japanese yen (JPY) to the US dollar (USD), the Chinese yuan (CNY) and the European euro (EUR). The endowments, described export earnings in the lecture, are:

Supply of Currencies

	USD	CNY	EUR	JPY
US	0	700	90	15,000
$China$	300	0	200	5,200
$Europe$	650	850	0	450
$Japan$	790	2100	670	0

As before we assume that all demand for a currency comes from its home exporters. Thus US exporters supply their foreign currency to the exchange market (that is 700 CNY, 90 EUR and 15,000 JPY) to demand all the 300 + 650 + 790 = 1740 USD. Let p_{CNY} denote the exchange rate of CNY in terms of USD. In words p_{CNY} the dollar price of a yuan. Similarly let p_{EUR} denote the exchange rate of EUR in terms of USD, and p_{JPY} the exchange rate of JPY in terms of USD.

1. In a competitive equilibrium the foreign exchange rates are set so that all the foreign currency markets clear. Display what the matrix above will look like after all trading has taken place in a competitive equilibrium, by filling in its 16 elements. Your new matrix is called the **Demand for Currencies**.

2. Write down the equations that equate supply with demand in the yuan market, the euro market and the yen market.

3. Show that the market for dollars will clear, that is the supply of dollars equals the demand for dollars, if p_{CNY}, p_{EUR} and p_{JPY} are set so that the other three markets clear. (This is called *Walras law*.)

4. Solve for the competitive equilibrium exchange rates p_{CNY}, p_{EUR} and p_{JPY}.

Question 5 (6 points: 1 point for each part.))

At the beginning of her life a consumer worker chooses how much to consume at each age $t = 0, 1, 2, \ldots, T$. Denote by c_t her consumption at age t, let $\ln(c_t)$ denote the natural logarithm of c_t and suppose β is her subjective discount factor for the future, where we assume $0 < \beta < 1$. Also let W_t denote the current value of her wealth at the beginning of period t and y_t denote her income in that period (or at that age). We assume she maximizes her lifetime utility function, denoted by:

$$\left[\sum_{t=0}^{T} \beta^t \ln(c_t)\right]$$

subject to a lifetime budget constraint that $W_{T+1} \geq 0$ (which is mathematics for saying she can't die indebted), that $W_0 = 0$ (which is mathematics for saying she is born with nothing) and that:

$$W_{t+1} = (W_t - c_t + y_t)(1 + r_t)$$

for all $t = 1, 2, \ldots, T$ (which is mathematics for saying that the amount of wealth you have at age $t + 1$ equals the amount she had at age t less consumption in that period but plus income, all scaled up by a one-year-risk-free-return on assets,or debt, carried forward):

1. Recalling the derivative of $\ln(c_t)$ is $\frac{1}{c_t}$, using the first order condition for the optimization problem, what is the relationship between c_t^o and c_{t+1}^o? Rewrite this condition to illustrate the *fundamental equation of portfolio choice.*

2. Suppose the interest is constant. Does consumption increase or decrease over time. Explain using the equation you have derived in the first part of this question.

3. Now suppose the interest rate is declining over time. How does this affect your answer to the second part of the question?

4. Now suppose $T = 2$ (lumping all the springtime years of life together and all the autumnal years of life together). Draw and fully label a diagram for income (y_0, y_1) in both periods, consumption (c_0, c_1) in both periods, with indifference curves (that is consumption pairs (c_0, c_1) that deliver the same lifetime utility $\ln(c_0) + \beta \ln(c_1) = \overline{u}$ say for different values of $\overline{u}$) along with the optimal consumption pair (c_0^o, c_1^o) for the two period specialization.

5. Show on the diagram (or preferably draw another one so things are not too cluttered) and explain what happens if at $t = 0$, the beginning of her life y_1 doubles over what the consumer worker originally anticipated. Does this result generalize to the case where $T > 2$?

6. Now consider what would happen in this two period specialization (where $T = 2$) if the income (y_0, y_1) for both periods did not change but that the interest rate is lower than before. You may focus on two special cases, illustrating both on your diagrams: one case is when $y_0 = 0$ but $y_1 > 0$ and the other is when $y_0 > 0$ but $y_1 = 0$. How does consumption change in both periods?